



RELIABILITY ANALYSIS OF A COMPLEX SYSTEM WITH REPAIR MACHINE AND CORRELATED FAILURE AND REPAIR TIMES

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Abstract: The present paper deals with the reliability analysis of two non – identical unit parallel systems with correlated failure and repair times and Repair machine failure. Of the two non – identical units, one is priority unit and other one is ordinary. The repair of failed units is done by Repair machine, while the failed Repair Machine is repaired by repairman. The repair machine is given preventive maintenance after a random period of operation except when both the units are in failure mode. The random period of operation after which the repair machine is given preventive maintenance and the time of completion of preventive maintenance are independent exponential variates whereas failure and repair times of both the units are correlated random variables having joint density as bivariate exponential. The different measures of system effectiveness are obtained by using regenerative point technique.

Keywords: Parallel system; Repair machine; Reliability; Availability; Busy period; Expected number of repairs.

I. INTRODUCTION

A huge amount of literature is available in the field of reliability theory on the analysis of two unit priority system models. Various authors including (1-4, 6) have analyzed two unit system models assuming failure and repair times of the units as independently distributed random variables. But it has been found in many practical situations that failure and repair times are correlated random variables. With this concept of correlated failure and repair times various authors including (1,3, 4) considered system models assuming bivariate exponential distribution of failure and repair times. Further in system models authors have assumed that the machine device is used for repairing the failed units remains good forever. But in real situations this assumption is practicable and the repair machine may also fail during its working process. In case of nuclear reactors and marine equipments, robots are used for repair purposes and a robot again being a machine may also fail while performing its intended task. The concept of repair machine was introduced by Gupta and Chaudhary (5) in a two unit cold standby system with independent failure and repair times.

In the present study we investigate and analyze a two non-identical unit parallel system model with a repair machine having correlated failure and repair times of units. Initially both the units work and repair machine is in good condition. Repair machine is used to repair the failed unit but if during the repair of units, repair machine fails then the repair of failed unit is discontinued and repair machine is taken up for its repair, and after the repair of repair machine the repair of failed unit is done afresh. The repair machine is given preventive maintenance after a random period of operation except when both the units are in failure mode. The priority unit is given preference in repair over non-priority unit. The random period of operation after which the repair machine is given preventive maintenance and the time of completion of preventive maintenance are independent exponential variates whereas failure and repair times of both the units are correlated random variables having joint density of the form

$$f_i(x, y) = \lambda_i \mu_i (1 - r_i) e^{-(\lambda_i x + \mu_i y)} I_0 \left(2 \sqrt{\lambda_i \mu_i r_i x y} \right) ; x, y, \mu_i, \lambda_i > 0, 0 \leq r_i < 1$$

Where,

$I_0(2\sqrt{\lambda_i \mu_i r_i x y})$ is modified Bessel function of type one and order zero and is defined as

$$I_0(z) = \sum_{K=0}^{\infty} \frac{(z/2)^K}{(K!)^2}$$

By using regenerative point technique the following measures of systems effectiveness are obtained:

1. Reliability of the system and Mean time to system failure.
2. Expected up time of the system (0,t) and in steady state.
3. Expected busy period of the repair machine and repairman.
4. Expected number of repairs by repair machine and repairman.
5. Net expected profit earned by the system in (0, t) and in steady state

II. SYSTEM DESCRIPTION AND ASSUMPTIONS

1. The system consists of two non –identical unit working in parallel form named as priority and non-priority unit with a repair machine (R.M.).
2. Initially both the units are working.
3. Both the units having two modes – Normal (N) and Total failure (F).
4. The Repair machine repairs a failed unit but during repair it can also fail. In such a situation, the repair of a failed unit is discontinued and the repairman starts the repair of R.M. as a single repairman is always available with system.
5. After a random period of time preventive maintenance is given to R.M. but when both the units are failed then preventive maintenance is not given to R.M.
6. The failure and repair times of both the priority and non-priority units are taken to be correlated random variables having bivariate exponential distribution.
7. The failure and repair times of R.M. are taken to be exponential distribution with different parameters.
8. The random period of operation after which preventive maintenance is given to R.M. and period of completion of preventive maintenance both are exponentially distributed random variables with different parameters.

III. NOTATIONS AND SYMBOLS

X_i : Random variables representing failure time of priority unit/ non-priority unit. $i=1,2$ resp.

Y_i : Random variables representing repair time of priority unit/ non-priority unit. $i=1,2$ resp.

$f_i(x, y)$: Joint p.d.f of (X_i, Y_i) is

$$= \lambda_i \mu_i (1 - r_i) e^{-(\lambda_i x + \mu_i y)} I_0(2\sqrt{\lambda_i \mu_i r_i xy}) ; x, y, \mu_i, \lambda_i > 0, 0 \leq r_i < 1, i = 1, 2$$

$g_i(x)$: Marginal p.d.f of X_i

$$= \lambda_i (1 - r_i) e^{-\lambda_i (1 - r_i) x} ; x, \lambda_i > 0, 0 \leq r_i < 1, i = 1, 2$$

$h_i(y)$: Marginal p.d.f of Y_i

$$= \mu_i (1 - r_i) e^{-\mu_i (1 - r_i) y} ; y, \mu_i > 0, 0 \leq r_i < 1, i = 1, 2$$

$k_i(y|x)$: Conditional p.d.f of Y_i given $X_i = x$

$$= \mu_i e^{-(\mu_i y + \lambda_i r_i x)} I_0(2\sqrt{\lambda_i \mu_i r_i xy}) ; x, y, \mu_i, \lambda_i > 0, 0 \leq r_i < 1, i = 1, 2$$

α_1 : Rate of giving preventive maintenance to Repair machine

β_1 : Rate of completion of preventive maintenance

α_2 : Failure rate of Repair machine

β_2 : repair rate of Repair machine

SYMBOLS FOR THE STATES OF THE SYSTEM

N_{10} / N_{20} : Priority/non priority unit is in normal mode and operative

F_{1r} / F_{2r} : Priority/ non-Priority unit is under repair

F_{1w} / F_{2w} : Priority/ non-Priority unit is waiting for repair

RM_0 / RM_g : Repair machine is operative/ good and non- functioning

RM_r / RM_p : Repair machine is under repair/ preventive maintenance.

With the help of the above symbols the possible states of the system are:

$$S_0 = [N_{10}, N_{20}, RM_g], \quad S_1 = [F_{1r}, N_{20}, RM_0], \quad S_2 = [N_{10}, F_{2r}, RM_0]$$

$$S_3 = [N_{10}, N_{20}, RM_p], \quad S_4 = [F_{1w}, N_{20}, RM_r], \quad S_5 = [F_{1r}, F_{2w}, RM_0]$$

$$S_6 = [F_{1w}, N_{20}, RM_p], \quad S_7 = [N_{10}, F_{2w}, RM_r], \quad S_8 = [F_{1w}, F_{2w}, RM_r]$$

$$S_9 = [N_{10}, F_{2w}, RM_p], \quad S_{10} = [F_{1w}, F_{2w}, RM_p]$$

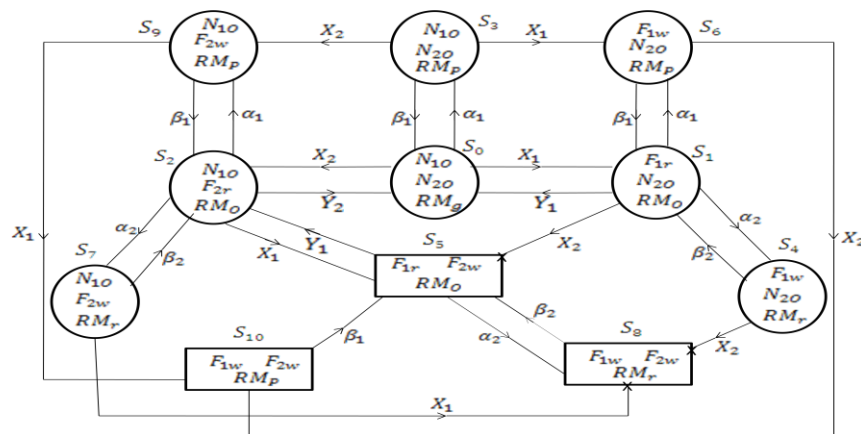


FIG. 1. TRANSITION DIAGRAM

IV. TRANSITION PROBABILITIES AND SOJOURN TIMES

A. STEADY STATE PROBABILITIES:

First we find the following conditional direct and indirect steady-state probabilities of transition:

$$p_{10|x} = \int_0^\infty dK_1(u|x)e^{-[a_1+a_2+\lambda_2(1-r_2)]u} = k_1^*[a_1 + a_2 + \lambda_2(1-r_2)|x]$$

$$p_{12|x}^{(5)} = \lambda_2(1-r_2) \int_0^\infty e^{-[a_1+a_2+\lambda_2(1-r_2)]u} \bar{K}_1(u|x) du \int_u^\infty \frac{dK_1(v|x)e^{-a_2v}}{\bar{K}_1(u|x)e^{-a_2u}}$$

$$= \frac{\lambda_2(1-r_2)}{[a_1+\lambda_2(1-r_2)]} [k_1^*(a_2|x) - k_1^*(a_1 + a_2 + \lambda_2(1-r_2)|x)]$$

Similarly,

$$p_{14|x} = \frac{a_2}{[a_1+a_2+\lambda_2(1-r_2)]} [1 - k_1^*(a_1 + a_2 + \lambda_2(1-r_2)|x)]$$

$$p_{16|x} = \frac{a_1}{[a_1+a_2+\lambda_2(1-r_2)]} [1 - k_1^*(a_1 + a_2 + \lambda_2(1-r_2)|x)]$$

$$p_{18|x}^{(5)} = \frac{\lambda_2(1-r_2)}{[a_1+\lambda_2(1-r_2)]} [1 - k_1^*(a_2|x)] - \frac{\lambda_2(1-r_2)}{[a_1+\lambda_2(1-r_2)][a_1+a_2+\lambda_2(1-r_2)]} [1 - k_1^*(a_1 + a_2 + \lambda_2(1-r_2)|x)]$$

$$p_{20|x} = k_2^*[a_1 + a_2 + \lambda_1(1-r_1)|x]$$

$$p_{25|x} = \frac{\lambda_1(1-r_1)}{[a_1+a_2+\lambda_1(1-r_1)]} [1 - k_2^*(a_1 + a_2 + \lambda_1(1-r_1)|x)]$$

$$p_{27|x} = \frac{a_2}{[a_1+a_2+\lambda_1(1-r_1)]} [1 - k_2^*(a_1 + a_2 + \lambda_1(1-r_1)|x)]$$

$$p_{29|x} = \frac{a_1}{[a_1+a_2+\lambda_1(1-r_1)]} [1 - k_2^*(a_1 + a_2 + \lambda_1(1-r_1)|x)]$$

$$p_{52|x} = k_1^*(a_2|x)$$

$$p_{58|x} = 1 - k_1^*(a_2|x)$$

Unconditional steady state probabilities of transition are:

$$p_{01} = \frac{\lambda_1(1-r_1)}{[a_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}, \quad p_{02} = \frac{\lambda_2(1-r_2)}{[a_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}, \quad p_{03} = \frac{a_1}{[a_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}$$

$$p_{30} = \frac{\beta_1}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}, \quad p_{36} = \frac{\lambda_1(1-r_1)}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}, \quad p_{39} = \frac{\lambda_2(1-r_2)}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}$$

$$p_{41} = \frac{\beta_2}{[\beta_2+\lambda_2(1-r_2)]}, \quad p_{45}^{(8)} = \frac{\lambda_2(1-r_2)}{[\beta_2+\lambda_2(1-r_2)]}, \quad p_{61} = \frac{\beta_1}{[\beta_1+\lambda_2(1-r_2)]}$$

$$p_{6,10} = \frac{\lambda_2(1-r_2)}{[\beta_1+\lambda_2(1-r_2)]}, \quad p_{72} = \frac{\beta_2}{[\beta_2+\lambda_1(1-r_1)]}, \quad p_{75}^{(8)} = \frac{\lambda_1(1-r_1)}{[\beta_2+\lambda_1(1-r_1)]}$$

$$p_{92} = \frac{\beta_1}{[\beta_1+\lambda_1(1-r_1)]}, \quad p_{9,10} = \frac{\lambda_1(1-r_1)}{[\beta_1+\lambda_1(1-r_1)]}, \quad p_{85} = p_{10,5} = 1$$

also,

$$p_{10} = \int_0^\infty p_{10|x} g_1(x) dx$$

$$= \lambda_1(1-r_1) \int_0^\infty k_1^*(a_1 + a_2 + \lambda_2(1-r_2)|x) e^{-[\lambda_1(1-r_1)]x} dx$$

$$= \frac{\mu_1(1-r_1)}{[a_1+a_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}$$

Similarly,

$$p_{14} = \frac{a_2}{[a_1+a_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}, \quad p_{12}^{(5)} = \frac{\lambda_2\mu_1(1-r_1)(1-r_2)}{[a_2+\mu_1(1-r_1)][a_1+a_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}$$

$$p_{16} = \frac{a_1}{[a_1+a_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}, \quad p_{18}^{(5)} = \frac{a_2\lambda_2(1-r_2)}{[a_2+\mu_1(1-r_1)][a_1+a_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}$$

$$p_{20} = \frac{\mu_2(1-r_2)}{[a_1+a_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]}, \quad p_{25} = \frac{\lambda_1(1-r_1)}{[a_1+a_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]}$$

$$p_{27} = \frac{a_2}{[a_1+a_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]}, \quad p_{29} = \frac{a_1}{[a_1+a_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]}$$

$$p_{52} = \frac{\mu_1(1-r_1)}{[a_2+\mu_1(1-r_1)]}, \quad p_{58} = 1 - \frac{\mu_1(1-r_1)}{[a_2+\mu_1(1-r_1)]} \quad (1-26)$$

It can be easily seen that the following results hold good:

$$p_{01} + p_{02} + p_{03} = 1$$

$$p_{20} + p_{25} + p_{27} + p_{29} = 1$$

$$p_{41} + p_{45}^{(8)} = 1$$

$$p_{61} + p_{6,10} = 1$$

$$p_{92} + p_{9,10} = 1$$

$$p_{10} + p_{12}^{(5)} + p_{14} + p_{16} + p_{18}^{(5)} = 1$$

$$p_{30} + p_{36} + p_{39} = 1$$

$$p_{52} + p_{58} = 1$$

$$p_{72} + p_{75}^{(8)} = 1$$

$$p_{85} = p_{10,5} = 1 \quad (27-36)$$

B. MEAN SOJOURN TIMES:

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i , in state S_i , we observe that as long as the system is in state S_i , there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$$\Psi_i = E[T_i] = \int P(T_i > t) dt$$

First we obtain the following conditional mean sojourn times:

$$\begin{aligned}\psi_{1|x} &= \int_0^\infty e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]t} \bar{K}_1(t|x) dt \\ &= \frac{1}{[\alpha_1+\alpha_2+\lambda_2(1-r_2)]} [1 - k_1^*(\alpha_1 + \alpha_2 + \lambda_2(1-r_2)|x)] \\ \psi_{2|x} &= \frac{1}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)]} [1 - k_2^*(\alpha_1 + \alpha_2 + \lambda_1(1-r_1)|x)] \\ \psi_{5|x} &= \int_0^\infty e^{-\alpha_2 t} \bar{K}_1(t|x) dt = \frac{1}{\alpha_2} [1 - k_1^*(\alpha_2|x)]\end{aligned}$$

Unconditional mean sojourn times are given by

$$\begin{aligned}\Psi_0 &= \frac{1}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]} & \Psi_3 &= \frac{1}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]} & \Psi_4 &= \frac{1}{[\beta_2+\lambda_2(1-r_2)]} \\ \Psi_6 &= \frac{1}{[\beta_1+\lambda_2(1-r_2)]} & \Psi_7 &= \frac{1}{[\beta_2+\lambda_1(1-r_1)]} & \Psi_9 &= \frac{1}{[\beta_1+\lambda_1(1-r_1)]} \\ \Psi_8 &= \frac{1}{\beta_2} & \Psi_{10} &= \frac{1}{\beta_1}\end{aligned}$$

Also,

$$\Psi_1 = \int_0^\infty \Psi_{1|x} g_1(x) dx = \frac{1}{[\alpha_1+\alpha_2+\mu_1(1-r_1)+\lambda_2(1-r_2)]}$$

Similarly,

$$\Psi_2 = \frac{1}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]} \quad \Psi_5 = \frac{1}{[\alpha_2+\mu_1(1-r_1)]} \quad (37-47)$$

V. ANALYSIS OF RELIABILITY AND MTSF

Let the random variable T_i be the time to system failure when system starts up from state $S_i \in E_i$, then the reliability of the system is given by

$$R_i(t) = P[T_i > t]$$

Using the definition of $R_i(t)$ relations among $R_i(t)$ can be developed, taking their Laplace transforms and solving the resultant set of equations for $R_0^*(s)$, we get

$$R_0^*(s) = N_1(s)/D_1(s) \quad (48)$$

Where,

$$\begin{aligned}N_1(s) &= (Z_0^* + q_{03}^* Z_3^* + q_{03}^* q_{36}^* Z_6^* + q_{03}^* q_{39}^* Z_9^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*)(1 - q_{27}^* q_{72}^* - q_{29}^* q_{92}^*) + (Z_1^* + q_{14}^* Z_4^* + q_{16}^* Z_6^*)(q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)(1 - q_{27}^* q_{72}^* - q_{29}^* q_{92}^*) + (Z_2^* + q_{27}^* Z_7^* + q_{29}^* Z_9^*)(q_{02}^* + q_{03}^* q_{39}^* q_{92}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*) \\ \text{and}\end{aligned}$$

$$D_1(s) = (1 - q_{03}^* q_{30}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*)(1 - q_{27}^* q_{72}^* - q_{29}^* q_{92}^*) - q_{20}^*(q_{02}^* + q_{03}^* q_{39}^* q_{92}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*) - q_{10}^*(q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)(1 - q_{27}^* q_{72}^* - q_{29}^* q_{92}^*)$$

To get MTSF, we use the well known formula

$$E(T_0) = \int R_0(t) dt = \lim_{s \rightarrow 0} R_0^*(s) = N_1(0)/D_1(0) \quad (49)$$

Where,

$$N_1(0) = (\Psi_0 + p_{03}\Psi_3 + p_{03}p_{36}\Psi_6 + p_{03}p_{39}\Psi_9)(1 - p_{14}p_{41} - p_{16}p_{61})(1 - p_{27}p_{72} - p_{29}p_{92}) + (\Psi_1 + p_{14}\Psi_4 + p_{16}\Psi_6)(p_{01} + p_{03}p_{36}p_{61})(1 - p_{27}p_{72} - p_{29}p_{92}) + (\Psi_2 + p_{27}\Psi_7 + p_{29}\Psi_9)(p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61})$$

and

$$D_1(0) = (1 - p_{03}p_{30})(1 - p_{14}p_{41} - p_{16}p_{61})(1 - p_{27}p_{72} - p_{29}p_{92}) - p_{20}(p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61}) - p_{10}(p_{01} + p_{03}p_{36}p_{61})(1 - p_{27}p_{72} - p_{29}p_{92})$$

Since we have $q_{ij}^*(0) = p_{ij}$ & $\lim_{s \rightarrow 0} Z_i^*(s) = \int Z_i(t) dt = \Psi_i$

VI. AVAILABILITY ANALYSIS

Define $A_i(t)$ as the probability that the system is up at epoch 't' when it initially starts from regenerative state S_i .

To obtain recurrence relations among pointwise availabilities $A_i(t)$ we use the simple probabilistic arguments.

Taking the Laplace transform and solving the resultant set of equations for $A_0^*(s)$, we have

$$A_0^*(s) = N_2(s)/D_2(s) \quad (50)$$

Where,

$$\begin{aligned}N_2(s) &= [(Z_0^* + q_{03}^* Z_3^* + q_{03}^* q_{36}^* Z_6^* + q_{03}^* q_{39}^* Z_9^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*) + (q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)(Z_1^* + q_{14}^* Z_4^* + q_{16}^* Z_6^*)] \mathbf{B}_1 + [q_{12}^{(5)*}(q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)(1 - q_{58}^* q_{85}^*) + (q_{02}^* + q_{03}^* q_{39}^* q_{92}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*)(1 - q_{58}^* q_{85}^*) + q_{52}^*(q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)(q_{14}^* q_{45}^{(8)*} + q_{18}^{(5)*} q_{85}^* + q_{16}^* q_{6,10}^* q_{10,5}^*) + q_{52}^* q_{03}^* (q_{36}^* q_{6,10}^* q_{10,5}^* + q_{39}^* q_{9,10}^* q_{10,5}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*)] [Z_2^* + q_{27}^* Z_7^* + q_{29}^* Z_9^*] \\ \text{And}\end{aligned} \quad (51)$$

$$\begin{aligned}D_2(s) &= [(1 - q_{03}^* q_{30}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*) - q_{10}^*(q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)] \mathbf{B}_1 - q_{20}^* [q_{12}^{(5)*}(q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)(1 - q_{58}^* q_{85}^*) + (q_{02}^* + q_{03}^* q_{39}^* q_{92}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*)(1 - q_{58}^* q_{85}^*)] + q_{52}^*(q_{01}^* + q_{03}^* q_{36}^* q_{61}^*)(q_{14}^* q_{45}^{(8)*} + q_{18}^{(5)*} q_{85}^* + q_{16}^* q_{6,10}^* q_{10,5}^*) + q_{52}^* q_{03}^* (q_{36}^* q_{6,10}^* q_{10,5}^* + q_{39}^* q_{9,10}^* q_{10,5}^*)(1 - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*) \\ \text{Where,}\end{aligned} \quad (52)$$

$$\mathbf{B}_1 = (1 - q_{27}^* q_{72}^* - q_{29}^* q_{92}^*)(1 - q_{58}^* q_{85}^*) - q_{52}^* (q_{25}^* + q_{27}^* q_{75}^{(8)*} + q_{29}^* q_{9,10}^* q_{10,5}^*)$$

The steady state Availability will be given by

$$A_0 = \lim_{t \rightarrow \infty} A_0(t) = \lim_{s \rightarrow 0} s A_0^*(s) = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)} = \lim_{s \rightarrow 0} N_2(s) \lim_{s \rightarrow 0} \frac{s}{D_2(s)}$$

Where

$$N_2(0) = [(\Psi_0 + p_{03}\Psi_3 + p_{03}p_{36}\Psi_6 + p_{03}p_{39}\Psi_9)(1 - p_{14}p_{41} - p_{16}p_{61}) + (p_{01} + p_{03}p_{36}p_{61})(\Psi_1 + p_{14}\Psi_4 + p_{16}\Psi_6)]B_2 + [p_{12}^{(5)}(p_{01} + p_{03}p_{36}p_{61})(1 - p_{58}p_{85}) + (p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61})(1 - p_{58}p_{85}) + p_{52}(p_{01} + p_{36}p_{61})(p_{14}p_{45}^{(8)} + p_{18}^{(5)}p_{85} + p_{16}p_{6,10}p_{10,5}) + p_{52}p_{03}(p_{36}p_{6,10}p_{10,5} + p_{39}p_{9,10}p_{10,5})(1 - p_{14}p_{41} - p_{16}p_{61})][\Psi_0 + p_{27}\Psi_7 + p_{29}\Psi_9] \quad (53)$$

Since $D_2(0) = 0$, by using L'Hospital rule we have

Hence on using L'Hospital's rule, A_0 becomes

$$A_0 = N_2(0)/D_2'(0) \quad (54)$$

Therefore,

$$D_2'(0) = (m_{01} + m_{02} + m_{03})A + (m_{10} + m_{12}^{(5)} + m_{14} + m_{16} + m_{18}^{(5)})B + (m_{20} + m_{25} + m_{27} + m_{29})C + (m_{30} + m_{36} + m_{39})D + (m_{41} + m_{45}^{(8)})E + (m_{52} + m_{58})F + (m_{61} + m_{6,10})G + (m_{72} + m_{75}^{(8)})H + m_{85}I + (m_{92} + m_{9,10})J + m_{10,5}K$$

Using the relation $\sum_j m_{ij} = \Psi_i$, we get

$$D_2'(0) = \Psi_0A + \Psi_1B + \Psi_2C + \Psi_3D + \Psi_4E + \Psi_5F + \Psi_6G + \Psi_7H + \Psi_8I + \Psi_9J + \Psi_{10}K \quad (55)$$

Where,

$$\begin{aligned} A &= p_{52}p_{20}(1 - p_{14}p_{41} - p_{16}p_{61}) \\ B &= p_{52}p_{20}(p_{01} + p_{03}p_{36}p_{61}) \\ C &= p_{52}(1 - p_{03}p_{30})(1 - p_{14}p_{41} - p_{16}p_{61}) - p_{52}p_{10}(p_{01} + p_{03}p_{36}p_{61}) \\ D &= p_{52}p_{20}p_{03}(1 - p_{14}p_{41} - p_{16}p_{61}) \\ E &= p_{52}p_{20}p_{14}(p_{01} + p_{03}p_{36}p_{61}) \\ F &= [(1 - p_{03}p_{30})(1 - p_{14}p_{41} - p_{16}p_{61}) - p_{10}(p_{01} + p_{03}p_{36}p_{61})][1 - p_{27}p_{72} - p_{29}p_{92}] - p_{20}p_{12}^{(5)}(p_{01} + p_{03}p_{36}p_{61}) - p_{20}(p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61}) \\ G &= p_{52}p_{20}p_{01}p_{16} + p_{52}p_{20}p_{03}p_{36}(1 - p_{14}p_{41}) \\ H &= p_{27}p_{52}[(1 - p_{03}p_{30})(1 - p_{14}p_{41} - p_{16}p_{61}) - p_{10}(p_{01} + p_{03}p_{36}p_{61})] \\ I &= p_{58}[(1 - p_{03}p_{30})(1 - p_{14}p_{41} - p_{16}p_{61}) - p_{10}(p_{01} + p_{03}p_{36}p_{61})][1 - p_{27}p_{72} - p_{29}p_{92}] - p_{20}p_{12}^{(5)}p_{58}(p_{01} + p_{03}p_{36}p_{61}) - p_{20}p_{58}(p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61}) + p_{20}p_{18}^{(5)}p_{52}(p_{01} + p_{03}p_{36}p_{61}) \\ J &= p_{52}p_{29}[(1 - p_{03}p_{30})(1 - p_{14}p_{41} - p_{16}p_{61}) - p_{10}(p_{01} + p_{03}p_{36}p_{61})] + p_{52}p_{20}p_{03}p_{39}(1 - p_{14}p_{41} - p_{16}p_{61}) \\ K &= p_{52}p_{29}p_{9,10}[(1 - p_{03}p_{30})(1 - p_{14}p_{41} - p_{16}p_{61}) - p_{10}(p_{01} + p_{03}p_{36}p_{61})] + p_{52}p_{20}p_{16}p_{6,10}(p_{01} + p_{03}p_{36}p_{61}) + p_{52}p_{20}p_{03}(p_{36}p_{6,10} + p_{39}p_{9,10})(1 - p_{14}p_{41} - p_{16}p_{61}) \end{aligned}$$

using (53) and (55) in (54), we get the expression for A_0 .

The expected up time of the system during $(0, t]$ is given by

$$\mu_{up}(t) = \int_0^t A_0(u) du$$

$$\text{So that, } \mu_{up}^*(s) = A_0^*(s)/s$$

VII. BUSY PERIOD ANALYSIS

BUSY PERIOD ANALYSIS OF REPAIRMAN/REPAIR MACHINE

Define $B_i(t)/B_i^{RM}(t)$ as the probability that the repairman/repair machine is busy in the repair of the failed repair machine /unit when the system initially starts from state $S_i \in E$. Using probabilistic arguments, the value of $B_0(t)$ can be obtained in its L.T as:

$$B_0^*(s) = N_3(s)/D_2(s) \quad (56)$$

Where,

$$N_3(s) = [q_{03}(Z_3^* + q_{36}^*Z_6^* + q_{39}^*Z_9^*)(1 - q_{14}^*q_{41}^* - q_{16}^*q_{61}^*) + (q_{01}^* + q_{03}^*q_{36}^*q_{61}^*)(q_{14}^*Z_4^* + q_{16}^*Z_6^*)]B_1 + [q_{12}^{(5)*}(q_{01}^* + q_{03}^*q_{36}^*q_{61}^*)(1 - q_{58}^*q_{85}^*) + (q_{02}^* + q_{03}^*q_{39}^*q_{92}^*)(1 - q_{14}^*q_{41}^* - q_{16}^*q_{61}^*)(1 - q_{58}^*q_{85}^*) + q_{52}^*(q_{01}^* + q_{03}^*q_{36}^*q_{61}^*)(q_{14}^*q_{45}^{(8)*} + q_{18}^{(5)*}q_{85}^* + q_{16}^*q_{6,10}^*q_{10,5}^*) + q_{52}^*q_{03}^*(q_{36}^*q_{6,10}^*q_{10,5}^* + q_{39}^*q_{9,10}^*q_{10,5}^*)(1 - q_{14}^*q_{41}^* - q_{16}^*q_{61}^*)][q_{27}^*Z_7^* + q_{29}^*Z_9^*] \quad (57)$$

Where,

$$B_1 = (1 - q_{27}^*q_{72}^* - q_{29}^*q_{92}^*)(1 - q_{58}^*q_{85}^*) - q_{52}^*(q_{25}^* + q_{27}^*q_{75}^{(8)*} + q_{29}^*q_{9,10}^*q_{10,5}^*)$$

In the steady state, the probability that the repairman will be busy is given by

$$B_0 = \lim_{t \rightarrow \infty} B_0(t) = \lim_{s \rightarrow 0} s B_0^*(s) = N_3(0)/D_2'(0) \quad (58)$$

Where,

$$N_3(0) = [p_{03}(\Psi_3 + p_{36}\Psi_6 + p_{39}\Psi_9)(1 - p_{14}p_{41} - p_{16}p_{61}) + (p_{01} + p_{03}p_{36}p_{61})(p_{14}\Psi_4 + p_{16}\Psi_6)]\mathbf{B}_2 + [p_{12}^{(5)}(p_{01} + p_{03}p_{36}p_{61})(1 - p_{58}p_{85}) + (p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61})(1 - p_{58}p_{85}) + p_{52}(p_{01} + p_{03}p_{36}p_{61})(p_{14}p_{45}^{(8)} + p_{18}^{(5)}p_{85} + p_{16}p_{6,10}p_{10,5}) + p_{52}p_{03}(p_{36}p_{6,10}p_{10,5} + p_{39}p_{9,10}p_{10,5})(1 - p_{14}p_{41} - p_{16}p_{61})][p_{27}\Psi_7 + p_{29}\Psi_9] \quad (59)$$

Where,

$$\mathbf{B}_2 = (1 - p_{27}p_{72} - p_{29}p_{92})(1 - p_{58}p_{85}) - p_{52}(p_{25} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5})$$

Similarly,

$$B_0^{RM*}(s) = N_3^{RM}(s)/D_2(s) \quad (60)$$

Where,

$$N_3^R(s) = Z_1^*(q_{01}^* + q_{03}^*q_{36}^*q_{61}^*)\mathbf{B}_1 + [Z_2^*(1 - q_{58}^*q_{85}^*) + Z_5^*(q_{25}^* + q_{27}^*q_{75}^{(8)*} + q_{29}^*q_{9,10}^*q_{10,5}^*)][q_{12}^{(5)*}(q_{01}^* + q_{03}^*q_{36}^*q_{61}^*) + (q_{02}^*q_{03}^*q_{39}^*q_{92}^*)(1 - q_{14}^*q_{41}^* - q_{16}^*q_{61}^*)] + [q_{52}^*Z_2^* + Z_5^*(1 - q_{27}^*q_{72}^* - q_{29}^*q_{92}^*)][(q_{01}^* + q_{03}^*q_{36}^*q_{61}^*)(q_{14}^*q_{45}^{(8)*} + q_{18}^{(5)*}q_{85}^* + q_{16}^*q_{6,10}^*q_{10,5}^*) + q_{03}^*(q_{36}^*q_{6,10}^*q_{10,5}^* + q_{39}^*q_{9,10}^*q_{10,5}^*)(1 - q_{14}^*q_{41}^* - q_{16}^*q_{61}^*)] \quad (61)$$

$$\text{Where, } \mathbf{B}_1 = (1 - q_{27}^*q_{72}^* - q_{29}^*q_{92}^*)(1 - q_{58}^*q_{85}^*) - q_{52}^*(q_{25}^* + q_{27}^*q_{75}^{(8)*} + q_{29}^*q_{9,10}^*q_{10,5}^*)$$

In the steady state, the probability that the repair machine will be busy is given by

$$B_0^{RM} = \lim_{t \rightarrow \infty} B_0^{RM}(t) = \lim_{s \rightarrow 0} s B_0^{RM*}(s) = N_3^{RM}(0)/D_2'(0) \quad (62)$$

Where,

$$N_3^{RM}(0) = \Psi_1(p_{01} + p_{03}p_{36}p_{61})\mathbf{B}_2 + [\Psi_2(1 - p_{58}p_{85}) + \Psi_5(p_{25} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5})][p_{12}^{(5)}(p_{01} + p_{03}p_{36}p_{61}) + (p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61})] + [\Psi_2p_{52} + \Psi_5(1 - p_{27}p_{72} - p_{29}p_{92})][(p_{01} + p_{03}p_{36}p_{61})(p_{14}p_{45}^{(8)} + p_{18}^{(5)}p_{85} + p_{16}p_{6,10}p_{10,5}) + p_{03}(p_{36}p_{6,10}p_{10,5} + p_{39}p_{9,10}p_{10,5})(1 - p_{14}p_{41} - p_{16}p_{61})] \quad (63)$$

$$\text{Where, } \mathbf{B}_2 = (1 - p_{27}p_{72} - p_{29}p_{92})(1 - p_{58}p_{85}) - p_{52}(p_{25} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5})$$

The expected busy period of the repairman during (0, t] is given by

$$\mu_b(t) = \int_0^t B_0(u) du$$

$$\text{So that, } \mu_b^*(s) = B_0^*(s)/s$$

The expected busy period of the repair machine during (0, t] is given by

$$\mu_b^{RM}(t) = \int_0^t B_0^{RM}(u) du$$

$$\text{So that, } \mu_b^{RM*}(s) = B_0^{RM*}(s)/s$$

VIII. EXPECTED NUMBER OF REPAIRS

EXPECTED NUMBER OF REPAIRS BY REPAIRMAN/REPAIR MACHINE

Let us define $V_i(t)/V_i^{RM}(t)$ as the expected number of repairs by repairman/ of repair machine during the time interval (0, t] when the system initially starts from regenerative state S_i . Using the definition of $V_i(t)/V_i^{RM}(t)$ the recursive relations among $V_i(t)$ can be easily developed, using L.S.T. and solving for $\tilde{V}_0(s)$, we get

$$\tilde{V}_0(s) = N_4(s)/D_2(s) \quad (64)$$

Where,

$$N_4(s) = [\tilde{Q}_{03}(\tilde{Q}_{30} + \tilde{Q}_{36}\tilde{Q}_{61} + \tilde{Q}_{39}\tilde{Q}_{92})(1 - \tilde{Q}_{14}\tilde{Q}_{41} - \tilde{Q}_{16}\tilde{Q}_{61}) + (\tilde{Q}_{01} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{61})(\tilde{Q}_{14}\tilde{Q}_{41} + \tilde{Q}_{16}\tilde{Q}_{61})]\mathbf{B}_1 + [\tilde{Q}_{12}^{(5)}(\tilde{Q}_{01} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{61}) + (\tilde{Q}_{02} + \tilde{Q}_{03}\tilde{Q}_{39}\tilde{Q}_{92})(1 - \tilde{Q}_{14}\tilde{Q}_{41} - \tilde{Q}_{16}\tilde{Q}_{61})]\mathbf{C}_1 + [\tilde{Q}_{52}(\tilde{Q}_{27}\tilde{Q}_{72} + \tilde{Q}_{29}\tilde{Q}_{92} - \tilde{Q}_{25}) + (1 - \tilde{Q}_{27}\tilde{Q}_{72} - \tilde{Q}_{29}\tilde{Q}_{92})]\mathbf{D}_1 \quad (65)$$

Where,

$$\mathbf{B}_1 = (1 - \tilde{Q}_{27}\tilde{Q}_{72} - \tilde{Q}_{29}\tilde{Q}_{92})(1 - \tilde{Q}_{58}\tilde{Q}_{85}) - \tilde{Q}_{52}(\tilde{Q}_{25} + \tilde{Q}_{27}\tilde{Q}_{75}^{(8)} + \tilde{Q}_{29}\tilde{Q}_{9,10}\tilde{Q}_{10,5})$$

$$\mathbf{C}_1 = \tilde{Q}_{58}\tilde{Q}_{85}(\tilde{Q}_{25} + \tilde{Q}_{27}\tilde{Q}_{75}^{(8)} + \tilde{Q}_{29}\tilde{Q}_{9,10}\tilde{Q}_{10,5}) + (1 - \tilde{Q}_{58}\tilde{Q}_{85})(\tilde{Q}_{27}\tilde{Q}_{72} + \tilde{Q}_{29}\tilde{Q}_{92} + \tilde{Q}_{27}\tilde{Q}_{75}^{(8)} + \tilde{Q}_{29}\tilde{Q}_{9,10}\tilde{Q}_{10,5})$$

$$\mathbf{D}_1 = (\tilde{Q}_{01} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{61})(\tilde{Q}_{14}\tilde{Q}_{45}^{(8)} + \tilde{Q}_{18}^{(5)}\tilde{Q}_{85} + \tilde{Q}_{16}\tilde{Q}_{6,10}\tilde{Q}_{10,5}) + \tilde{Q}_{03}(\tilde{Q}_{36}\tilde{Q}_{6,10}\tilde{Q}_{10,5} + \tilde{Q}_{39}\tilde{Q}_{9,10}\tilde{Q}_{10,5})(1 - \tilde{Q}_{14}\tilde{Q}_{41} - \tilde{Q}_{16}\tilde{Q}_{61})$$

In the long run the expected number of repairs per unit of time by the repairman is given by

$$V_0 = \lim_{t \rightarrow \infty} \frac{[V_0(t)]}{t} = \lim_{s \rightarrow 0} s^2 \tilde{V}_0^*(s) = N_4(0)/D_2'(0) \quad (66)$$

Where,

$$N_4(0) = [p_{92}(p_{30} + p_{36}p_{61} + p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61}) + (p_{01} + p_{03}p_{36}p_{61})(p_{14}p_{41} + p_{16}p_{61})]\mathbf{B}_2 + [p_{12}^{(5)}(p_{01} + p_{03}p_{36}p_{61}) + (p_{02} + p_{03}p_{39}p_{92})(1 - p_{14}p_{41} - p_{16}p_{61})]\mathbf{C}_2 + [p_{52}(p_{27}p_{72} + p_{29}p_{92} - p_{25}) + (1 - p_{27}p_{72} - p_{29}p_{92})]\mathbf{D}_2 \quad (67)$$

Where,

$$\mathbf{B}_2 = (1 - p_{27}p_{72} - p_{29}p_{92})(1 - p_{58}p_{85}) - p_{52}(p_{25} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5})$$

$$\mathbf{C}_2 = p_{58}p_{85}(p_{25} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5}) + (1 - p_{58}p_{85})(p_{27}p_{72} + p_{29}p_{92} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5})$$

$$D_2 = (p_{01} + p_{03}p_{36}p_{61})(p_{14}p_{45}^{(8)} + p_{85}p_{18}^{(5)} + p_{16}p_{6,10}p_{10,5}) + p_{03}(p_{36}p_{6,10}p_{10,5} + p_{39}p_{9,10}p_{10,5})(1 - p_{14}p_{41} - p_{16}p_{61})$$

Similarly,

$$\tilde{V}_0^{RM}(s) = N_4^{RM}(s)/D_2(s) \quad (68)$$

Where,

$$N_4^{RM}(s) = \tilde{Q}_{10}(\tilde{Q}_{01} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{61})B_1 + [\tilde{Q}_{52}(\tilde{Q}_{01} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{61})(\tilde{Q}_{14}\tilde{Q}_{45}^{(8)} + \tilde{Q}_{85}\tilde{Q}_{18}^{(5)} + \tilde{Q}_{16}\tilde{Q}_{6,10}\tilde{Q}_{10,5}) + \tilde{Q}_{52}\tilde{Q}_{03}(\tilde{Q}_{36}\tilde{Q}_{6,10}\tilde{Q}_{10,5} + \tilde{Q}_{39}\tilde{Q}_{9,10}\tilde{Q}_{10,5})(1 - \tilde{Q}_{14}\tilde{Q}_{41} - \tilde{Q}_{16}\tilde{Q}_{61}) + \tilde{Q}_{12}^{(5)}(\tilde{Q}_{01} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{61})(1 - \tilde{Q}_{58}\tilde{Q}_{85})][1 + \tilde{Q}_{20} - \tilde{Q}_{27}\tilde{Q}_{72} - \tilde{Q}_{29}\tilde{Q}_{92}] + \tilde{Q}_{12}^{(5)}(\tilde{Q}_{01} + \tilde{Q}_{03}\tilde{Q}_{36}\tilde{Q}_{61})[\tilde{Q}_{20}(1 - \tilde{Q}_{58}\tilde{Q}_{85}) + \tilde{Q}_{52}(\tilde{Q}_{25} + \tilde{Q}_{27}\tilde{Q}_{75}^{(8)} + \tilde{Q}_{29}\tilde{Q}_{9,10}\tilde{Q}_{10,5})] \quad (69)$$

Where,

$$B_1 = (1 - \tilde{Q}_{27}\tilde{Q}_{72} - \tilde{Q}_{29}\tilde{Q}_{92})(1 - \tilde{Q}_{58}\tilde{Q}_{85}) - \tilde{Q}_{52}(\tilde{Q}_{25} + \tilde{Q}_{27}\tilde{Q}_{75}^{(8)} + \tilde{Q}_{29}\tilde{Q}_{9,10}\tilde{Q}_{10,5})$$

In the long run the expected number of repairs per unit of time by the repair machine is given by

$$V_0^{RM} = \lim_{t \rightarrow \infty} \frac{[V_0^{RM}(t)]}{t} = \lim_{s \rightarrow 0} s^2 V_0^{RM*}(s) = N_4^{RM}(0)/D_2'(0) \quad (70)$$

$$N_4^{RM}(0) = p_{10}(p_{01} + p_{03}p_{36}p_{61})B_2 + [p_{52}(p_{01} + p_{03}p_{36}p_{61})(p_{14}p_{45}^{(8)} + p_{85}p_{18}^{(5)} + p_{16}p_{6,10}p_{10,5}) + p_{52}p_{03}(p_{36}p_{6,10}p_{10,5} + p_{39}p_{9,10}p_{10,5})(1 - p_{14}p_{41} - p_{16}p_{61}) + p_{12}^{(5)}(p_{01} + p_{03}p_{36}p_{61})(1 - p_{58}p_{85})][1 + p_{20} - p_{27}p_{72} - p_{29}p_{92}] + p_{12}^{(5)}(p_{01} + p_{03}p_{36}p_{61})[p_{20}(1 - p_{58}p_{85}) + p_{52}(p_{25} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5})] \quad (71)$$

$$\text{Where, } B_2 = (1 - p_{27}p_{72} - p_{29}p_{92})(1 - p_{58}p_{85}) - p_{52}(p_{25} + p_{27}p_{75}^{(8)} + p_{29}p_{9,10}p_{10,5})$$

IX. PROFIT FUNCTION ANALYSIS

Two profit functions $P_1(t)$ and $P_2(t)$ can easily be obtained for the system model under study with the help of characteristics obtained earlier. The expected total profits incurred during $(0, t]$ are:

$$P_1(t) = \text{Expected total revenue in } (0, t] - \text{Expected total expenditure in } (0, t] \\ = K_0\mu_{up}(t) - K_1\mu_b(t) - K_2\mu_b^M(t) \quad (72)$$

Similarly,

$$P_2(t) = K_0\mu_{up}(t) - K_3V_0(t) - K_4V_0^M(t) \quad (73)$$

Where,

K_0 is revenue per unit up time.

K_1 is the cost per unit time for which repair man is busy in repair of the Repair machine.

K_2 is the cost per unit time for which Repair machine is busy in repair of the failed unit.

K_3 is per unit repair cost.

K_4 is repair cost for Repair machine.

The expected total profits per unit time, in steady state, is

$$P_1 = \lim_{t \rightarrow \infty} [P_1(t)/t] = \lim_{s \rightarrow 0} s^2 P_1^*(s)$$

$$P_2 = \lim_{t \rightarrow \infty} [P_2(t)/t] = \lim_{s \rightarrow 0} s^2 P_2^*(s)$$

So that,

$$P_1 = K_0A_0 - K_1B_0 - K_2B_0^{RM} \quad (74)$$

$$P_2 = K_0A_0 - K_3V_0 - K_4V_0^{RM} \quad (75)$$

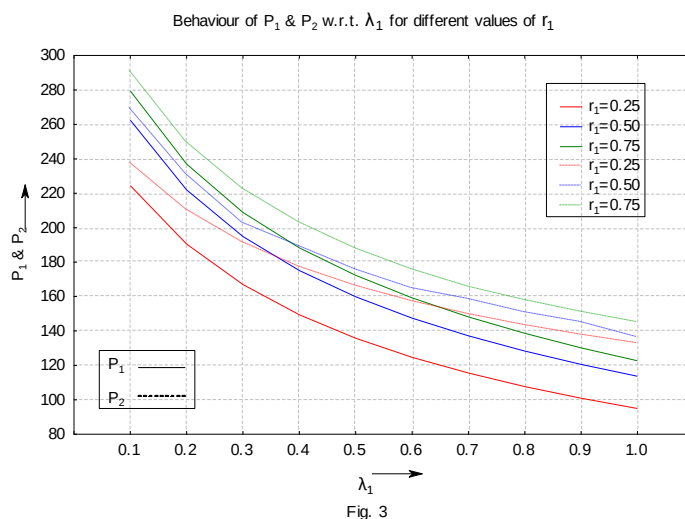
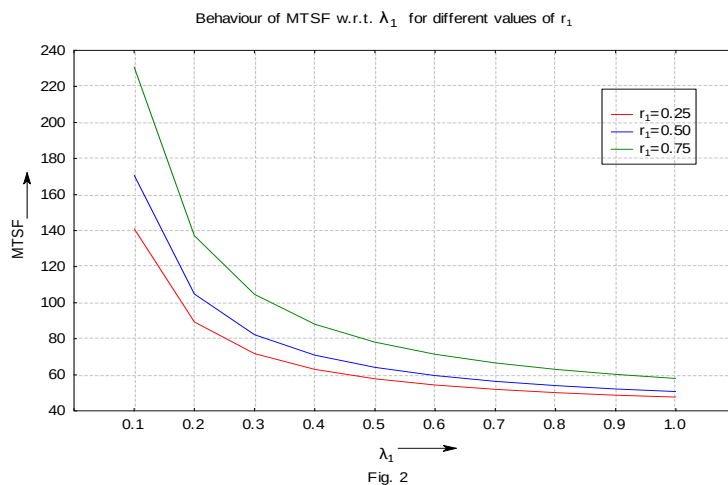
X. GRAPHICAL STUDY OF THE SYSTEM MODEL

For more concrete study of system behavior, we plot MTSF and Profit functions with respect to λ_1 (failure rate of priority unit) for different values of r_1 .

Fig. 2 shows the variations in MTSF in respect of λ_1 for different values of r_1 as 0.25, 0.50 and 0.75 while the other parameters are fixed as $\lambda_1=0.05, \mu_1=0.8, \mu_2=0.7, \alpha_1=0.05, \alpha_2=0.06, \beta_1=0.4, \beta_2=0.5, r_2=0.50$. It is observed from the graph that MTSF decreases with the increase in the failure parameter λ_1 and increases with the increase in r_1 .

Fig. 3 represents the change in profit function P_1 and P_2 w.r.t. λ_1 for different values of r_1 as 0.25, 0.50 and 0.75 while the other parameters are fixed as $\lambda_1=0.05, \mu_1=0.8, \mu_2=0.7, \alpha_1=0.05, \alpha_2=0.06, \beta_1=0.4, \beta_2=0.5, r_2=0.50, K_0=1000, K_1=300, K_2=250, K_3=350, K_4=200$.

From the graph it is seen that both the profit functions decrease with the increase in failure rate λ_1 and increase with the increase in r_1 . It is also observed that profit function P_2 is always higher as compared to profit function P_1 for fixed values of λ_1 and r_1 .



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