



Correlation between electrical resistivity and water content of sand – a statistical approach

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Abstract: The civil engineering structures are built in, on or with soils and its engineering behaviour has a direct bearing with water content. Cohesion-less soil such as saturated fine to medium sand is most susceptible and least resistant to liquefaction and may cause damage to the overlying structures. Electrical resistivity measurements are nowadays widely used for geotechnical characterisation of soil as they are fast, cost effective and non destructive. The paper presents the results of electrical resistivity tests conducted on prepared samples of sand under varying moisture contents using direct current electric supply and attempts to establish correlation between water content of soil with its electrical resistivity by detailed statistical analyses of test data. The correlation may be useful in determining the water content by observing electrical resistivity. The study by observing variation in electrical resistivity and thereby water content at different point of time could be useful in studying the underground seasonal variation in water content of soil and effects therein due to ground motions on occurrence of earthquake and liquefaction resistance potential of sub surface sand layers. Results indicate that the non linear regression model is determined to be a more precise analytical model.

Keywords: Degree of saturation, electrical resistivity, geophysical methods, geotechnical characterization, sand, water content.

I. Introduction

The physical characteristics and properties of soil are of prime concern for civil engineers and need to be determined for design and construction of all the civil structures as they are built in, on or with soils. The suitability of soil for use as foundation or construction material is evaluated by determination of its index and engineering properties.

The engineering behaviour of soil such as compaction, strength or load bearing capacity, stability of earth works and foundations, settlement etc. has a direct bearing with the water content present in it. The natural water content/moisture content also characterises the performance of sub surface soil under the action of load. The water content may vary from trace quantity to the saturation limit. The techniques which are used for their determination are laboratory tests, in-situ tests and geophysical methods. The undisturbed or disturbed samples are acquired for tests from field through borehole sampling and this method of sampling is found to be invasive, costly and time consuming. The sample is then tested in the laboratory by oven-drying method or other recommended methods. There are a few in situ, on site quick methods of water content determination but they are described as either crude or have certain limitations.

The Geophysical techniques are nowadays considered as an attractive soil testing tool for geotechnical characterisation as they are fast, cost effective, non destructive and disturb the soil structure to least. The electrical resistivity method of soil exploration is one of such geophysical techniques that are useful in demarcating subsurface properties and estimating the depths of soil/bedrock surfaces [7].

The soils and rocks are essentially nonconductive, except some metallic ores. The electrical conduction in electrolytic solutions, moist soil and water bearing rocks takes place as a result of the movement of ions. The ability to transmit ions is governed by the electrical resistivity a basic property of all the materials and has been demonstrated to be an effective predictor of various soil properties including salinity [12] porosity and water content [3], [13], [2]. For soils, electrical resistivity depends on many factors such as porosity, electrical resistivity of the pore fluid, composition of the solids, degree of saturation, particle shape and orientation, and pore structure [6], [10], [1], [15], [14].

The earthquake prone zones having high water table and soil as fine to medium sands, the stratification is vulnerable to liquefaction and may cause damages to the overlying structures by settlement, failure of foundation and lateral movement of slopes etc. Liquefaction resistance of such soils in partially saturated condition is sensitive to the slight change of the saturation. Reference [8] reported that the liquefaction resistance of partially saturated sand having about 70% of the degree of saturation is more than three times to that of the completely saturated sand. Hence determination of water content/degree of saturation of such soils would be helpful in assessing the liquefaction countermeasure potential. The electrical resistivity along with the

depth and thickness of partially saturated sand layer by Vertical Electrical Sounding (VES) is one of the fast methods of assessing water content determination in such a case.

The paper presents an attempt to propose a correlation between electrical resistivity and water content, obtained through laboratory tests, on sand samples. These samples are prepared at a constant dry density & varying water content from dry to saturation condition. The electrical resistivity is determined in the laboratory under direct current (DC) supply of electrical potential. The DC electric supply is commonly used for determination of electrical resistivity in the field.

These correlations may be useful in determining the water content, with the help of in situ electrical resistivity values measured by Vertical Electrical Sounding method. This will be helpful in studying the underground seasonal variation in water content and probable effects therein due to ground motions on occurrence of earthquake.

An attempt has been made in the past by [4] to correlate water content and electrical resistivity during the study to find out the effects of earthquake in the Istanbul and Golcuk areas after its occurrence in northwest Turkey in the year 1999. The electrical resistivity was measured by VES method by McPHAR resistivity equipment on the field and water content was determined in the laboratory on the field samples. In the study carried out on sandy soil [4], the relationship between soil water content and electrical parameters was obtained by exponential curvilinear model and is given by $w = 51.764 e^{-0.0188R}$ where R = Electrical Resistivity and regression R Square = 78%. The ranges of the results varied between 1 - 50 ohm-m for resistivity and 20 - 60% for water content.

In the present study the correlation between electrical resistivity and water content has been established by experimental studies on laboratory samples of fine sand by taking measurement of electrical resistivity at varying water content and analysis of results by statistical method.

II. Electrical Resistivity Method

The electrical resistivity is the resistance measured between two opposing surfaces of a 1m^3 cube of homogeneous soil material usually measured in $\Omega\text{-m}$ or $\Omega\text{-cm}$. If a material has a Resistance = r (Ohm), Cross Sectional Area = A (m^2) and a Length = L (m) then Electrical Resistivity = ρ ($\Omega\text{-m}$) is given by

$$\rho = \frac{rA}{L} \quad (1)$$

The usual practice is to apply direct or alternating current between two electrodes and measure the difference of potential between two other electrodes implanted in the ground that do not carry current. The theory and field methods used for resistivity surveys are based on the use of Direct Current, because it allows greater depth of investigation than Alternating Current (AC) and it avoids the complexities caused by effects of ground inductance and capacitance and resulting frequency dependence of resistivity.

The two general methods viz. Vertical Electrical Sounding (VES) and Electrical Profiling (EP) are employed in practice for making resistivity observations. First method is used for investigating the change of apparent resistivity at a location with depth while Electric Profiling method is employed for investigating its change up to a depth involved in longitudinal direction. These techniques employ a 4-electrode configuration and two types of electrode spacing arrangements are commonly in use:

- (a) Wenner Arrangement (b) Schlumberger Arrangement

a). Wenner Arrangement:

The four equally spaced (Distance = a) electrodes are driven in the ground along a straight line and a current of electricity (I) is established between the two outer electrodes. The potential voltage (V) is then measured between the two inner probes. A simple Ohm's Law equation determines the resistance (r). The Apparent Resistivity (ρ) of the ground is given by:

$$\rho = 2\pi ar \quad \text{or} \quad \rho = \frac{2\pi a^2 V}{I} \quad (2)$$

b). Schlumberger Arrangement:

In this type of arrangement two external Current electrodes have spacing = $l/2$ and internal Potential electrodes have a spacing of $b/2$ from centre of the four electrode arrangement. Theoretically $l \gg b$ but for practical application good results can be obtained if $l \geq 5b$. The apparent Resistivity is given by:

$$\rho = \frac{\pi \left[\left(\frac{l}{2} \right)^2 - \left(\frac{b}{2} \right)^2 \right] V}{bI} \quad (3)$$

Field operations are faster with the Schlumberger array because the outer electrodes are only moved as against the movement of all the four electrodes in the Wenner array. However the Wenner array requires less instrument sensitivity and reduction of data is marginally easier.

III. Model Preparation and Testing Procedure

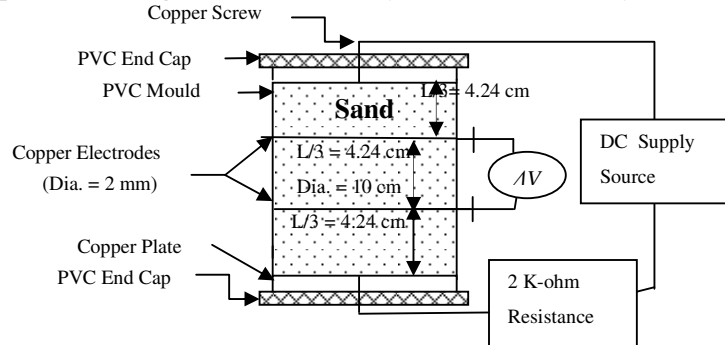
The clean, oven dried fine sand (Specific Gravity = 2.7) passing through a 425 micron IS sieve and bore-well water (pH = 7.50) is taken for preparing all the specimens. The specific gravity of oven dried sand is determined by pycnometer method and pH of water was measured in the pH meter in the laboratory. The predefined

quantity of water is added in sand and is kept in plastic bags for 24 hours for maturing. This wet sand of known water content is used for filling in the P.V.C. mould in three layers for performing electrical resistivity tests. The weight of wet sand required to fill the mould is calculated by its bulk density (γ) given by $\gamma = \gamma_d (1 + w)$ where 'w' is the water content of the sample and ' γ_d ' is the dry density of sand which is kept constant throughout the experiment and is 15.5 kN/m^3 .

To measure the vertical electrical resistivity of compacted sand in the laboratory the test set up as shown in Figure 1 was used. It is similar to that used by Zeyad S. et al Ref. [16] with some modifications. The set up consists of a cylinder having a size of compaction mould (diameter = 10 cm, length = 12.7 cm) and made of polyvinyl chloride (PVC) of very high resistivity. The two ends of the cylinder are pressed through copper plates of a dia. of 10 cm which work as end electrodes to carry current to induce one dimensional electric field through the specimen. The electric power supply is used at a constant frequency of 50 Hz.

Two copper electrodes (Dia.= 2 mm) are pressed through the small holes in the sand specimen (Cross Sectional Area = A Sq.m.) at a spacing of $L = 0.0424 \text{ m}$ from two ends to divide the specimen into three equal parts and to simulate Wenner four electrode arrangement.

Fig.1 Setup for Measuring Electrical Resistivity of Sand based on Zeyad S. et al (1996)



The resistivity test is performed by applying current by electric source through two end copper plates in the circuit. The supplied DC voltage is varied from 0-30 volts in a step of 5 volts and electrical resistivity of soil is measured. The voltage drop (ΔV) between these two electrodes is measured for determining the electrical resistivity of soil. A voltmeter having an accuracy of 0.001 V to measure minimum electrical resistivity of about 1 ohm-m is used for measuring the potential difference between the copper rods on establishing an electrical field. The current (I) is measured indirectly by measuring a voltage drop across a 2 k Ω resistor inserted in series with the specimen using ohm's law. Assuming that the electric field is one-dimensional and using ohm's law the vertical electrical resistivity (ρ) in $\Omega\text{-m}$ is then computed from:

$$\rho = \frac{rA}{L} = \frac{\Delta VA}{IL} \quad (4)$$

The small quantity of sand from the specimen is taken out and water content is determined by oven drying method which is defined as the ratio of amount of water present to its dry mass. The other sand samples are prepared likewise for resistivity tests at different water contents varying from dry to saturated condition.

Within the scope of this study, 12 test samples of sand at a constant dry density of 15.5 kN/m^3 are taken for analysis, which are having water content varying from dry state to saturation state.

IV. Statistical Analysis of Experimental Results

A. Descriptive Statistics

The electrical resistivity of oven dried sand sample and at a low water content of 1.02% could not be determined under 0-30 volts of DC electric supply and therefore these samples are not taken into account for statistical analysis. For the remaining ten samples the values obtained for electrical resistivity at 30V of DC electric supply in the laboratory at varying water content have been used for analysis. The general properties of tested samples are given in Table I.

Table I General properties of tested samples

Soil Classification	Fine Sand
Water Contents - w (%)	4.17 – 27.85
Degree of Saturation S_r (%)	15.19 – 99.66
Dry Density - γ_d (g/cc)	1.55
Electrical Resistivity (30V-DC) – $\Omega\text{-m}$	49.723 – 297.666

The sample size is 10 and confidence level was selected by 95%. Descriptive statistical parameters for testing results are given in Table II. Calculated electrical resistivity (ER) value for 30V DC supply varied between 49.723–297.666 ohm-m and the mean is 140.913. The Standard deviation of electrical resistivity is 89.498. The skewness characterises the degree of asymmetry of a distribution around its mean and its value is +0.928 for electrical resistivity, which implies that the asymmetry degree of sample distribution around arithmetic mean is

closer to positive value and tail extends toward more positive value. Since kurtosis of a data set has a negative value of -0.480 it characterises a relatively flat distribution of electrical resistivity.

Table II Descriptive statistics of experimental results

Parameter	WC (%)	DoS (%)	ER (30V DC) Ohm-m
Mean	14.874	53.368	140.913
Geometric Mean	13.068	46.931	117.980
Standard Error	2.360	8.453	28.302
Median	13.045	46.646	108.677
Mode	-----	-----	-----
Standard Deviation	7.462	26.731	89.498
Sample Variance	55.679	714.542	8009.849
Kurtosis	-0.572	-0.620	-0.480
Skewness	0.488	0.484	0.928
Range	23.680	84.470	247.943
Minimum	4.170	15.190	49.723
Maximum	27.850	99.660	297.666
Sum	148.737	533.677	1409.126
Count	10	10	10
Confidence Level (95.0%)	5.338	19.122	64.023

Since the sample size for determination of ER is less than 30 and is small, the assumption of normality is not true and t-distribution is applied. The null hypothesis (H_0) that the difference between sample mean $\bar{X}$ and population mean μ (Mu) is not significant for two tailed test, the range of population mean is calculated from the sample means for t critical value [2.262]. Consequently, for tested sand samples representative values for 95% confidence level and 9 degree of freedom for water content lying between [9.536,20.212] the electrical resistivity will lie between [76.89, 204.936].

The Pearson correlation coefficient is a measure of how electrical resistivity varies with water content. The correlation coefficient [-0.869] is very good and implies that these two variables move together and the negative sign shows that the small values of water content are associated with large values of electrical resistivity.

B. Linear Regression Model of Electrical Resistivity with Water Content:

A linear regression analysis was performed for the mapping of electrical resistivity with respect to water content (variable). The obtained relation as a result of regression analysis for electrical resistivity is as follows:

$$\rho = -10.424 w + 295.952 + \varepsilon \quad \text{Where } \varepsilon = \text{Standard Error} \quad (5)$$

Descriptive statistical parameters of performed linear regression analysis are presented in Table III. The R^2 value for water content and electrical resistivity is 0.76 which indicates a good correlation between both data sets. In order to determine whether these results with such a very good R^2 value occurred by chance, the F-statistic is used. The calculated F value for 95% confidence level [24.691] is much higher than F critical value of 5.318 obtained by F distribution curve. Thus the hypothesis that there is no relationship between ER and WC may be rejected. The probability of a higher F value occurring by chance [0.00109] is also extremely small. It can be concluded that the regression equation is useful in predicting the assessed value of ER.

Table III Linear regression statistics for electrical resistivity with water content

Summary Output:

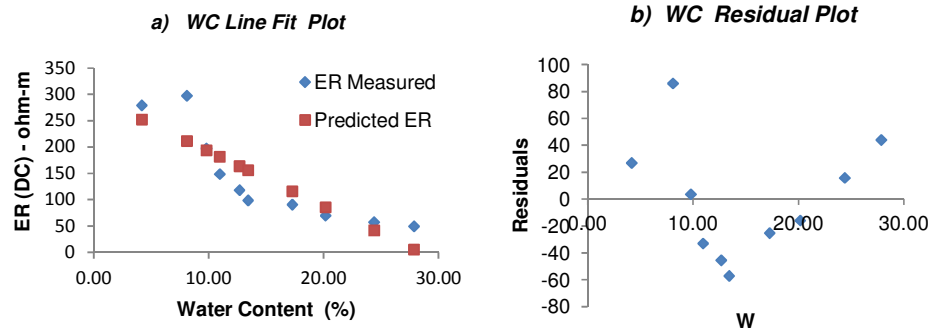
Parameter	Value
Multiple R	0.870
R Square	0.755
Adjusted R Square	0.725
Standard Error (ε)	46.959
Observations	10
P Value	0.00109
Numerator Degree of Freedom	1
Denominator Degree of Freedom	8
F (For 95%)	24.691
F _{critical} (For 95%,1 and 8)	5.318
t _{critical} (for 95% and 8)	2.306

To determine whether water content slope coefficient is useful in estimating the assessed value of ER, t statistics is used and information is presented in Table 4. The absolute t value for variable WC [4.969] is greater than t-critical value of 2.306 (for 95% confidence level). This demonstrates that **WC is an important variable and plays an important role in the developed linear regression model**. For the illustration of the performance of linear regression model, scatter and residual plots are given in Fig.2a and 2b.

Table IV. Summary of t-statistics for electrical resistivity

	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
WC vs. ER:					
Intercept	34.555	8.565	2.663E-05	216.2688	375.6353
WC	2.0977	-4.9690	0.001094	-15.2611	-5.5863

Fig. 2 Scatter and residual plots of WC variable for ER linear regression model



C. Non Linear Regression Model of Electrical Resistivity with Water Content variable:

The non linear behaviour of model data has also been considered and regression analysis was conducted to establish relation between the electrical resistivity and water content taking water content as variable and electrical resistivity as variable.

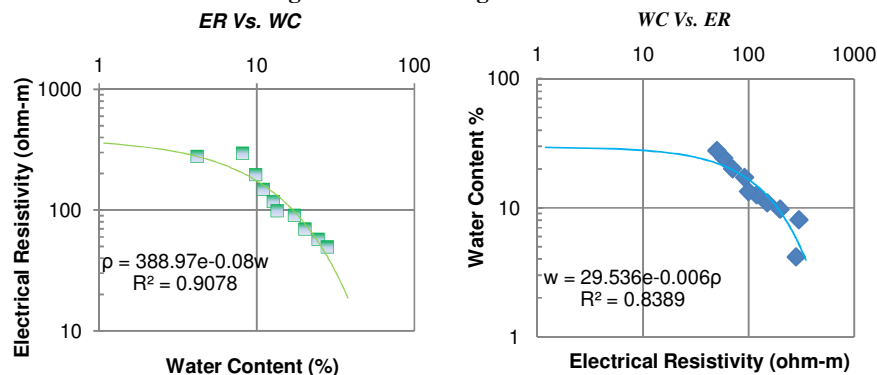
C.1 Electrical Resistivity vs. Water Content (variable)

The analysis produced better result and the regression statistics is shown in the Table V. The equation obtained of an exponential curve that fits the data and gives values that describes the curve shown in Fig.3a is given below:

$$\rho = 388.97 e^{-0.08w} + \varepsilon \quad (6)$$

The R^2 value for correlation between ER and WC is 0.908 which indicates a good correlation between both data sets. The calculated F value (95% confidence level) is [78.768] which is much higher than critical F value [5.32] obtained by F distribution curve. The probability of a higher F value occurring by chance 0.0000205 is also extremely small and hence **it demonstrates that the regression equation is useful in predicting the assessed value of Electrical Resistivity.**

Fig. 3 Non-linear regression model



3a) Plot of WC variable for ER

3b) Plot of ER variable for WC

Table V Logarithmic regression statistics for water content with electrical resistivity

Parameter	WC Variable	ER Variable
Multiple R	0.9528	0.9159
R Square	0.9078	0.8389
Standard Error (ε)	0.2023	0.2407
Observations	10	10
P Value	0.0000205	0.000197
Numerator Degree of Freedom	1	1
Denominator Degree of Freedom	8	8
F (For 95%)	78.768	41.662
F _{critical} (For 95%,1 and 8)	5.318	5.318
t _{critical} (for 95% and 8)	2.306	2.306

C.2 Water Content vs. Electrical Resistivity (variable):

Further, the statistical analysis of data has been performed to establish the correlation between water content and considering electrical resistivity to be a variable. This will be helpful in determining the water content by

measuring the electrical resistivity on the field by VES. The regression statistics is shown in the Table 5. The equation obtained of an exponential curve that fits the data and gives values that describes the curve shown in Fig. 3b is given below:

$$w = 29.536 e^{-0.006\rho} + \epsilon \quad (7)$$

The non linear regression model gives the R^2 value of 0.839 which indicates a good correlation between both data sets. The calculated F value (95% confidence level) is [41.662] which is much higher than critical F value [5.32] obtained by F distribution curve (Table V). The probability of a higher F value occurring by chance 0.00019 is also extremely small and hence **it demonstrates that the regression equation is useful in predicting the assessed value of Water Content.**

V. Results and Discussion

The electrical resistivity of sand sample in the dry state and at very low water content of 1.02% was not reported during the laboratory test. The non-reporting of electrical resistivity in the dry state is due to non conduction of electric current. Electrical conduction in clean sands and gravels occurs primarily in liquid contained in the pores [5]. "Solid soil components are generally electrical insulators, the conduction of electrical current only lies on phenomenon occurring in water [11], [9]. Also, the other sample having very low water content did not show the electrical resistivity value and is attributed to having water content below critical value " S_{cr} " corresponding to the minimum amount of water required to maintain a continuous film of water surrounding the solid particles [10].

The experimental results obtained through laboratory tests on sand samples for geo-electrical values corresponding to different water content under 30V DC supply of voltage at a constant dry density of samples and there statistical analysis showed that water content and electrical resistivity showed a very good correlation and electrical resistivity of sand is found to be decreasing on increase of water content. The value for ER fell by 248 ohm-m on increase of water content by 23.68%.

The experimental study has shown that the correlation $\rho = 388.97e^{-0.08w} + \epsilon$ or $w = 29.536 e^{-0.006\rho} + \epsilon$ between water content and electrical resistivity, established through non linear regression model by statistical approach has very good R square value and is therefore determined to be a more precise analytical model as compared to linear model. .

VI. Conclusions

This experimental study thus demonstrated that an increase in the soil water content results in a decrease in the electrical resistivity of soil and a good correlation exists between them. The resistivity is thus a good indirect predictor of water content. The non linear regression model developed is proved to be a better predictor as compared to the linear model. The water content of soil can be ascertained by measuring its electrical resistivity and using the correlation.

VII. Notations

- ρ = Electrical Resistivity of the soil (ohm-m)
- a = Distance between two consecutive probes in Wenner Array (m)
- b = Spacing of Internal Potential electrodes in Schlumberger Arrangement (m)
- l = Spacing between External Current electrodes in Schlumberger Arrangement (m)
- A = Cross Sectional area of test specimen (m^2)
- I = Current flowing between outer electrodes (Ampere)
- L = Spacing between two internal electrodes in Test Setup (m)
- r = Resistance (ohm)
- R^2 = Regression value (Percentage)
- S_r = Degree of Saturation of soil (Percentage)
- ΔV = Voltage drop between the two inner copper electrodes of model (Volts)
- V = Potential Difference between internal electrodes (Volts)
- w = Water content of soil (Percentage)
- γ = Bulk Density of sand in kN/m^3
- γ_d = Dry Density of sand in kN/m^3
- ϵ = Standard Error

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