



Numerical Analysis of Resistance and Conductance Controlled the Dynamical Behavior of Chaotic Circuit

Abo-Talib Abbas, Noori H.N. Al-Hashimi, Forat Ahmed

Department of Physics, College of Education for pure science, University of Basra, Basra, IRAQ

Abstract: In this paper, we have analysis one of the Chaotic Circuit Numerically with the non-linear type of Memductance, Memristor flux-controlled, the effect of the resistance and conductance on dynamics behavior have been study and in particular the area of chaos. It is found that the dynamical behavior existing in all cases when the resistance and/or conductance are changes and that the area of chaos is sensitive to the value of these changes.

Keywords: Chua's circuit; Lagrange stability; Attraction-repulsion function; chaos; bifurcation

I. Introduction

Chua in 1983 [1] design an electronic circuit showed some dynamical behavior, this circuit has been known as Chua circuit. Chua was trying to get circuits containing three elements can storage the energy, with three-point balance. Chua circuits contain also a non-linear part of type piecewise linear. The existence of chaotic attractors from the Chua circuit had been confirmed numerically by Matsumoto [2], observed experimentally by Zhong and Ayrom [3], and proved rigorously in Chua, et al. [4], where he was observed the following phenomena: period-doubling route; period-adding Sequence; quasi-periodic route; and chaos. The Chua Circuit is the simplest electronic circuit exhibiting chaos, and many well known bifurcation phenomena, as verified from numerous laboratory experiments, computer simulations, and rigorous mathematical analysis see for example [5-9]. After Chua circuit successfully expressed a verity of dynamical behavior, researcher focused on the benefit of this circuit in practical applications. Many researchers has succeeded in using this circuit in a lot of applications in many important areas, see for example, [10, 11]. These applications are as follow;; hiding many of the information to be concealed and not leakage to other parties, the use of the circuit in the control of information systems and control mechanism reorient, and the use of machines in musical mechanics in addition of many other applications. Chua model has been change several times by adding many electronic parts or add an external source [12] or transform the original circuit of the circle from three-dimensional to the four-dimensional [13] or five-dimensional [14] by adding or deleting one of the basic elements constituting the circuit. It's worth to say that all of these changes have shown many of variety in dynamical behavior. In this paper we will consider a chaos circuits shown in the figure (1). Where R represents the non-linear resistance, G_N represent the impedance, C_1 and C_2 capacitance of first and second capacitor, L self-inductance of the coil used, ϕ is the magnetic flux, q represents the electric charge, V_{c1} and V_{c2} voltage across the capacitor respectively, i_l the current across the coil. In order to study the stability of or instability of this circuit a non-linear part has been added to this circuit represent by the symbol M , which is a type of Memristor flux-controlled memductance [15] as shown in Figure (2). Some computer simulations on the modified Chua's circuit with the suitable function are used together with the effect of the linear resistance and inductance on the dynamical behavior of the chaotic circuit is analysis.

II. Theory

Usually, any dynamical system can be described in differential equations; the center manifold theory [16] is first applied to obtain a locally invariant small-dimensional manifold—a center manifold. Then additional nonlinear transformations are introduced to further simplify the center manifold to a normal form. To find the “form” of a normal form, first a homogeneous polynomial vector field of degree k is found in a space complementary to the range of the so called “homological operator”. Then the original vector field is decomposed into two parts: one of them, called the non-resonant terms, is eliminated and the other, called the resonant terms, is kept in the normal form. This simple form can be used conveniently for analyzing the local dynamic behavior of the original system. For a practical system, not only the possible qualitative dynamical behavior of the system of concern, but also the quantitative relationship between the normal forms and the equations of the original system needs to be established. Normal forms are, in general, not uniquely defined and finding a normal form for a given system of differential equations is not a simple task. In particular, finding the explicit formulas for normal forms in terms of the coefficients of the original nonlinear system is not easy. Therefore, the crucial part in computing a normal form is the computational efficiency in finding the coefficients of the normal forms and the

corresponding nonlinear transformations. Furthermore, the algebraic manipulation becomes very involved as the order of approximation increases. The idea of normal form theory is to use successive nonlinear transformations to derive a new set of differential equations by removing as many nonlinear terms from the system as possible. The terms remaining in the normal form are called the resonant terms. If the Jacobian matrix of the linearized system evaluated at equilibrium can be transformed into diagonal form, then the bases of the nonlinear transformations are decoupled from each other. However, for a general singular vector field such as a system with non-semi simple double-zero or triple-zero Eigen values, these bases are coupled. Such a coupling makes computation of the normal forms complicated. First, we introduce a general approach for computing normal forms based on the work of [16]. Consider a system described by the general nonlinear ordinary differential equation, produced by applying the Kirchhoff's law around the first and second capacitor and around the coil and the resistance, this system can be describe by the following differential equations:

$$\frac{d\varphi}{dt} = v_{c1} \quad (1)$$

$$\frac{dv_{c1}}{dt} = \frac{1}{c_1} \{i_1 - w(\varphi)v_{c1}\} \quad (2)$$

$$\frac{dv_{c2}}{dt} = -\frac{1}{c_2} \{i_1 - G_N v_{c2}\} \quad (3)$$

$$\frac{di_1}{dt} = \frac{1}{L} \{v_{c2} - Ri_1 - v_{c1}\} \quad (4)$$

Where $w(\varphi)$ is the function of the non-linear parts shown in figure (2) which one can assume is follow the behavior of the differential equation of the form:

$$w(\varphi) = \frac{dq(\varphi)}{dt} = -\alpha + 3b\varphi^2 \quad (5)$$

Where α , and β are some constant. It is worth noticing that the equations governing the circuit are symmetrical with respect to the origin, i.e., they are invariant under the transformation $(v_{c1}, v_{c2}, i_L) \rightarrow (-v_{c1}, -v_{c2}, -i_L)$, this means that the system is symmetric with respect to the origin which can be proved via the above transformation. Since the nonlinearity of the Chua's circuit is a piecewise-linear function, i.e. has the form:

$$f(x) = \max\{l_1(x) \dots \dots l_k(x)\} \quad (6)$$

Where l_i are linear functions, then the set of points on or above the graph is an intersection of closed half spaces and hence it is convex, which means that the function $f(x)$ is convex. Based on this property the circuit can be divided into a set of separate regions. Analyzing the behavior of the system in each of these regions is helpful to understand the global behavior of the circuit. Now if one assumes a study state condition, i.e.

$$\frac{dv_{c1}}{dt} = \frac{dv_{c2}}{dt} = \frac{di_L}{dt} = 0, \quad (7)$$

And

$$v_{c1} = 0, \quad (8)$$

Equations (2-4) reflect a unique equilibrium point at the origin which is $S_0 = (v_{c1}, v_{c2}, i_L, \varphi) = (0,0,0, \varphi_0)$. If these conditions do not satisfies, on can expected some of dynamical behavior will arises on both side of the equilibrium point and a chaotic properties will be clear. Now the above system can be rewritten in a matrix form as $V = AV + cont. w(\varphi)$ where:

$$V = \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & \frac{1}{c_1} \\ 0 & G_N & -\frac{1}{c_2} \\ -\frac{1}{L} & \frac{1}{L} & -\frac{R}{L} \end{bmatrix}, \quad cont. = \begin{bmatrix} -\frac{\beta}{c_1} \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

And $w(\varphi) = v_{c1} \exp(\gamma v_{c1}^2)$, where $w(\varphi)$ is bounded when $\gamma < 0$.

III. Result and Discussion

The description of the dynamic behavior of the circuit adopted in this study have been analyzed through the relationship between the voltage passing through the capacitance C_2 as a function of the values of C_2 at a constant value of the capacitance C_1 for two cases:

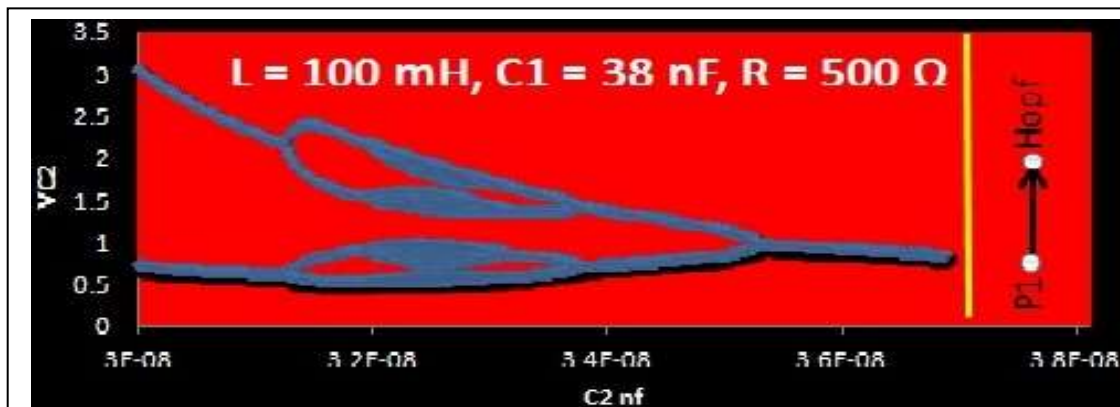
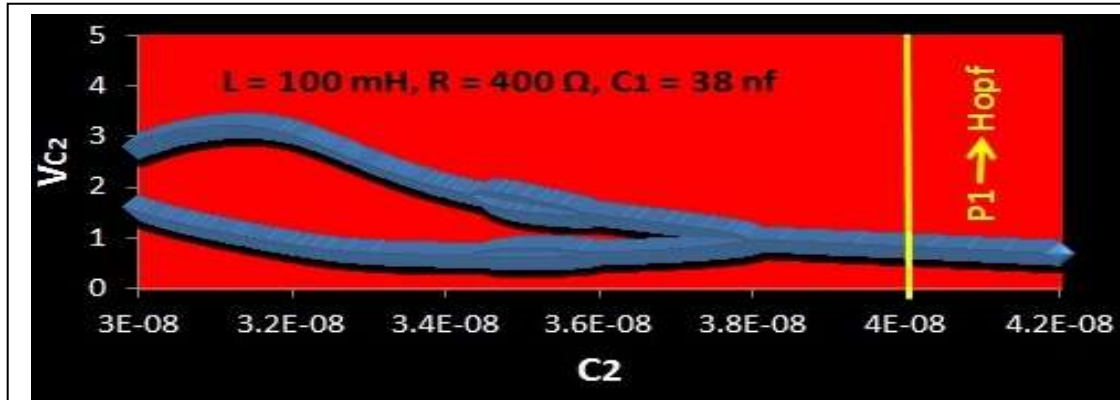
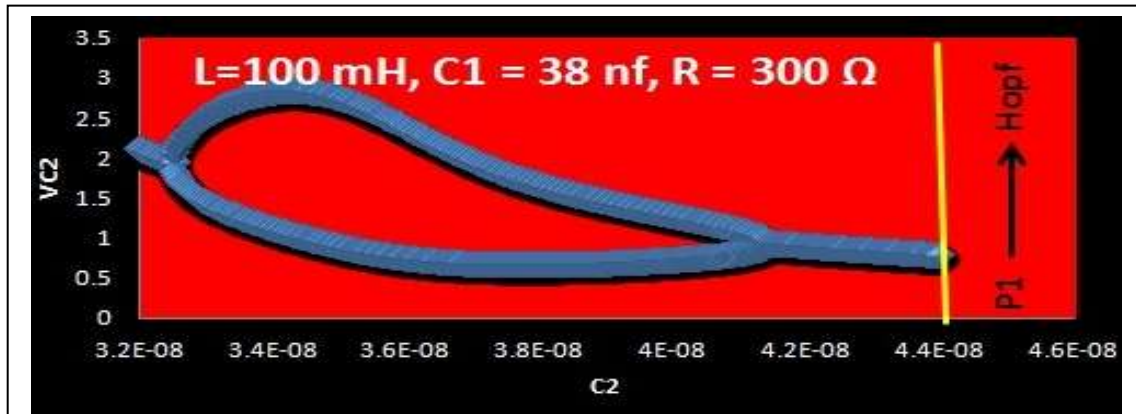
A) The effect of linear resistance on the stability of chaotic Circuit: The impact of change the linear resistance on the stability of chaotic circuit under study is described in a figures (1) to (4). In these figures the value of the conductance is fixed to the value of 100 mH, and the value of C_1 is fixed to the value of 38 nf. The study is carried out for difference values of linear resistance, so one can record the following observations:

- Figure (1): Linear resistance is 300Ω: An existence a verity of dynamical behaviors in term of two kind of periodicity of the form P1 and P2, with no sign of chaotic area. We further note that for a value of $C_2 \geq 44 \text{ nf}$, the properties of periodic P1 is recorded which extended up to Hopf -bifurcation which means that the circuit behave as a DC circuit, which appears in the Phase space as afixed point.
- Figure (2): Linear resistance is 400Ω: almost the same description above applies to this figure except of the existence of periodic behavior of the type P1, P2, P4. We further note that for a value of $C_2 \geq 40 \text{ nf}$, the properties of periodic P1 is recorded which extended up to Hopf -bifurcation.

- Figure (3): Linear resistance is 500Ω : the presence of chaotic areas is recorded and the properties of periodic P1 is appear when $C_2 \geq 37\text{ nf}$,
- Figure (4): Linear resistance is 600Ω : the presence of chaotic areas is recorded and the properties of periodic P1 is appear when $C_2 \geq 32\text{ nf}$,

B) The effect conductance on the stability of chaotic Circuit: The impact of change the conductance on the stability of chaotic circuit under study is described in a figures (5) to (7). In these figures the value of the linear resistance is fixed to the value of 300Ω , and the value of C_1 is fixed to the value of 28 nf . The study is carried out for difference values of conductance, so one can record the following observations:

- Figure (5): the value of conductance is 100 mH ; the presence of chaotic areas is recorded for wide range of C_2 , and the properties of periodic P1 is appear when $C_2 \geq 44\text{ nf}$,
- Figure (6): the value of conductance is 90 mH ; the existence of the chaotic area for wide range of C_2 , and the properties of periodic P1 is appear when $C_2 \geq 37\text{ nf}$,
- Figure (7): the value of conductance is 80 mH ; in this figure one can record a completely periodic behavior with the existence of periodic properties of the type P1, P2, P4. Further we note that no sigh of chaotic area.



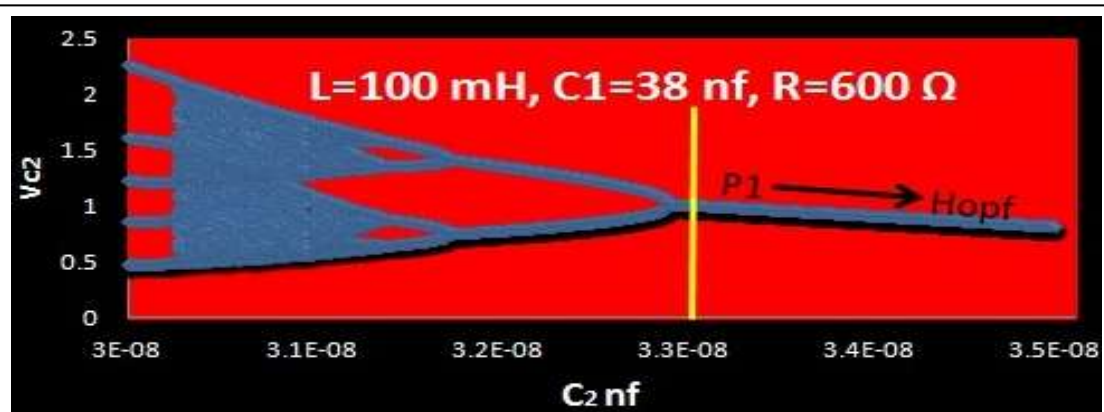


Figure (4) dynamical bifrucation for R=600 Ω

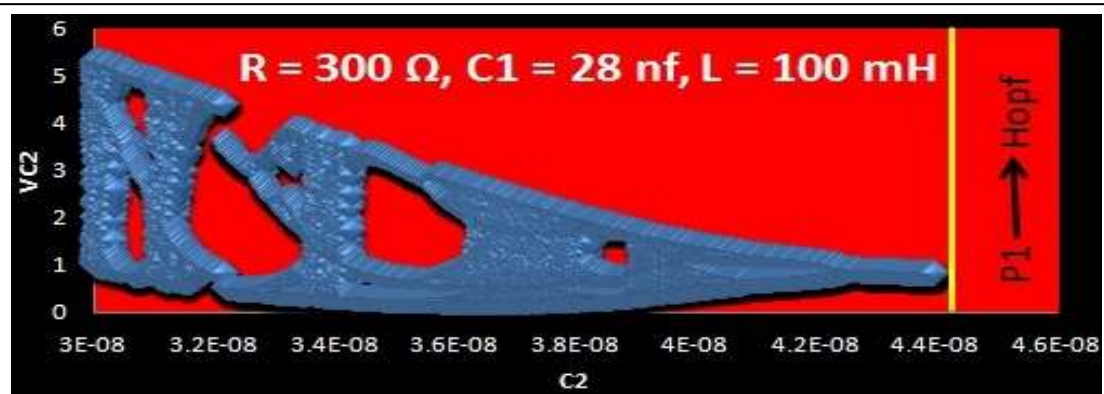


Figure (5) dynamical bifrucation for L=100 mH

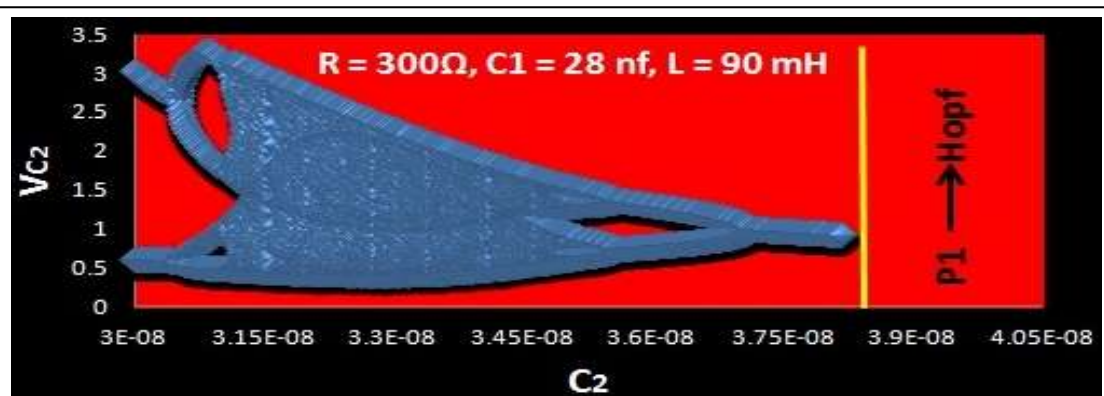


Figure (6) dynamical bifrucation for L=90 mH

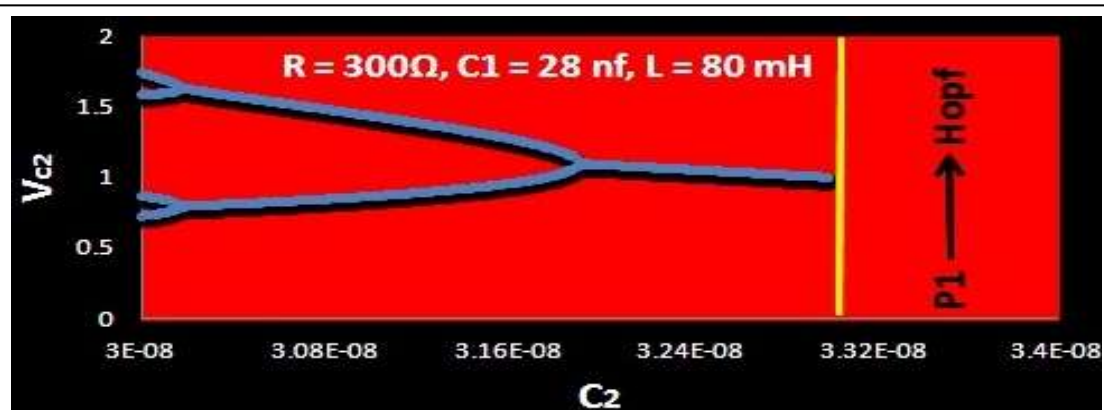


Figure (7) dynamical bifrucation for L=80 mH

VI. References

- [1] Matsumoto T., "A Chaotic attractor from Chua's circuit"; IEE Transactions on circuits and systems; VOL.CAS-31,NO.12[1984].
- [2] Mulukutla M. and aissi ch., "Implementation of the Chua's circuit and its applications"; The Louisiana BORSF contract NO. LEQSF(1999-2001)-RD-A-47.
- [3] Zhong G.Q. and Ayrom F., (1985), "Periodicity and Chaos in Chua's Circuit", IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS, VOL. CAS-32, NO. 5, MAY 1985
- [4] Chua, L. O., Komuro, M., and Matsumoto, T., (1986), "The double scroll family, IEEE Transactions on Circuits and Systems", 33 : 1072-1118.
- [5] Sprott, J. C. (2003), "Chaos and Time-Series Analysis", Oxford University Press, Oxford.
- [6] Stewart, I (2000), "The Lorenz Attractor Exists", Nature, 406 : 948-949.
- [7] Tsuneda, A. (2005), "A Gallery of Attractors from smooth Chua's Equation", International Journal of Bifurcation and Chaos, 15 : 1-50.
- [8] John Guckenheimer (2007), "Bifurcation", Scholarpedia, 2(6):1517.
- [9] Munuzuri, A.P., and Chua, L.O. (1997), "Stationary Structures in a Discrete Bistable Reaction-Diffusion System", International Journal of Bifurcation and Chaos, 12 : 2807-2825.
- [10] Ogorzalek M. J.,(1993), "Controlling chaos in Chua's circuit," International symposium on nonlinear theory and its applications"; (NOLTA 93) Hawaii. U. S. A.
- [11] Gottfried M. K., Insook C., Nicholas W., Robin B., Alfred H., (1993), "Musical signals from Chua's circuit"; IEE Transactions on circuits and systems: Analog and digital-signal processing. VOL 40.NO.10.
- [12] Lakshmanan, M. Murali, K., (1993) "Experimental chaos from non-autonomous electronic circuits"; Philosophical Transactions: Physical Sciences and Engineering; 353, 1701, pp. 33-46
- [13] Barboza, R., (2008), "Hyperchaos in a Chua's circuit with two new added branches"; International Journal of Bifurcation and Chaos 18, 4, pp. 1151-1159.
- [14] Murali, K. And Lakshmanan, M., (1992), "Effect of sinusoidal excitation on the Chua's circuit"; IEEE Transactions on Circuits And Systems I39, 4, pp.264-270.
- [15] Christos K. Volos, Ioannis M. Kyprianidis, Ioannis N. Stouboulos, (2010), "Memristors: A new approach in nonlinear circuits design"; proc. Of the 14th WSEAS Int. Conference on communication, pp. 25-30.
- [16] Han, Maoan, Yu, Pei; "Normal Forms, Melnikov Functions and Bifurcations of Limit Cycles"; Applied Mathematic science Vol. 181; 2012