



MEASURING THE FUZZINESS OF PRACTICAL DISTRIBUTED FUZZY SETS

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Abstract: Membership function of a fuzzy set can be generalized using distribution or generalized function. For distributed fuzzy sets defined over intervals on the real line, the membership functions can be generalized using distributions. The practical distributed fuzzy sets are defined by specifying a partition of $[a, b]$ and the average membership value m_i where $0 \leq m_i \leq 1$. The operations of the usual fuzzy sets are extended to PDFuS also. In this paper we introduce the concept of relative index of fuzziness for a PDFuS

Keywords: distributed fuzzy sets, fuzzy sets, generalized functions, measure of fuzziness, membership functions, practical distributed fuzzy sets, relative index of fuzziness.

I. INTRODUCTION

In 1965, Lotfi A. Zadeh[4] introduced the concept of fuzzy sets. The membership function of an usual fuzzy set is a function from a set to the interval $[0, 1]$. This can be generalized in several ways. For example, an L Fuzzy set has a membership function where the range is a partially ordered set [2].

In our paper [5], we had introduced the concept of distributed fuzzy sets which are defined over intervals on the real line. For a distributed fuzzy set membership function can be a generalized function or a distribution. In our previous paper [6], we had introduced the concept of practical distributed fuzzy sets. The practical distributed fuzzy sets differ from the usual fuzzy sets in a crucial manner. In an ordinary fuzzy set the membership function is known at every point while in a PDFuS only the average membership value over certain intervals is known. However we have seen that [7] intersection, union, aggregation can be defined for the PDFuS in exactly the same manner as in the case of fuzzy set. So what is the difference between these two?

The answer is that the PDFuS is vaguer than the fuzzy set. So while using PDFuS in applications of fuzzy logic we have to take this into account. So first there must be a way of measuring the fuzziness of a PDFuS.

For fuzzy sets several measures of fuzziness have been defined. These must be suitably modified for the PDFuS. As an example consider a PDFuS on the interval $[0, 4]$ with the average membership value $1/4$. If we take $\mu(x) = 1/4$ on the interval then the index of fuzziness in terms of Euclidean distance would be

$\sqrt{\int_0^4 ((\mu(x) - \mu_c(x))^2) dx}$ where $\mu_c(x) = 0$ if $\mu(x) \leq 1/2$ and $\mu_c(x) = 1$ if $\mu(x) > 1/2$. So on calculation the index is $1/2$. So for a fuzzy set with $\mu(x) = 1/4$ on the interval $[0, 4]$, the index of fuzziness (Euclidean) would be $1/4$. However it would not be meaningful to assign the same index number to a PDFuS with the average membership value $1/4$. This PDFuS may correspond to the crisp set $[x, x+1]$ for any x such that $0 \leq x \leq 3$. Or the PDFuS may represent a fuzzy set whose membership function $\mu(x)$ satisfies $\int_0^4 \mu(x) dx = 1$. So while attempting to measure the fuzziness of a PDFuS this additional type of vagueness or uncertainty must be taken into account.

So we define a new index number called the relative index of fuzziness. First the axioms and definitions corresponding to the measures of fuzziness [7] of usual fuzzy sets are listed. Then the concept of relative index of fuzziness of a PDFuS is introduced.

II. PRELIMINARIES

We briefly recall the definition of the practical distributed fuzzy sets.

Consider a sequence of functions $\{f_n\}$ in $C_0^\infty(R)$ with the following properties with respect to the interval $[a, b]$.

$$P1 : f_n(x) = 0 \quad \text{out side} \quad [a - 1/n, b + 1/n].$$

$$P2 : f_n(x) = 1 \quad \text{in} \quad [a + 1/n, b - 1/n].$$

$$P3 : 0 \leq f_n(x) \leq 1.$$

It is well known that such a sequence exists.

Definition 2.1: Let $\{f_n\}$ be a sequence of functions in $C_0^\infty(R)$ satisfying P1, P2, and P3 with respect to the interval $[a, b]$ and let T be a distribution. T is said to define a distributed fuzzy set on $[a, b]$ if

- i. T is a positive distribution
- ii. $\lim_{n \rightarrow \infty} T(f_n)$ exists and is the same for any sequence of functions $\{f_n\}$ in $C_0^\infty(R)$ satisfying the P1, P2, and P3 with respect to the interval $[a, b]$.
- iii. $\lim_{n \rightarrow \infty} T(f_n) \leq b - a$

Definition 2.2: Suppose T defines a distributed fuzzy set over an interval $[a, b]$. Then it is said to have average membership value m over the interval $[a, b]$ where $m = \frac{1}{b-a} \lim_{n \rightarrow \infty} T(f_n)$

The average membership value of T over the interval $[a, b]$ will be denoted by $Avmg_T([a, b])$.

Definition 2.3: A Practical distributed fuzzy set A on an interval $[a, b]$ is defined by specifying

- i. a partition $a = a_0 < a_1 < a_2 < \dots < a_n = b$ and
- ii. the average membership value m_i in each interval $[a_{i-1}, a_i]$ where m_i lies between 0 and 1.

The average membership value of A in the interval $[a_{i-1}, a_i]$ is denoted by $avm_A([a_{i-1}, a_i])$.

III. MEASURES OF FUZZINESS FOR FUZZY SETS

Definition 3.1: A measure of fuzziness is a function $f: P(X) \rightarrow R$ where $P(X)$ denotes the set of all fuzzy subsets of X . That is the function f assigns a value $f(A)$ to each fuzzy subset of X which characterizes the degree of the fuzziness of A .

Definition 3.2: Every measure of fuzziness must satisfy

axiom 1: $f(A) = 0$ if and only if A is a crisp set.

axiom 2: If A and B are two fuzzy subsets of X , then $f(a) \leq f(b)$ whenever A is less fuzzy than B .

Definition 3.3: For an ordinary fuzzy set A , a measure of fuzziness called the index of fuzziness is defined in terms of a metric distance (Hamming or Euclidean) of A from any of the nearest crisp set for which

$$\mu_c(x) = \begin{cases} 0 & \text{if } \mu_A(x) \leq 1/2 \\ 1 & \text{if } \mu_A(x) > 1/2 \end{cases}$$

Definition 3.4 When Hamming distance is used the measure of fuzziness is defined by,

$$f(A) = \int_a^b |\mu_A(x) - \mu_c(x)| dx$$

Definition 3.5: For Euclidean distances, measure of fuzziness is given by,

$$f(A) = (\int_a^b (|\mu_A(x) - \mu_c(x)|)^2 dx)^{1/2}$$

Definition 3.6: For Minkowski's class of functions, the index of fuzziness is defined by,

$$f(A) = (\int_a^b (|\mu_A(x) - \mu_c(x)|)^w dx)^{\frac{1}{w}}$$

IV. MEASURES OF FUZZINESS FOR PDFuS

Definition 4.1: Let I be an interval and P be a partition of I. Let A be a PDFuS defined on (I, P). Let X be the collection of all such (I,P,A) and f be a function from X to the set of all non-negative real numbers. Then to be a good measure of fuzziness of a PDFuS f should satisfy:

axiom 1: $f(I, P, A) = 0$ if and only if A is a crisp set

axiom 2: If A and B are fuzzy sets defined on the same interval I then $f(I, P, A) \leq f(I, P, B)$ for some partition P of I if B is considered to be fuzzier than A.

Note: Of course axiom 2 depends on the interpretation of "fuzzier".

Definition 4.2: Let A be a distributed fuzzy set defined on an interval I = [a,b], then the index of fuzziness for the DFuS is defined by : $f(I, A) = (\int_a^b (|m - \mu_c(m)|)^w dx)^{1/w}$ where,

$$\mu_c(m) = \begin{cases} 0 & \text{if } m \leq \frac{1}{2} \\ 1 & \text{if } m > \frac{1}{2} \end{cases}$$

and m is the average membership value of T over the interval [a,b],

Then

$$\begin{aligned} f(I, A) &= [|m - \mu_c(m)|^w (b-a)]^{1/w} \\ &= |m - \mu_c(m)| (b-a)^{1/w} \end{aligned}$$

Definition 4.3: Suppose we have a PDFuS A defined over a partition $a_0 < a_1 < \dots < a_n$ with the average membership value m_k on the interval $I_k = [a_{k-1}, a_k]$. Suppose that within interval $[a_{k-1}, a_k]$ it is desired to know the average membership value on subintervals of length ϵ_k (that is ϵ_k is the desired accuracy in I_k). Then the relative index of the fuzziness is given by

$$f(I, P, A) = \sum_{k=1}^n |m_k - \mu_c(m_k)| (\text{length of } I_k)^{\frac{1}{w} (\frac{\text{length of } I_k}{\epsilon_k})}$$

where,

$$\mu_c(m_k) = \begin{cases} 0 & \text{if } m_k \leq \frac{1}{2} \\ 1 & \text{if } m_k > \frac{1}{2} \end{cases}$$

Example 4.1: Consider a PDFuS A on I=[0,5] and let P={0,1,2,3,4,5} be a partition on I. Let $m_1=0.5$, $m_2=0.7$ and $m_3=0.1$, $m_4=0.9$, $m_5=0.2$ and let the desired accuracy $\epsilon_k=0.5$. Then the relative index of fuzziness (Euclidean) is given by,

$$f(I, P, A) = \sum_{k=1}^n |m_k - \mu_c(m_k)| (\text{length of } I_k)^{\frac{1}{w} (\frac{\text{length of } I_k}{\epsilon_k})} \text{ where } w=2.$$

$$\text{Hence } f(I, P, A) = |0.5-0|(1/0.5) + |0.7-1|(1/0.5) + |0.1-0|(1/0.5) + |0.9-1|(1/0.5) + |0.2-0|(1/0.5)$$

$$\begin{aligned} &= (0.5+0.3+0.1+0.1+0.2)(1/0.5) \\ &= 2.4 \end{aligned}$$

Note(i):The relative index of the fuzziness(Minkowski) satisfies the axiom1:

By definition 4.3, $f(I, P, A) = \sum_{k=1}^n |m_k - \mu_c(m_k)| (\text{length of } I_k)^{\frac{1}{w} (\frac{\text{length of } I_k}{\epsilon_k})}$

Hence $f(I, P, A) = 0$ if and only if $|m_k - \mu_c(m_k)| = 0$ for every k.

That is $f(I, P, A) = 0$ if and only if $m_k = \mu_c(m_k)$ for every k.

That is $f(I, P, A) = 0$ if and only if $m_k = 1$ or 0 for every k .

That is $f(I, P, A) = 0$ if and only if A is a crisp set.

Hence axiom 1 is satisfied.

(ii) The Minkowski index of fuzziness may not satisfy axiom 2.

V. CONCLUSION

In this paper we have introduced some measures for measuring the fuzziness of the PDFuS. All measures of fuzzy sets can be suitably modified for the PDFuS also.

Any PDFuS can be treated as an ordinary fuzzy set, where membership function is constant in each subinterval of the partition. Hence PDFuS can be used in fuzzy logic just like any ordinary fuzzy set. However while verifying the validity of the conclusion we have to look into the relative index of fuzziness of the PDFuS. Hence the concept introduced in this paper is very useful in application of PDFuS in fuzzy logic.

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