



Shortest Path Problem under Rough and Uncertain Environment

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Abstract: Shortest path problem in a network where the weights of edges are imprecise, many authors have developed methods considering the weights as fuzzy numbers. Stochastic approach has also been used for the purpose. In this paper, shortest path problem under uncertain environment has been studied, where the edges are attached with rough weights. Two shortest paths; α -optimistic and α -pessimistic are obtained. As a compromise shortest path, uncertain linear and uncertain zigzag distributions are used to find two other shortest paths as L-shortest path and Z-shortest path. Uncertain reliability of path is introduced and most reliable shortest path by finding k-shortest paths of minimum number of edges is obtained with a compromise of the weight.

Key words: Shortest path problem, k-shortest path, Rough variable, uncertain distribution, Rough set

1. Introduction

The shortest path problem is one of the most fundamental problems in graph theory which has many applications in the diversified field like transportation engineering, communication engineering, computer science, operations research etc. In a network, the shortest path problem aims at finding the path from one source node to the destination node with minimum weight, where some weight is attached to each edge connecting a pair of nodes. In classical graph theory, the weights attached to different edges are supposed to be definite. Many algorithms have been developed to find the shortest path where the weights are definite and deterministic in nature. In many practical applications, the parameters determine the weights of the edges are not precise and uncertain due to lack of proper information. To deal the impreciseness of the weights, fuzzy numbers are used by many authors and different methods for finding fuzzy shortest path have been proposed. The ranking and comparison of fuzzy numbers can be done in varieties ways leading to different methods to solve fuzzy shortest path problem. The paper due to Dubois and Prade[1] is one of the first on this area where they have extended the classic Floyd and Ford-Moore-Bellman algorithms under fuzzy environment. Thereafter, authors like Klein[2] obtained the shortest path corresponding to the threshold of membership degree; Yagar[3] studied the problem in terms of possibilistic production system; Okada and Soper [4] introduced the concept of non-dominated path. Xiaoyu Ji *et al* [5] using the credibility measure proposed three concepts of fuzzy shortest path: expected shortest path, α -shortest path and the most shortest path and formulated three models for the fuzzy shortest path according to different decision criteria. Yong Deng *et al* [8] proposed generalised Dijkstra's algorithm using graded mean integration representation of fuzzy numbers to find the shortest path.

To deal with the uncertainties, probability theory has also been used to find the shortest path [6-7]. However, randomness is not the only type of uncertainty in the real world problem and sometimes, the probability distributions of the weights of the edges are difficult to acquire due to lack of historical data.

In this paper, to tackle the issue of uncertain weights of the edges, rough variable is used where each edge is assigned a rough value as per the concept defined by Liu[12]. In the process, two shortest paths are obtained corresponding to α -pessimistic value and α -optimistic value of the corresponding rough weights. Further for a compromise solution, two other approaches are proposed, where linear and zigzag uncertain distributions are used. Eventually, four shortest paths are obtained where one can be selected considering the decision criteria used by the decision maker. Further, as the weights are uncertain in nature, reliability of the paths is also very important to study. We have defined uncertain reliability of the path and obtained the most reliable shortest path under uncertain weights by finding k-shortest paths having minimum number of edges.

The rest of the paper is organised as follows: Section 2 gives a brief introduction and important definitions and concepts of the basic theory used in the proposed work. The problem description is given in section 3. Two methods are proposed to find compromise solutions in section 4. Reliability analysis of shortest paths is discussed in section 5. One numerical example is given in section 6 to illustrate the efficiency of the proposed method. Lastly, the conclusion is given in section 7.

II. Preliminaries

In this section, some basic concepts on rough set, uncertainty theory and rough variable related to the work are given.

A. Rough set

Rough set theory developed by Pawlak [8-9] is a mathematical tool for dealing with uncertain and incomplete data without any prior knowledge about the data. We deal only with the available information provided by the data to generate conclusion. Let U be a finite non empty set called the universal and let R be a binary relation defined on U . Let R be an equivalence relation and $R(x)$ be the equivalence class of the relation which contains x . R shall be referred as indiscernibility relation.

For any $X \subseteq U$, the lower and upper approximations of X are defined by

$$\underline{R}(X) = \{x \in U : R(x) \subseteq X\}, \quad \bar{R}(X) = \{x \in U : R(x) \cap X \neq \emptyset\}$$

The lower approximation $\underline{R}(X)$ is exact set contained in X so that the object in $\underline{R}(X)$ are members of x with certainty on the basis of knowledge in R , where the objects in the upper approximation $\bar{R}(X)$ can be classified as possible members of X . The difference between the upper and lower approximation of X will be called as R -boundary of X and is defined by $BN_R(X) = \bar{R}(X) - \underline{R}(X)$.

The set X is R -exact if $BN_R(X) = \emptyset$, otherwise the set is R -rough set.

B. Uncertainty theory

B. Liu [11], [12] has developed uncertainty theory which is considered as a new approach to deal with indeterminacy factors when there is a lack of observed data. In this section, some basic concepts of uncertainty theory has been reviewed which shall be used in this paper.

B.1 Uncertainty measure

Let L be a δ -algebra on a nonempty set Γ . A set function $M : L \rightarrow [0,1]$ is called an uncertain measure if it satisfies the following axioms

Axiom 1: (Normality axiom) $M(\Gamma) = 1$ for the universal set Γ

Axiom 2: (Duality axiom) $M(\Lambda) + M(\Lambda^c) = 1$ for every event Λ

Axiom 3: (sub additive axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \dots$ we have

$$M\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} M(\Lambda_i)$$

The triplet (Γ, L, M) is called an uncertainty space.

Axiom 4: (Product measure) Let (Γ_k, L_k, M_k) be uncertainty spaces for $k = 1, 2, \dots$. The product uncertain measure is an uncertain measure satisfying

$$M\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \prod_{k=1}^{\infty} M(\Lambda_k)$$

where, Λ_k an arbitrary chosen events for L_k for $k = 1, 2, \dots$ respectively.

B.2 Uncertain variable

An uncertain variable ξ is essentially a measurable function from an uncertainty space to the set of real numbers. Let ξ be an uncertain variable. Then the uncertainty distribution of ξ is defined as $\phi(x) = M\{\xi \leq x\}$ for any real number x .

An uncertain variable ξ is called linear if it has linear uncertainty distribution $L(a, b)$

$$\phi(x) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } x > b \end{cases}$$

where, a and b are real numbers with $a < b$.

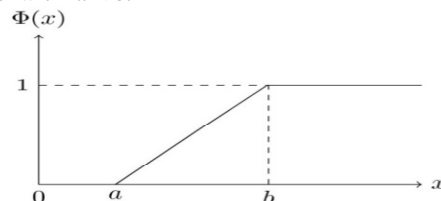


Figure 1. Linear uncertainty distribution

An uncertain variable is said to be zigzag if it has distribution $Z(a, b, c)$

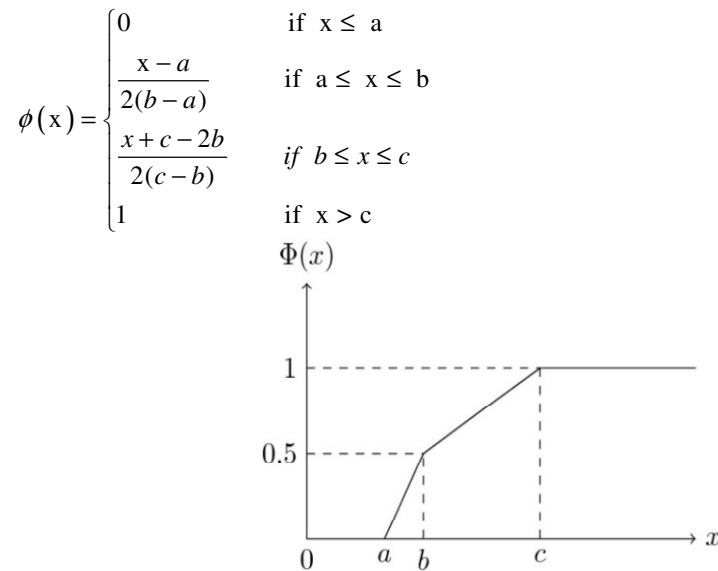


Figure 2. Zigzag uncertainty distribution

An uncertain distribution ϕ is said to be regular if its inverse function $\phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$.

The linear uncertainty distribution $L(a, b)$ is regular and its inverse uncertainty distribution is $\phi^{-1}(\alpha) = (1 - \alpha)a + (\alpha)b$

The zigzag uncertainty distribution $Z(a, b, c)$ is also regular and its inverse uncertainty distribution is

$$\phi^{-1}(\alpha) = \begin{cases} (1 - 2\alpha)a + 2\alpha b & \text{if } \alpha < 0.5 \\ (2 - 2\alpha)b + (2\alpha - 1)c & \text{if } \alpha \geq 0.5 \end{cases}$$

C. Rough variable

The concept of rough variable is introduced by Liu[12] as uncertain variable. The following definitions are based on Liu[12].

Definition 1: Let Λ be a non empty set, $\mathcal{A}$ be σ -algebra of subsets of Λ , Δ be an element in $\mathcal{A}$, and π be a non negative, real-valued, additive set function on $\mathcal{A}$. The quadruple $(\Lambda, \Delta, \mathcal{A}, \pi)$ is called a rough space.

Definition 2: A rough variable ξ on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$ is a measurable function from Λ to the set of real numbers $\mathfrak{R}$ such that for every Borel set B of $\mathfrak{R}$, we have $\{\lambda \in \Lambda \mid \xi(\lambda) \in B\} \in \mathcal{A}$. Then the lower and upper approximation of the rough variable ξ are defined as follows

Definition 3: Let $(\Lambda, \Delta, \mathcal{A}, \pi)$ be a rough space. Then the upper and lower trust of event A is defined by

$$T_{\underline{r}}(A) = \frac{\pi\{A\}}{\pi\{\Lambda\}} \quad \text{and} \quad T_{\overline{r}}(A) = \frac{\pi\{A \cap \Delta\}}{\pi\{\Lambda\}}$$

The trust of the event A is defined as

$$Tr(A) = \frac{1}{2}(T_{\underline{r}}(A) + T_{\overline{r}}(A))$$

The trust measure satisfies the followings:

$$Tr(\Lambda) = 1, \quad Tr(\emptyset) = 0$$

$$Tr(A) \leq Tr(B) \quad \text{where } A \subseteq B$$

$$Tr(A) + Tr(A^c) = 1$$

Definition 4: Let ξ_1, ξ_2 be rough variables defined on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$. Then their sum and product are defined as

$$(\xi_1 + \xi_2)(\lambda) = \xi_1(\lambda) + \xi_2(\lambda)$$

$$(\xi_1 \cdot \xi_2)(\lambda) = \xi_1(\lambda) \cdot \xi_2(\lambda)$$

Definition 5: Let ξ be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$ and $\alpha \in (0, 1]$ then

$$\xi_{\sup}(\alpha) = \sup \{r \mid \text{Tr} \{ \xi \geq r \} \geq \alpha \}$$
 is called α -optimistic value of ξ .
$$\xi_{\inf}(\alpha) = \inf \{r \mid \text{Tr} \{ \xi \leq r \} \geq \alpha \}$$
 is called α -pessimistic value of ξ .

Definition 6: Let ξ be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$. The expected value of ξ is defined by

$$E(\xi) = \int_0^\infty \text{Tr} \{ \xi \geq r \} dr - \int_{-\infty}^0 \text{Tr} \{ \xi \leq r \} dr$$

Definition 7: The trust density function $f: \mathbb{R} \rightarrow [0, \infty)$ of a rough variable ξ is a function such that $\phi(x) = \int_{-\infty}^x f(y)dy$ holds for all $x \in (-\infty, \infty)$, where ϕ is trust distribution of ξ . If $\xi = ([a, b], [c, d])$

be a rough variable such that $c \leq a < b \leq d$, then the trust distribution $\phi(x) = \text{Tr} \{ \xi \leq x \}$ is

$$\phi(x) = \begin{cases} 0 & \text{if } x \leq c \\ \frac{x-c}{2(d-c)} & \text{if } c \leq x \leq a \\ \frac{[(b-a)+(d-c)]x + 2ac - ad - bc}{2(b-a)(d-c)} & \text{if } a \leq x \leq b \\ \frac{x+d-2c}{2(d-c)} & \text{if } b \leq x \leq d \\ 1 & \text{if } x \geq d \end{cases}$$

And the trust density function is defined as

$$f(x) = \begin{cases} \frac{1}{2(d-c)} & \text{if } c \leq x \leq a \text{ or } b \leq x \leq d \\ \frac{1}{2(b-c)} + \frac{1}{2(d-c)} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

α -optimistic value to $\xi = ([a, b], [c, d])$ is

$$\xi_{\sup}(\alpha) = \begin{cases} (1-2\alpha)d + 2\alpha c, & \text{if } \alpha \leq \frac{d-b}{2(d-c)}; \\ 2(1-\alpha)d + (2\alpha-1)c, & \text{if } \alpha \geq \frac{2d-a-c}{2(d-c)}; \\ \frac{d(b-a)+b(d-c)-2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.} \end{cases}$$

α -pessimistic value to $\xi = ([a, b], [c, d])$ is

$$\xi_{\inf}(\alpha) = \begin{cases} (1-2\alpha)c + 2\alpha d, & \text{if } \alpha \leq \frac{a-c}{2(d-c)}; \\ 2(1-\alpha)c + (2\alpha-1)d, & \text{if } \alpha \geq \frac{b+d-2c}{2(d-c)}; \\ \frac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.} \end{cases}$$

The expected value of ξ is $E(\xi) = \frac{1}{4}(a + b + c + d)$

III. Problem description

Let $G = (V, E)$ be a directed graph, where $V = \{v_1, v_2, v_3, \dots, v_n\}$ be the set of nodes and E be the set of directed edges. Each edge is denoted by an ordered pair (i, j) , where $i, j \in V$.

It is supposed that there is only one directed edge (i, j) from the node i to the node j . Let the vertex 1 be the source node and node t be the destination node. We define a path p_{ij} from the node i to node j as the sequence of alternate nodes and edges as $\{i, (i, i_1), i_1, (i_1, i_2), i_2, \dots, (i_{k-1}, i_k), i_k, (i_k, j), j\}$.

Let $w_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$ be the rough number associated with the edge (i, j) as the uncertain weight of the edge. The weight may be the distance, time required to traverse through the edge, cost of travelling etc. Then the uncertain shortest path problem is to find one path from the source node 1 to destination node t such that the sum of weights of the edges in the path is minimum. When the weights attached to the edges are crisp, then the famous Dijkstra's algorithm can be applied to find such a path. In this case, as the weights are rough number and comparison of weights as in case of crisp number is not possible. Therefore it is necessary to convert the rough number to its equivalent crisp number so that the said algorithm can be used to find the shortest path. The one possible technique is to compare the weights of the edge by considering the corresponding α -optimistic values and α -pessimistic values of the rough weights.

Let α_{ij} and β_{ij} be the α -optimistic values and α -pessimistic values of the rough weights $w_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$, where $c_{ij} \leq a_{ij} < b_{ij} \leq d_{ij}$, considering the trust level (level of confidence) as α in the interval $(0, 1]$

One shortest path can be obtained from the source node 1 to the destination node t considering the α -optimistic values α_{ij} , $i, j = 1, 2, 3, \dots, n$ and $i \neq j$ which can be considered as optimistic shortest path. Similarly, another shortest path can be obtained using the α -pessimistic values β_{ij} , which can be considered as pessimistic shortest path. The two paths may coincide but the total weights of the arcs in the path shall be different; one corresponding to the optimistic values and the other corresponding to the pessimistic values.

Here two problems shall arise: (i) If the paths are different, the decision maker always likes to find one path instead of choosing one from two possible paths, (ii) As the weights are uncertain in nature, the decision maker must be sure about the reliability of the path under uncertain environment. In the next two sections, we proposed some approaches to get the compromise solution to the problem and the most reliable shortest path.

IV. Compromise solution

In the previous discussion, two shortest paths are obtained considering the α -pessimistic values and α -optimistic values of the rough weights. But, sometimes the decision maker may want to have one shortest path with certain compromise with the optimistic or pessimistic values. In this section we propose two compromise shortest paths.

A. L-shortest path

To deal with uncertainties of the weights attached to the edges, rough weights are attached to each edge.

Let α_{ij} = α -optimistic value of the rough weight w_{ij} attached to the edge (i, j)

β_{ij} = α -pessimistic value of the rough weight w_{ij} attached to the edge (i, j)

Then the interval $(\alpha_{ij}, \beta_{ij})$ can be considered as an uncertain interval in which the value of the actual weight lies with trust level α .

We define an uncertain variable ξ in the interval $(\alpha_{ij}, \beta_{ij})$ whose distribution function is defined as a linear regular uncertain distribution as defined by Liu[11]

$$\phi(x) = \begin{cases} 0 & \text{if } x \leq \alpha_{ij} \\ \frac{x - \alpha_{ij}}{\beta_{ij} - \alpha_{ij}} & \text{if } \alpha_{ij} \leq x \leq \beta_{ij} \\ 1 & \text{if } x > \beta_{ij} \end{cases}$$

The inverse linear uncertain distribution will give a crisp value w_{ij} as

$$\phi^{-1}(w_{ij}, \alpha) = (1 - \alpha)\alpha_{ij} + \alpha\beta_{ij}$$

In the above process, a set of deterministic weights can be generated from the rough weights of the edges.

Using these crisp values, we can obtain a shortest path using Dijkstra's algorithm which can be considered as a compromise path, where the total weight of the path may be greater than the optimistic value and less than the pessimistic value. This path may be considered as L-shortest path.

4.2 Z-shortest path

Another compromise shortest path can be obtained using zigzag uncertain distribution.

If $w_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$ be the rough weight attached to the edge (i, j) , then the expected value of w_{ij} is defined as

$$\gamma_{ij} = E(w_{ij}) = (a_{ij} + b_{ij} + c_{ij} + d_{ij})/4$$

So, the rough weight of the edge corresponds to a triplet $(\alpha_{ij}, \gamma_{ij}, \beta_{ij})$, where α_{ij}, β_{ij} are α -optimistic and α -pessimistic values respectively.

Corresponding to this triplet we can define zigzag uncertainty distribution $Z(\alpha_{ij}, \gamma_{ij}, \beta_{ij})$ whose distribution function is defined as (Liu[11])

$$\phi(x) = \begin{cases} 0 & \text{if } x \leq \alpha_{ij} \\ \frac{(x - \alpha_{ij})}{2(\gamma_{ij} - \alpha_{ij})} & \text{if } \alpha_{ij} \leq x \leq \gamma_{ij} \\ \frac{(x + \beta_{ij} - 2\gamma_{ij})}{2(\beta_{ij} - \gamma_{ij})} & \text{if } \gamma_{ij} \leq x \leq \beta_{ij} \\ 1 & \text{if } x \geq \beta_{ij} \end{cases}$$

The inverse zigzag uncertain distribution will generate a crisp value w_{ij} as

$$\phi^{-1}(w_{ij}, \alpha) = \begin{cases} (1 - 2\alpha)\alpha_{ij} + 2\alpha\gamma_{ij} & \text{if } \alpha < 0.5 \\ (2 - 2\alpha)\gamma_{ij} + (2\alpha - 1)\beta_{ij} & \text{if } \alpha \geq 0.5 \end{cases}$$

In the above process, a set of deterministic weights can be generated from the rough weights of the edges.

Using these crisp weights and using Dijkstra's algorithm, we can obtain a shortest path which can also be considered as a compromise shortest path. This path may be considered as Z-shortest path.

V. Reliability analysis

Out of the shortest paths proposed, the decision maker may prefer the one which is more reliable or less uncertain.

Y. H. Kim *et al* [13] have studied the reliability of a directed path in an oriented network. Initially, they have determined the directed paths from the connection matrix and computed reliability using the basic theory of probability of events.

If R_{stj} denotes the j th path from source v_s to destination v_t , then the probability of success P_j , of the path R_{stj} is given by the product of the reliabilities of the links in series which identify the path R_{stj} .

$$P_j = \prod_{l_i \in R_{stj}} p_i, \text{ where } \{l_i \in R_{stj}\} \text{ is the set of links in path } R_{stj}$$

If the path do not contain any common links, (that is, the path are parallel), then the reliability of the system is obtained directly from

$$R = 1 - \prod_{i=1}^k (1 - P_i)$$

where, k is the number of links from v_s to v_t .

As per the above definition of Kim *et al*[13], we define the uncertain reliability of path in a network as below. Considering the rough variable as $([a, b], [c, d])$ with $c \leq a < b \leq d$, we define the measure of uncertainty as

$$\text{Uncertain measure} = M = \frac{q - p}{d - c}$$

where, q and p are the α -pessimistic and α -optimistic values respectively. It is obvious that if the difference between the α -pessimistic and α -optimistic values is less, then the value is more reliable.

We define the uncertain reliability of the rough variable as

$$\text{Uncertain reliability} = P = 1 - \frac{|q - p|}{|c - d|}$$

so that $0 < P < 1$

The uncertain reliability of the path is defined as

$$R = 1 - \prod_{i=1}^k (1 - P_i)$$

where, k is the number of edges in the path from the source node to destination node and P_i is the uncertain reliability of the i th edge in the path.

Further, to find the most reliable path, we find k -shortest paths of minimum number of edges by using any standard algorithm. We consider the minimum number of edges in the k -shortest paths, because if number of edges increases, the weight of the path shall increase substantially though the reliability of the path increases. So

for better comparison of the paths so far as reliability is concerned, we select k -shortest paths of minimum number of edges. The reliability of the each type of paths can be obtained and the decision maker has got one more choice to select the most reliable path (having maximum reliability) with a compromise of the weights.

VI. Numerical example

In this section, we have given one numerical example to illustrate our proposed shortest paths. Xiaoyu *et al* [5] have taken one numerical example, where they have taken trapezoidal fuzzy number as the uncertain weight of the edges which has also been used by Yong Deng *et al* [8] in their paper. We have taken up the same problem for a better comparison. They have taken (a, b, c, d) as trapezoidal fuzzy number where $a < b < c < d$. We have consider the rough variable as $([b, c], [a, d])$. The graph is given in Fig-1, the rough weights and the related parameters are given in Table 1. The source node is 1 and the destination node is 23.

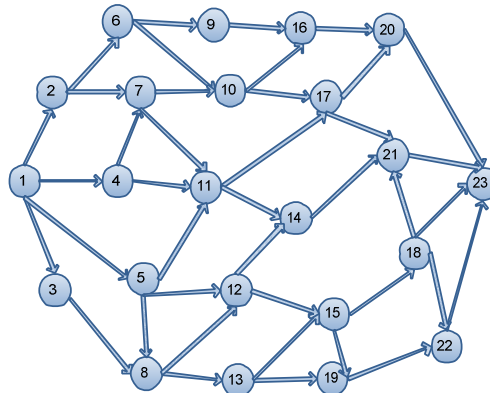


Figure 3: Network

Table 1 Different values ($\alpha=0.8$)

Edge	Weight	α -optimistic value	α -pessimistic value	Average value	$\Phi^{-1}(\alpha)$ for $L(a, b)$	$\Phi^{-1}(\alpha)$ for $Z(a, b, c)$	Reliability
(1,2)	([13,15],[12,17])	13.2857	15	14.25	14.657	14.7	0.6571
(1,3)	([11,13],[9,15])	11.1	12.9	12	12.54	12.54	0.7
(1,4)	([10,12],[8,13])	10	11.7143	10.75	11.3714	11.3286	0.6571
(1,5)	([8,9],[7,10])	8.05	8.95	8.5	8.77	8.77	0.7
(2,6)	([10,15],[5,16])	9.4	13.937	11.5	13.03	12.962	0.5875
(2,7)	([11,12],[6,13])	8.8	11.775	10.5	11.18	11.265	0.575
(3,8)	([11,16],[10,17])	11.75	15.25	13.5	13.5	14.55	0.5
(4,7)	([20,22],[17,24])	19.8	21.822	20.75	21.4178	21.393	0.711
(4,11)	([10,13],[6,14])	9.2	12.4	10.75	11.76	11.74	0.6
(5,8)	([9,11],[6,13])	8.8	10.822	9.75	10.4178	10.393	0.7111
(5,11)	([10,13],[7,14])	9.8	12.46	11	11.928	11.876	0.62
(5,12)	([13,15],[10,17])	12.8	14.86	13.75	14.4178	14.393	0.7111
(6,9)	([8,10],[6,11])	8	9.714	8.75	9.37143	9.3285	0.6571
(6,10)	([11,14],[10,15])	11.375	13.625	12.5	13.175	13.175	0.55
(7,10)	([10,12],[9,13])	10.2	11.8	11	11.48	11.48	0.6
(7,11)	([7,8],[6,9])	7.05	7.95	7.5	7.77	7.77	0.7
(8,12)	([8,9],[5,10])	7.0	8.833	8	8.4667	8.5	0.6333
(8,13)	([5,8],[3,10])	5.24	7.76	6.5	7.256	7.256	0.64
(9,16)	([7,9],[6,10])	7.2	8.8	8	8.48	8.48	0.6
(10,16)	([13,16],[12,17])	13.375	15.625	14.5	15.175	15.175	0.55
(10,17)	([19,20],[15,21])	17.4	19.8	18.75	19.32	19.38	0.6
(11,14)	([9,11],[8,13])	9.2857	11	10.25	10.657	10.7	0.657
(11,17)	([9,11],[6,13])	8.8	10.822	9.75	10.417	10.393	0.7111
(12,14)	([14,16],[13,18])	16	16.285	15.25	15.657	15.7	0.657
(12,15)	([14,15],[12,16])	13.6	14.88	14.25	16.624	14.628	0.68
(13,15)	([12,14],[10,15])	12	13.714	12.75	13.371	13.328	0.657
(13,19)	([18,19],[17,20])	18.05	18.95	18.5	18.77	18.77	0.7
(14,21)	([12,13],[11,14])	12.05	12.95	12.5	12.77	12.77	0.7
(15,18)	([9,11],[8,13])	9.285	11	10.25	10.657	10.7	0.657
(15,19)	([7,10],[5,12])	7.24	9.76	8.5	9.256	9.256	0.64
(16,20)	([12,14],[9,16])	11.8	13.822	12.75	13.4178	13.393	0.7111
(17,20)	([10,11],[7,12])	9	10.833	10	10.466	10.5	0.633
(17,21)	([7,8],[6,10])	7.12	8.4	7.75	8.144	8.14	0.68
(18,21)	([17,18],[15,19])	16.6	17.88	17.25	17.624	17.628	0.68

(18,22)	([5,7],[3,9])	5.1	6.9	6	6.54	6.54	0.7
(18,23)	([7,9],[5,11])	7.1	8.9	8	8.54	8.54	0.7
(19,22)	([16,17],[15,19])	16.12	17.4	16.75	17.144	17.14	0.68
(20,23)	([14,16],[13,17])	14.2	15.8	15	15.48	15.48	0.6
(21,23)	([15,17],[12,18])	14.4	16.65	15.5	16.2	16.19	0.625
(22,23)	([5,6],[4,8])	5.12	6.4	5.75	6.144	6.14	0.68

Using Dijkstra's algorithm, four shortest paths are obtained taking $\alpha = 0.8$. Using Yen algorithm [15] we have found k-shortest paths having same number of minimum edges. The table 2 shows the different shortest paths, their weights under different parameters and their reliability. So the decision maker has option to choose the one as per his choice.

Table 2: Types of Shortest paths

Type of path	Path	Weight	Reliability of the path
α- optimistic			
1 st shortest path	1-5-11-17- 21-23	48.17	0.9969
2nd shortest path	1-5-11-17-20-23	49.85	0.9951
3rd shortest path	1-5-12-15-18-23	50.84	0.9971
α- pessimistic			
1 st shortest path	1-5-11-17- 21-23	57.282	0.9960
2nd shortest path	1-5-12-15-18-23	58.59	0.9971
3rd shortest path	1-5-11-17-20-23	58.865	0.9951
Linear			
1 st shortest path	1-5-11-17- 21-23	55.459	0.9969
2nd shortest path	1-5-11-17-20-23	57.061	0.9951
3rd shortest path	1-5-12-15-18-23	59.008	0.9971
4th shortest path	1-5-11-14-21-23	60.325	0.9956
Zigzag			
1 st shortest path	1-5-11-17- 21-23	55.369	0.9969
2nd shortest path	1-5-11-17-20-23	57.019	0.9951
3rd shortest path	1-5-12-15-18-23	57.031	0.9971
4th shortest path	1-5-11-14-21-23	60.306	0.9956

From this numerical example, it is seen that in all four types one path 1-5-11-17-21-23 comes as the shortest path. But so far as reliability is concerned, another path 1-5-12-15-18-23 becomes the most reliable shortest path in all four types

VII. Conclusion

In this paper, we have proposed four types of shortest paths from the source node to destination node where the edges are attached with rough weights. Initially, α - optimistic and α - pessimistic shortest paths are obtained as two possible shortest paths. Further, two models are proposed for one compromise shortest path using linear and zigzag uncertain distribution. Then uncertain reliability of the path is defined and reliability of the paths is obtained finding k-shortest paths of minimum number of edges in all types. It is found that one path in all four cases comes to be the shortest path. But another path in all four cases comes to be the most reliable shortest path, though the weight is slightly high. So the option lies with the decision maker to choose either of the paths. Further, the result obtained in this work is found to be better than the result obtained by Xiaoyu *et al* [5] by suitably converting the fuzzy trapezoidal numbers in to rough number.

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