



A Modified Discrete Rossler System

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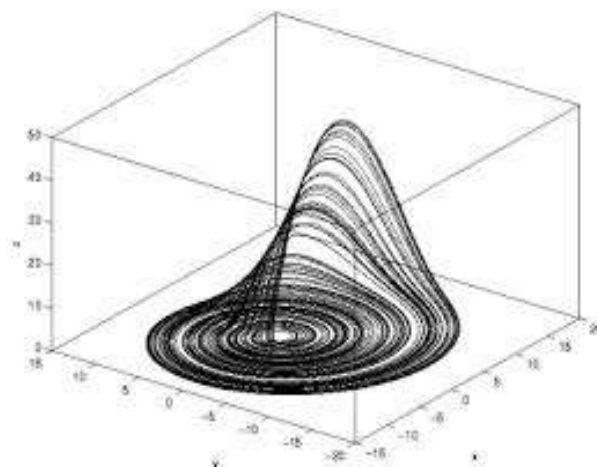
Abstract: This paper investigates a modified form of Rossler system. Existence of fixed points is analyzed and stability analysis is performed. Eigen values and eigen vectors are evaluated. Time plots and phase portraits are presented through numerical simulations for different parameter values. Periodic and Chaotic behavior are exhibited in the system for certain parameter values.

Keywords: difference equations, Rossler system, fixed points, stability,

I. Rossler System and Attractor

Chaos was first observed in a mathematical model of atmospheric convection by meteorologist Edward Lorenz in 1963. This system possesses a “strange attractor. Since the discovery of the first chaotic attractor, the Lorenz chaotic attractor, great progress has been achieved in the area of chaos. In the ensuing 40 years, much progress has been made in the study of chaotic systems. One of the famous attractor appears from a simple set of three differential equations in Rössler's system. In 1976, the Swiss mathematician *Otto Rössler* of the University of Tübingen had formulated a system of three differential equations as a model of a chemical reaction. Rössler analyzed such equations in order to achieve a qualitative understanding of the chaotic flow. Rossler system contains three non linear ordinary differential equations. The system exhibits chaotic dynamics. Lorenz system has two nonlinearities and Rössler has only one which enters in the third equation [3,4,5,6,7]. The defining equations of the Rossler system are

$$\begin{aligned}\dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + (x - c)z\end{aligned}\tag{1}$$



Rossler attractor for the parameter values: $a=0.432$, $b=2$ and $c=4$ is shown above.

II. Discrete Rossler System and Stability Analysis

Discrete dynamical systems governed by difference equations can be used to model and analyze many real-world problems. Discrete models can also provide efficient computational models of continuous models for numerical simulations [1]. The maps defined by simple difference equation can lead to rich complicated dynamics. We shall discuss some of the dynamical nature of the system (1) by specifying the parameters b and c in terms of a as follows:

$$b = a, c = 1 + \frac{1}{a}$$

This paper considers the following modified Discrete Rossler System as

$$\begin{aligned} x(n+1) &= x(n) - h[y(n) + z(n)] \\ y(n+1) &= y(n) + h[x(n) + ay(n)] \\ z(n+1) &= z(n) + h\left[ax(n) + x(n)z(n) - \left(1 + \frac{1}{a}\right)z(n)\right] \end{aligned} \quad (2)$$

The fixed points are obtained by solving the equations

$$\begin{aligned} x &= x - h[y + z] \\ y &= y + h[x + ay] \\ z &= z + h\left[ax + xz - \left(1 + \frac{1}{a}\right)z\right] \end{aligned}$$

Rössler system has two fixed points and they are $E_0 = (0, 0, 0)$ and $E_1 = \left(\frac{a - a^3 + 1}{a}, \frac{a^3 - a - 1}{a^2}, \frac{a - a^3 + 1}{a^2}\right)$.

The linear systems exhibit straightforward, predictable behavior. Most nonlinear systems of differential equations are impossible to solve analytically. The one exception to this occurs when we have equilibrium solutions. One of the most useful techniques for analyzing nonlinear systems qualitatively is the analysis of the behavior of the solutions near equilibrium points using linearization. The local stability analysis of the model can be carried out by computing the Jacobian corresponding to each equilibrium point [2]. The Jacobian matrix J for the system (2) is

$$J(x, y, z) = \begin{pmatrix} 1 & -h & -h \\ h & 1 + ah & 0 \\ h(a + z) & 0 & 1 + h\left[x - \left(1 + \frac{1}{a}\right)\right] \end{pmatrix}$$

Trace $J = 3 + h\left[a + x - \left(1 + \frac{1}{a}\right)\right]$ and

$$\text{Det } J = 1 + h\left[x - \left(1 + \frac{1}{a}\right)\right][h^2 + ha + 1] + h^2(1 + ha)(a + z)$$

The Jacobian matrix for the trivial Equilibrium E_1 is

$$J(E_0) = \begin{pmatrix} 1 & -h & -h \\ h & 1 + ah & 0 \\ ha & 0 & 1 - h\left(1 + \frac{1}{a}\right) \end{pmatrix}$$

Trace $J = 3 + h\left[a - \left(1 + \frac{1}{a}\right)\right]$ and $\text{Det } J = 1 - h\left(1 + \frac{1}{a}\right)[h^2 + ha + 1] + h^2a(1 + ha)$

The Jacobian matrix for the Interior Equilibrium E_2 is

$$J(E_1) = \begin{pmatrix} 1 & -h & -h \\ h & 1 + ah & 0 \\ \frac{h(a+1)}{a^2} & 0 & 1 - a^2h \end{pmatrix}$$

$$\text{Trace } J = 3 + ha(1-a) \text{ and } \text{Det } J = (1-a^2h)[h^2 + ha + 1] + h^2(1+ha)\left(1 + \frac{1}{a}\right)$$

III. Numerical Study

In this section, we perform numerical simulations for different choices of parameter values and determine eigen values and eigen vectors. The corresponding time plots and phase trajectory in 3-D are presented. For the equilibrium point E_0 consider the Rossler parameter values $h = 0.005$ and $a = 0.1$. The Eigen values are

$\lambda_{1,2} = 1.0002 \pm i0.0050$ and $\lambda_3 = 0.9450$ and the Eigenvectors are

$$v_1 = \begin{bmatrix} 0.7072 \\ -0.0385 - i0.7059 \\ 0.0064 - i0.0006 \end{bmatrix}, v_2 = \begin{bmatrix} 0.7072 \\ -0.0385 + i0.7059 \\ 0.0064 + i0.0006 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 0.0899 \\ -0.0081 \\ 0.9959 \end{bmatrix}$$

For E_1 , the Rossler parameter values of $h = 0.005$ and $a = 0.1$ yield the Eigen values $\lambda_{1,2} = 1.000 \pm i0.0527$ and $\lambda_3 = 1.0005$. Also the Eigen vectors are

$$v_1 = \begin{bmatrix} 0 + i0.0953 \\ 0.0090 - i0.0001 \\ 0.9954 \end{bmatrix}, v_2 = \begin{bmatrix} 0 - i0.0953 \\ 0.0090 + i0.0001 \\ 0.9954 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 0.0007 \\ -0.7071 \\ 0.7071 \end{bmatrix}. \text{ Also the numerical simulation is}$$

presented below for the Rossler's parameter value $h = 0.005$ and $a = 0.1$ see figure-1

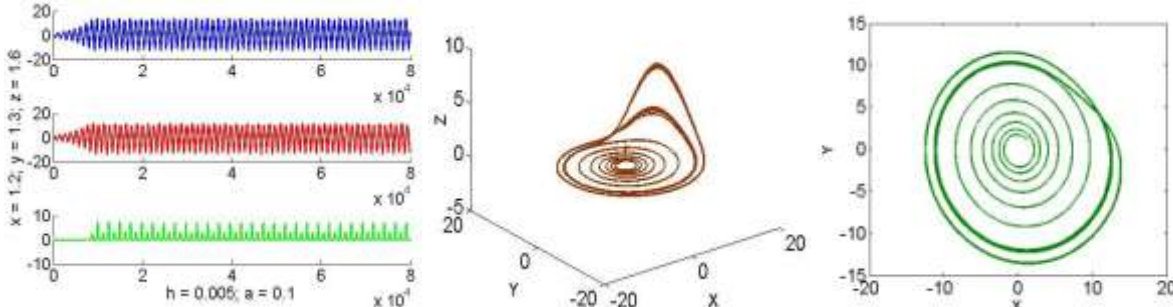


Figure 1: Time Plot and Phase Portrait for $h = 0.005$ and $a = 0.1$

For the equilibrium point E_0 we consider the Rossler parameter values $h = 0.01$ and $a = 0.45$ which yield Eigen values $\lambda_{1,2} = 1.0016 \pm i0.0098$ and $\lambda_3 = 0.9691$. Also the Eigen vectors are

$$v_1 = \begin{bmatrix} 0.7107 \\ -0.1998 - i0.6684 \\ 0.0874 - i0.0252 \end{bmatrix}, v_2 = \begin{bmatrix} 0.7107 \\ -0.1998 + i0.6684 \\ 0.0874 + i0.0252 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 0.2836 \\ -0.0802 \\ 0.9556 \end{bmatrix} \text{ For } E_1 \text{ with the Rossler parameter}$$

values $h = 0.01$ and $a = 0.45$ the Eigenvalues are $\lambda_{1,2} = 0.9994 \pm i0.0285$ and $\lambda_3 = 1.0037$. The Eigenvectors are

$$v_1 = \begin{bmatrix} -0.0179 - i0.3666 \\ -0.1236 + i0.0285 \\ -0.9215 \end{bmatrix}, v_2 = \begin{bmatrix} -0.0179 + i0.3666 \\ -0.1236 - i0.0285 \\ -0.9215 \end{bmatrix}, v_3 = \begin{bmatrix} 0.0556 \\ -0.7163 \\ 0.6956 \end{bmatrix}. \text{ Also the numerical simulation is}$$

presented below for the Rossler's parameter value $h = 0.01$ and $a = 0.45$ see figure-2

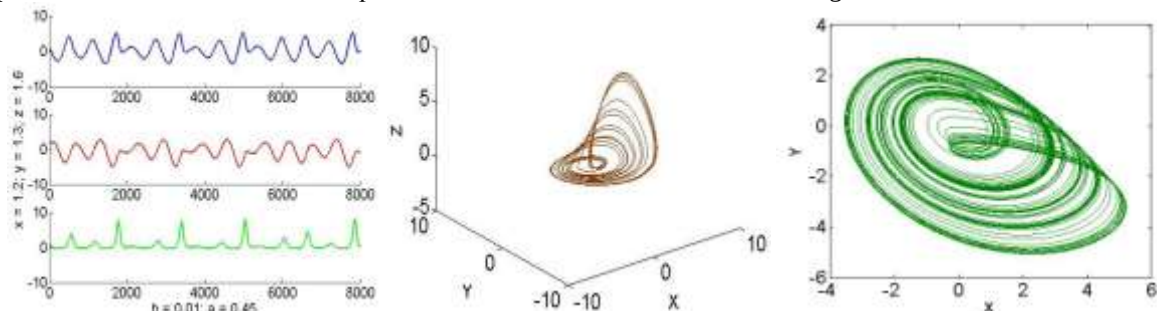


Figure 2: Time Plot and Phase Portrait for $h = 0.01$ and $a = 0.45$

For the equilibrium point E_0 consider the Rossler parameter values of $h = 0.001$ and $a = 0.6$ yield the Eigen values are $\lambda_{1,2} = 1.0001 \pm i0.0010$ and $\lambda_3 = 0.9992$ also the Eigen vectors are

$$v_1 = \begin{bmatrix} -0.7232 \\ 0.2809 + i0.5737 \\ -0.2070 + i0.1616 \end{bmatrix}, v_2 = \begin{bmatrix} -0.7232 \\ 0.2809 - i0.5737 \\ -0.2070 - i0.1616 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 0.5125 \\ -0.3650 \\ 0.7772 \end{bmatrix} \text{ For the equilibrium point } E_1$$

consider the Rossler parameter values of $h = 0.001$ and $a = 0.6$ yield the Eigenvalues are $\lambda_{1,2} = 0.999 \pm i0.0023$ and $\lambda_3 = 1.0004$ also the Eigenvectors are

$$v_1 = \begin{bmatrix} -0.0567 - i0.4525 \\ -0.1731 + i0.5737 \\ -0.2070 + i0.1616 \end{bmatrix}, v_2 = \begin{bmatrix} -0.0567 + i0.4525 \\ -0.1731 - i0.5737 \\ -0.2070 - i0.1616 \end{bmatrix}, v_3 = \begin{bmatrix} 0.5125 \\ -0.3650 \\ 0.7772 \end{bmatrix}. \text{ Also the numerical simulation is}$$

presented below for the Rossler's parameter value $h = 0.001$ and $a = 0.6$ see figure-3

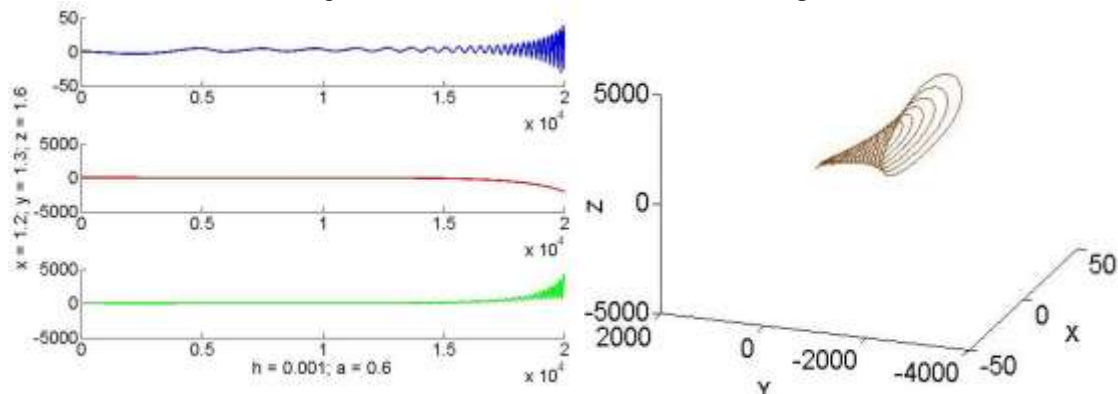


Figure 3: Time Plot and Phase Portrait for $h = 0.01$ and $a = 0.45$

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