



A direct mathematical method to calculate the efficiencies of $4\pi\text{NaI (Tl)}$ scintillation detector

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Abstract: A direct mathematical method is applied to calculate the total efficiency of a $4\pi\text{NaI(Tl)}$ scintillation detector for an arbitrarily positioned radiating point source. The $4\pi\text{NaI(Tl)}$ scintillation detector is extremely important to measure the activities of low radioactivity samples. The central square is left void, so the radiating source can be moved easily inside the detector in which the total efficiency is measured at different positions and energies. The attenuation of the photon by the detector active volume and the detector end-cap materials are also included in the integrals. The calculated values of the efficiency are found to be in good agreement with the experimental one.

Keywords: $4\pi\text{NaI (Tl)}$ scintillation detector, Total efficiency, Direct mathematical method, Parallelepiped detector.

I. Introduction

Scintillation detectors are widely used for gamma-ray detection. Gamma rays could be emitted from point, line, disc or volumetric sources. The detection of these radiations is done by different detectors [1-9]. These detectors were designed with different efficiencies, depending on the shape of the detector and on the material of its active medium. The total efficiency for the source – detector system is determined experimentally and theoretically [6]. In our present work we will take into consideration the photon path length through the detector active medium in order to determine the efficiency of the $4\pi\text{NaI (Tl)}$ scintillation detector [10]. The work described below involves the use of straight forward analytical formulae [10] for the computation of the $4\pi\text{NaI(Tl)}$ detector and total efficiency. Section 2 presents direct mathematical formulae for the total efficiency in the case of isotropic radiating axial point, non-axial point, plane and volumetric sources. Section 3 contains comparisons between the calculated efficiency using the formulae derived in this work with the published experimental and simulated values illustrating the validity of the present mathematical formulae. The conclusion is presented in section 4.

II. Mathematical viewpoint

The position of the isotropic point source is defined by (ρ, h) , where, ρ is the lateral displacement between the source and the detector axis, whereas h is the height of the source from the detector center. The polar (θ) and azimuthal (ϕ) angles represent the direction of the photon when it enters the detector active volume. The effective rays which enter the detector active volume traverse a distance d until it emerges from detector [10].

A. Axial point source placed inside the detector void part

The incident photon may enter the detector's inner side and emerges from see fig.1

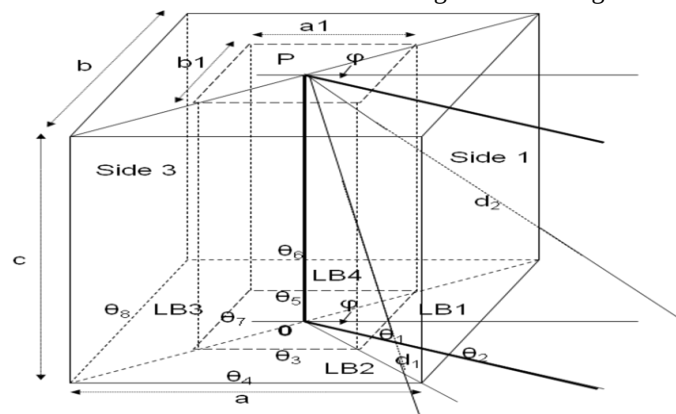


Fig. 1: Schematic view of $4\pi\text{NaI (Tl)}$ parallelepiped scintillator detector with central part left void and all the possible path lengths through the active medium of a parallelepiped ($a \times b \times c$) detector for a photon emitted from an isotropic axial radiating point source.

i) lower base 1(LB1) and 2 (LB2):

$$d_1^{a,b} = \left| \frac{a,b}{2 \sin(\theta) \cos(\varphi)} - \frac{c}{\cos(\theta)} \right| \quad (1)$$

ii) detector side (1) and (2):

$$d_2^{a,b} = \left| \frac{a,b - a_1, b_1}{2 \sin(\theta) \cos(\varphi)} \right| \quad (2)$$

The polar angle θ takes the steps

$$\theta_1^{a_1,a} = \tan^{-1} \left(\frac{a_1, a}{2c} \right) \quad (3)$$

$$\theta_2^{b_1,b} = \tan^{-1} \left(\frac{b_1, b}{2c} \right) \quad (4)$$

The azimuthal angle φ takes the steps

$$\phi_n = n \frac{\pi}{2} + \tan^{-1} \left(\frac{b}{a} \right) \quad n=0,2 \quad (5)$$

$$\phi_n = n \frac{\pi}{2} + \tan^{-1} \left(\frac{a}{b} \right) \quad n=1,3 \quad (6)$$

The total efficiency can be calculated by

$$\xi_1 = \int_0^{\phi_0} \int_{\theta_1^a}^{\theta_2^a} f_1^a d\theta d\phi + \int_0^{\phi_0} \int_{\theta_1^a}^{\theta_2^a} f_2^a d\theta d\phi + \int_{\phi_0}^{\phi_1} \int_{\theta_2^b}^{\theta_1^b} f_1^b d\theta d\phi + \int_{\phi_0}^{\phi_1} \int_{\theta_2^b}^{\theta_1^b} f_2^b d\theta d\phi + \int_{\phi_1}^{\phi_2} \int_{\theta_1^a}^{\theta_2^a} f_2^b d\theta d\phi \quad (7)$$

$$+ \int_{\phi_0}^{\phi_2} \int_{\theta_1^a}^{\theta_2^a} f_1^a d\theta d\phi + \int_{\phi_0}^{\phi_1} \int_{\theta_2^b}^{\theta_1^b} f_1^b d\theta d\phi + \int_{\phi_2}^{\phi_3} \int_{\theta_2^b}^{\theta_1^b} f_1^b d\theta d\phi + \int_{\phi_3}^{2\pi} \int_{\theta_1^a}^{\theta_2^a} f_1^a d\theta d\phi + \int_{\phi_0}^{2\pi} \int_{\theta_1^a}^{\theta_2^a} f_2^a d\theta d\phi$$

$$f_i^{a,b} = (1 - e^{-\mu d_i^{a,b}}) \sin(\theta) \cdot e^{-\frac{\mu_1 t}{\sin(\theta)}}, \quad i=1,2,\dots,4. \quad (8)$$

where, μ and μ_1 are the total attenuation coefficient of the detector and its housing, with thickness t , material for gamma-ray photon with energy E_γ , respectively.

B. Non-axial point source placed inside the detector void part

The incidence photon may enter the detector's inner side and emerges from see fig. 2

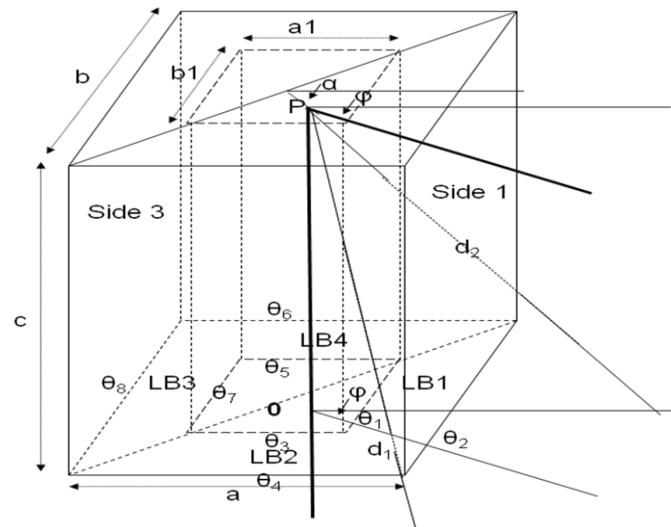


Fig. 2: Schematic view of all the possible path lengths through the active medium of a parallelepiped ($a \times b \times c$) detector for a photon emitted from an isotropic non-axial radiating point source.

i) lower base 1 (LB1):

$$d_1 = \frac{c}{\cos(\theta)} - \frac{a_1 + 2\rho \cos(\alpha)}{2 \sin(\theta) \cos(\phi)} \quad (9)$$

ii) detector side (1), (2),(3) and (4):

$$d_2^{a,b} = \left| \frac{a, b - a_1, b_1}{2 \sin(\theta) \cos(\phi)} \right| \quad (10)$$

iii) lower base 2 (LB2):

$$d_3 = \frac{-c}{\cos(\theta)} - \frac{a_1 - 2\rho \cos(\alpha)}{2 \sin(\theta) \cos(\phi)} \quad (11)$$

iv) lower base 3 (LB3) and 4 (LB4)

$$d_4^{\pm} = \frac{c}{\cos(\theta)} - \frac{b_1}{2 \sin(\theta) \sin(\phi)} \pm \frac{\rho \sin(\alpha)}{\sin(\theta)} \quad (12)$$

The final expression of the total efficiency of a non-axial point source at different positions is given by:

$$\begin{aligned} \xi_2 = & \int_0^{\theta_1^a} \int_{\phi_4^a}^{\phi_1^a} f_1 d\theta d\phi + \int_{\phi_4^a}^{\phi_1^a} \int_{\phi_4^a}^{\phi_1^a} f_1 d\theta d\phi + \int_0^{\frac{\pi}{2}} \int_{\phi_4^a}^{\phi_1^a} f_2 d\theta d\phi + \int_{\phi_4^a}^{\phi_1^a} \int_{\phi_4^a}^{\phi_1^a} f_2 d\theta d\phi + \int_{\phi_2^a}^{\phi_4^a} \int_{\phi_4^a}^{\phi_1^a} f_3 d\theta d\phi + \int_{\phi_2^a}^{\phi_4^a} \int_{\phi_4^a}^{\phi_1^a} f_4 d\theta d\phi + \int_{\phi_1^a}^{\phi_2^a} \int_{\phi_4^a}^{\phi_1^a} f_5 d\theta d\phi + \int_{\phi_1^a}^{\phi_2^a} \int_{\phi_4^a}^{\phi_1^a} f_6 d\theta d\phi \\ & + \int_{\phi_3^b}^{\phi_4^b} \int_{\phi_3^b}^{\phi_4^b} f_7 d\theta d\phi + \int_{\phi_3^b}^{\phi_4^b} \int_{\phi_3^b}^{\phi_4^b} f_8 d\theta d\phi \end{aligned} \quad (13)$$

The polar angle θ takes the steps

$$\theta_1^{a,a} = \tan^{-1} \frac{a_1, a - 2\rho \cos(\alpha)}{2.c.\cos(\phi)} \quad (14)$$

$$\theta_2^{b_1,b} = \tan^{-1} \frac{b_1, b - 2\rho \sin(\alpha)}{2.c.\cos(\phi)} \quad (15)$$

$$\theta_3^{b_1,b} = \tan^{-1} \frac{b_1, b + 2\rho \sin(\alpha)}{2.c.\sin(\phi)} \quad (16)$$

$$\theta_4^{a_1,a} = \tan^{-1} \frac{a_1, a + 2\rho \cos(\alpha)}{2.c.\cos(\phi)} \quad (17)$$

The azimuthal angle ϕ takes the steps

$$\phi_1 = \tan^{-1} \left(\frac{\frac{a}{2} - \rho \sin(\alpha)}{\frac{b}{2} - \rho \cos(\alpha)} \right) \quad (18)$$

$$\phi_2 = \frac{\pi}{2} + \tan^{-1} \left(\frac{\frac{b}{2} + \rho \cos(\alpha)}{\frac{b}{2} - \rho \sin(\alpha)} \right) \quad (19)$$

$$\phi_3 = \pi + \tan^{-1} \left(\frac{\frac{a}{2} + \rho \sin(\alpha)}{\frac{b}{2} - \rho \cos(\alpha)} \right) \quad (20)$$

$$\phi_4 = \frac{3\pi}{2} + \tan^{-1} \left(\frac{\frac{a}{2} - \rho \cos(\alpha)}{\frac{b}{2} + \rho \sin(\alpha)} \right) \quad (21)$$

C. Line source placed inside the detector void part

The efficiency of a $4\pi\text{NaI}$ (Tl) scintillation detector arising from a line of length ℓ cm is derived as [2]

$$\xi_L = \frac{\int_{h_1}^{h_2} \xi_1 dh}{h_2 - h_1} \quad (22)$$

where, ξ_1 is the total efficiency of an axial point source as identified before in equation (13). The total efficiency of the line source is determined at different positions along the axial axis of the detector.

D. Disc source placed inside the detector void part

The efficiency of $4\pi\text{NaI}$ (Tl) scintillation detector arising from a disc source is derived as [2]

$$\xi_D = \frac{\int_0^s \xi_2 \rho d\rho}{s^2} \quad (23)$$

Where, ξ_2 is the total efficiency of a non-axial point source as identified before in equation 23. The total efficiency of the disc source is determined at different positions along the axis of the detector.

III. Results and Discussion

The direct mathematical method is applied to determine the total efficiency of the $4\pi\text{NaI}$ (Tl) detector using an isotropic point source. The $4\pi\text{NaI}$ scintillation detector is shown in fig. 1 in which the central part is left void so the radiating source can be moved easily. The cross sectional size and the length of a NaI(Tl) crystal are $0.1 \times 0.1 \text{ m}^2$ and 0.4 m , respectively. The housing of the $4\pi\text{NaI}$ (Tl) detector is Stainless steel of $1 \times 10^{-3} \text{ m}$ thick. The solid angle of the detection geometry is high that more than 95% can reach the detector surface.

The total efficiency values of a point source placed at different positions along the axial axis and at different energies have been calculated for $4\pi\text{NaI}$ (Tl) scintillator detector by the direct mathematical method and compared with Byun [6] as shown in figs. 3-4. The total efficiency is also determined at lateral distances ρ from the central axis of the detector as shown in fig. 5. The figure shows no variation of the total efficiency as the lateral distance changes for the same height. Three different sources, a non-axial point, a line source of length $5 \times 10^{-2} \text{ m}$ and disc source of radius $2 \times 10^{-2} \text{ m}$ have been used in the theoretical calculations for this detector. Since the experimental values were in good consistent with direct mathematical method, we extended the study to line and disc source at different heights. The efficiency was studied at an energy $E=0.5 \text{ Mev}$, starting from the center where the height was considered zero until it reached the surface at a height of 0.2 m . Figs. 6-8 show that as the height of the non-axial, line and disc sources from the center increases the total efficiency decreases for different sources, which consists with the results obtained by Byun [5]. It is clear also that the total efficiency for disc source is greater than that of line and point sources. The percentage differences between the calculated values and the measured ones, which are given by equation (25), are shown in fig. 3.

$$\text{Diff}(\%) = \frac{(\varepsilon_{cal} - \varepsilon_{meas}) * 100}{\varepsilon_{meas}} \quad (25)$$

The squares in figs. 3 show the theoretical total efficiencies while the red circles show experimental total efficiencies.

IV. Conclusions:

Direct mathematical expressions to calculate the total efficiency of $4\pi\text{NaI(Tl)}$ scintillator detector have been derived in the case of axial point, non-axial point and extended disk and cylindrical sources. In addition, the attenuation of photons by the source container and the detector housing materials is also presented in simple straight forward mathematical expressions. The agreement between the results calculated in this work and the published values are very good, the high discrepancies being less than 4 % (point source).

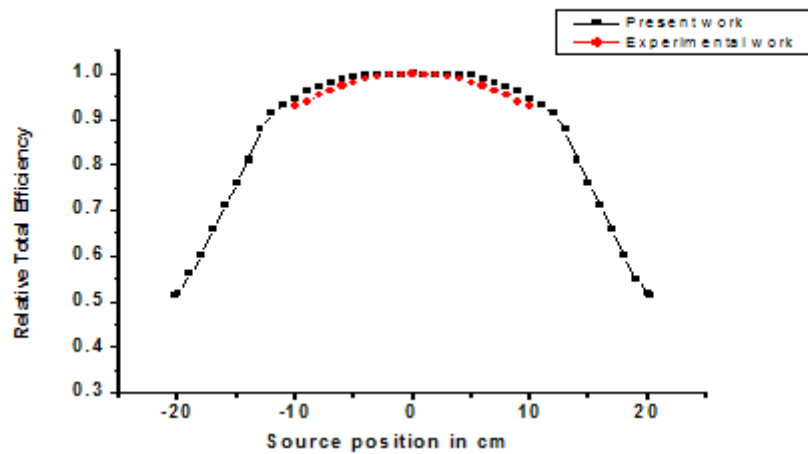


Fig. 3: Variation of relative total efficiency of a point source along the axial axis of the parallelepiped detector at energy 0.5 MeV.

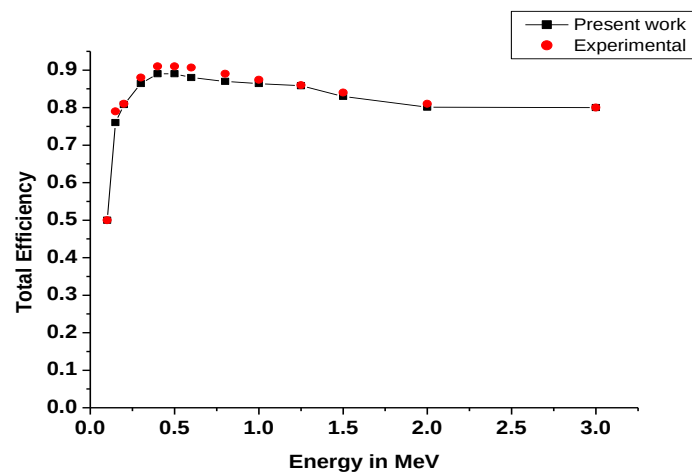


Fig. 4: Variation of total efficiency with photon's energy and its comparison with the experimental values.

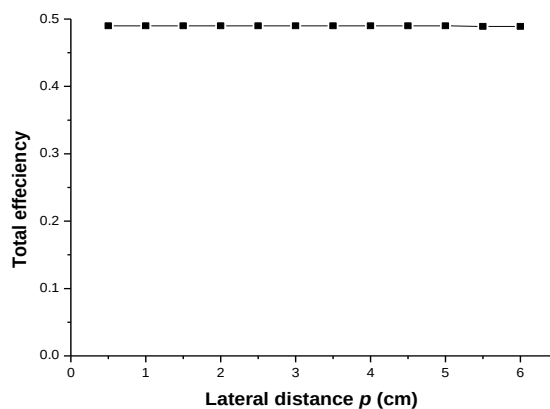


Fig. 5: Variation of total efficiency of a point source placed at the surface the detector versus the lateral distance (ρ).

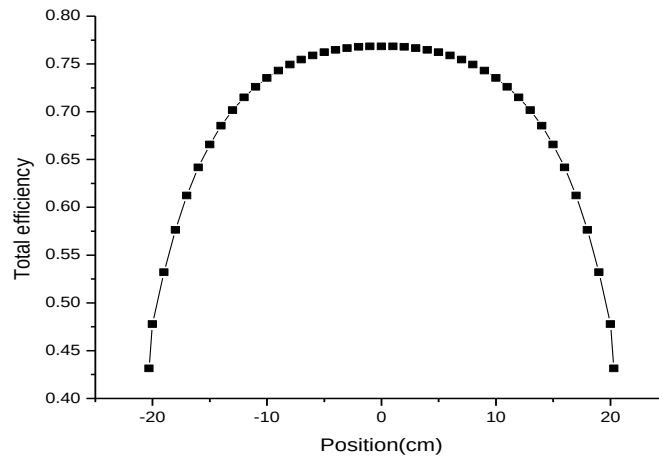


Fig. 6: Variation of total efficiency of a non-axial point source placed at different distances from the center of the detector.

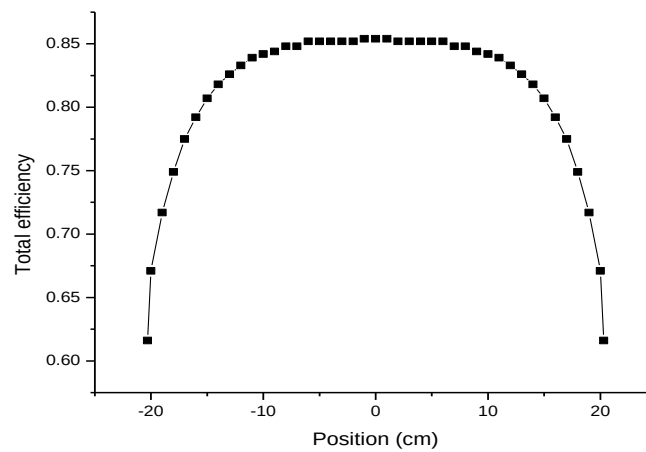


Fig. 7: Variation of total efficiency of a line source placed at different distances from the center of the detector.

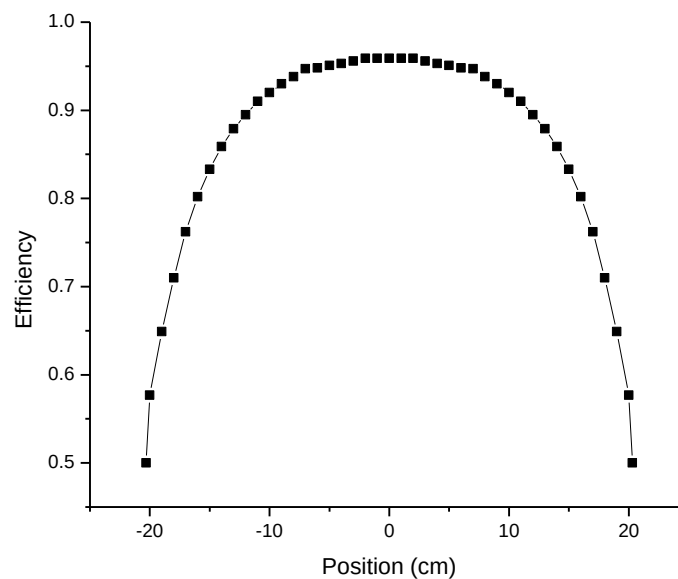


Fig. 8: Variation of total efficiency of a disc source placed at different distances from the center of the detector.

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