



Synchronization Response of an Indirectly Coupled Nonlinear Digital Resonator -A Simulation Study

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Abstract: Connecting two nonlinear digital resonators (NDR) in an indirect coupling configuration (Driver-Response), the synchronization scenario of the response NDR has been explored through numerical simulation. In this paper, the frequency control parameter of the response NDR is tuned with the help of an error signal generated by comparing the phases of the driver and the response NDRs. It has been observed that for a specific range of design parameters of the response NDR, the synchronization between two NDRs is possible for both periodic and chaotic modes of oscillation of the driver NDR.

Keywords: Nonlinear Digital Resonator; Tracking; Chaos; Auxiliary system; synchronization;

I. Introduction

In modern communication technology, the preference of digital systems compared to their analog counterparts is well documented in literature [1-3]. Digital systems have flexibility in design, reliability in long time application, negligible dc drift and easy implementation possibility in IC technology. So researchers and the system designers are interested in designing different digital systems which could perform the jobs of already available analog systems [4, 5]. In digital signal processing digital filters are most important elements. They find application in removing undesirable co-channel signals and additive noise, in spectral shaping etc. [1]. Digital Resonators (DRs) belong to a group of second order digital filters that exhibit natural oscillations depending upon its design parameter values and they can be used as signal sources. DRs are used as digital frequency synthesizers [1]. However, they cannot be synchronized to external signal as they are basically linear systems. Modifications of the DR with the introduction of controlled nonlinearities in the system make the system synchronizable to an external signal. Moreover presence of nonlinearity would enrich its dynamics. As for example the system exhibits bifurcation, chaos, quasi-periodicity etc. A discrete equivalent of DR having the nonlinearities of a Vander pol oscillator has been proposed by the authors [6]. In this paper, we consider two such nonlinear digital resonators (NDRs) in a uni-directionally coupled configuration and study the synchronization performance of the second NDR (consider as response system) both in stable periodic oscillation and chaotic oscillation of the first NDR (consider as driver system). Here the dynamics of the response NDR is controlled by parameter tuning with the help of an error signal generated by comparing the phases of the driver and the response NDRs and the principle of conventional phase locked loops is being used here for that indirect coupling. The design parameters of response system could have non-identical values from that of the driver. We have studied the synchronization response of the response NDR in a generalized sense and for this purpose, the auxiliary system method of nonlinear dynamics [7-9] have been followed.

The paper has been organized as follows. In Section II, the difference equations describing the system dynamics for an isolated NDR and a cascaded NDR system have been formulated. The difference equations in terms of some error variables have been formulated at the end of this section to study the generalized synchronization (GS) between the driver and the response NDR. The results of numerical simulation on the dynamics of cascaded system have been given in Section III. A brief of simulation results regarding the dynamics of single NDR in presence of external forcing signal are given in first part of section III for completeness of discussion. Finally, we concluded with our findings and future implementation possibilities of the work in section IV.

II. System equation formulation

The dynamical behavior of a NDR has been studied in [6] numerically with the help of difference equation that describes the time evolution of the system. For the sake of completeness, we include the description of the structure and system equations of that work in subsection A and then we describe the structure of the cascaded system and formulate the corresponding system equations in subsection B. The subsection C describes the “auxiliary system” method through which the GS between two NDR is examined.

A. System equation of the NDR:

The block diagram of a conventional DR is shown in Fig.1. The system equation that describes the dynamics of a DR in terms of design parameter c_1 and c_2 is given by,

$$y[n] = -c_1 y[n-1] - c_2 y[n-2] \quad (1)$$

The transfer function of this resonator in z domain is,

$$H(z) = \frac{z^2}{z^2 + c_1 z + c_2} \quad (2)$$

Thus, it is a second order system. One can easily understand the dynamics of such system by finding the poles and zeros from equation (2). The poles of the transfer function may be either real or complex depending upon c_1 and c_2 . Since we are interested in oscillatory behavior of the system, we use the complex poles only. The two complex conjugate poles could be written as,

$$(p, p^*) = -\frac{c_1}{2} \pm \frac{j}{2} \sqrt{4c_2 - c_1^2} \quad (3)$$

Here, $j = \sqrt{-1}$. If we represents the complex poles of the system as,

$$p = r e^{\pm j\omega T} \quad (4)$$

Where, r, ω and T are the amplitude, angular frequency and the sample interval of the digital system respectively. Then the design parameters c_1 and c_2 can be expressed in terms of r, ω and T as follows,

$$c_1 = -2r \cos \omega T \quad (5)$$

$$\text{and, } c_2 = r^2 \quad (6)$$

With the help of equation (5) and (6), the unit impulse response of the system obtained from (1) is written as,

$$y[n] = \frac{r^n}{\sin(\omega T)} \sin[(n+1)\omega T] \quad (7)$$

Therefore, the system is oscillatory. If we put $r=1$ we can get sustained oscillation in a DR. Thus, the parameters c_1 and c_2 should be as follows for sustained oscillation,

$$c_1 = -2 \cos \omega T \quad (8a)$$

$$\text{and, } c_2 = 1 \quad (8b)$$

The frequency of the oscillatory DR system is determined by c_1 and has the range $-2 < c_1 < 2$. Since c_1 must be less than 2 for oscillation, so for sustained oscillatory behavior of the system we have, $c_1 - c_2 < 1$.

It is well known that one can obtained the system equation of a Vanderpol oscillator by adding the nonlinear term $-\varepsilon(1-y^2)\frac{dy}{dt}$ to the equation of un-damped harmonic oscillator. Following the same we add a discrete equivalent of the above term to the DR and we get the following two first order difference equations that describe the discrete time Vanderpol oscillator.

$$y[n+1] = z[n] \quad (9a)$$

$$z[n+1] = (a - \varepsilon y^2[n])z[n] - (b - \varepsilon y^2[n])y[n] \quad (9b)$$

$$\text{Where, } a = c_1 + \varepsilon \quad (10a)$$

$$b = c_2 + \varepsilon \quad (10b)$$

Therefore, a and b are the frequency determining and oscillation controlling term of the NDR respectively. To get sustained oscillation in the system we have $a - b = c_1 - c_2 < 1$. The block diagram of such NDR represented by equation (9) is given in Fig 2. Note that, putting $\varepsilon = 0$ one can obtain the equation of DR.

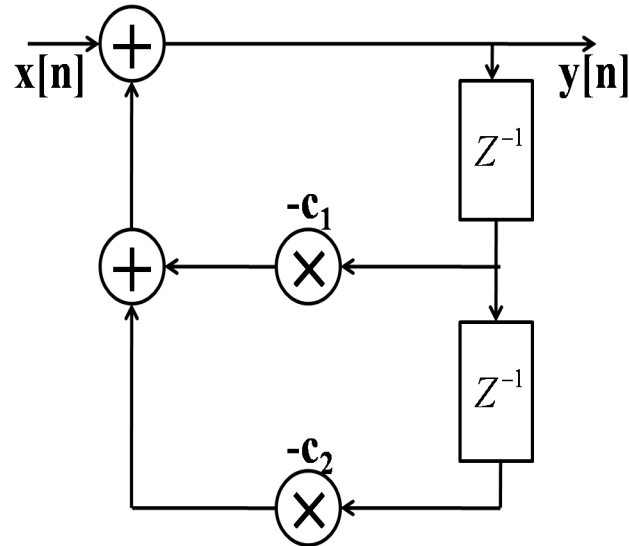


Figure 1 Block diagram of a conventional DR

B. System equations for two indirectly coupled NDRs:

Fig 3 shows the Driver-Response configuration of two NDR. Note that state variables and the design parameters are suffixed by 1 to indicate driver NDR and by 2 for that of response NDR. Both the systems have a common input signal (i.e. $x_1[n] = x_2[n] = x[n]$) which is a unit impulse to initiate the oscillation. In Fig 3 the indirect coupling section consists of a multiplier type phase detector (PD) and a first order low pass filter (LPF). This section produces an average of the instantaneous error signal between the output of two NDRs (driver and response). The frequency determining term (a_2) of the response NDR is then modified by this error signal. Here a first order digital LPF is used for averaging having transfer function as follows,

$$H(z) = \frac{(1-k)}{(1-kz^{-1})} \quad (11)$$

Therefore, one can derive the error signal as,

$$e[n+1] = ke[n] + (1-k)z_1[n]z_2[n] \quad (12)$$

Here, n denotes the sampling instant, z_1 and z_2 are the output state variable of the driver and response respectively. k is the design parameter of LPF. In the present case, the frequency parameter of the response i.e. a_2 is made to vary with the error signal $e[n]$. Thus, the proposed algorithm modifies the a_2 by the following relation,

$$a_2(1+pe[n]) \quad (13)$$

Here, p is a parameter used to quantify the strength of the error signal. Using this we derive the system equation that describes the system dynamics as,

$$y_1[n+1] = z_1[n] \quad (14a)$$

$$z_1[n+1] = (a_1 - \varepsilon_1 y_1^2[n])z_1[n] - (b_1 - \varepsilon_1 y_1^2[n])y_1[n] + x_1[n] \quad (14b)$$

$$y_2[n+1] = z_2[n] \quad (14c)$$

$$z_2[n+1] = (a_2(1+pe[n]) - \varepsilon_2 y_2^2[n])z_2[n] - (b_2 - \varepsilon_2 y_2^2[n])y_2[n] + x_2[n] \quad (14d)$$

Here, a_1, b_1, ε_1 are the frequency controlling, the oscillation controlling and nonlinear gain parameters of the driver respectively whereas, b_2 and ε_2 are the oscillation controlling and nonlinear gain parameters of the response NDR.

$$e_x[n+1] = 0 \quad (15a)$$

$$e_y[n+1] = e_z[n+1] \quad (15b)$$

$$e_z[n+1] = a_2 e_z[n+1] + p a_2 (e[n] z_2[n] - e'[n] z_2'[n]) - \varepsilon_2 (y_2^2[n] z_2[n] - y_2'^2[n] z_2'[n]) - b_2 e_y[n] + \varepsilon_2 (y_2^3[n] - y_2'^3[n]) + e_x[n] \quad (15c)$$

Using these equations we obtain a zone of design parameter for synchronization through numerical simulation.

III. Numerical simulation Results

The dynamics of the isolated system and the cascaded system is studied through numerical simulation of equation set (9) and (14) respectively. The dynamics is examined by observing the time series of state variable $z[n]$ and phase plane plot of $z[n]$ and $(z[n]-z[n-1])$. In all cases a large number of initial data are discarded to eliminate the transient response. The isolated system shows oscillation whose frequency depends on c_1 . However, if we put $x[n] = d \sin(nf\omega_0)$ i.e. the system is driven by some external forcing signal, the system enters into chaotic regimes through period-1, period-2 and period-4 oscillation for increasing value of d with properly chosen value of system design parameters. Here d and f are the strength of the external signal, the ratio of the frequency of the external signal to that of the natural frequency (ω_0) of the DR. All these dynamics are verified by doing FFT over 7000 time series data of $z[n]$. In case of chaotic oscillation a broad spectrum is obtained. This observation is shown in Fig 4.

Now, we study the tracking property of the response NDR when the driver NDR is in different periodic oscillation modes (period-1, 2 and 4) and also in chaotic mode. To study the synchronization in period-1 oscillation state, both the NDR are initiated by impulse input having different design parameter values. Different frequency control parameter value gives that the NDRs are oscillate at two different frequencies. Thus when the two systems are not synchronized or isolated from each other, the error signal that produces at the filter output shows the frequency difference between two systems but at synchronized condition such difference diminishes. Fig 5 shows such unsynchronized and the synchronized condition of the response NDR. The first part of the figure shows the time series plot of error, driver and response output signal whereas in the second part we plot the z_1 and z_2 state variables along x and y axis respectively. Our observation shows that there is no relationship between z_1 and z_2 state variables in unsynchronized case and the $e[n]$ shows the frequency difference in time series plot, but in synchronized case z_2 follows the z_1 with some constant phase difference and no frequency difference is observed in time series plot of $e[n]$. Note that filter cannot suppress the double frequency term of error signal. Response NDR gives a reasonable amount of frequency lock range over which it tracks the driver NDR and the range is increased as we increase p and ε_2 . These facts are shown in Fig 6(a) and 6(b) respectively. Numerical observation shows that the effect of p is better than that of ε_2 to increase the lock range. This is because, p has a direct impact on a_2 .

To study the synchronization in higher dynamics, the driver NDR is forced by a sinusoidal external signal of the form $x[n] = d \sin(nf\omega_0)$ as stated before and its design parameters are set to a critical value. The same input is applied to the response NDR but its design parameters are so chosen that it remains in period-1 oscillation throughout the observation. Under such condition when the strength of the external signal increased gradually the system dynamics of driver NDR enters into the chaotic regime through period doubling. Phase plane plot of the driver and the response in Fig 7 shows that the response NDR follows the driver in all cases such as period-2, 4 and chaotic state of oscillations. But the plot of z_1 and z_2 state variables along x and y axis respectively shows that there is no one to one correspond between driver and response. The response follows the driver in some complex manner. So they are not in identical synchronization but show a complex phase relationship between them.

The error equations (15a)–(15c), formulated using auxiliary method, are numerically solved to obtain the range of system parameter values for which the response NDR follows the driver. We find out the value of d and ε_2 leading to zero or a constant value of error variables by numerical solution of (15) with particular value of other design parameters. Then plot them in the $d - \varepsilon_2$ plane. The result is shown in Fig 8. The obtained result shows that the tracking capability of the response NDR gradually decreases as the driver NDR enters into the chaotic

region with the increase of d and response NDR follows the higher dynamics of driver for higher value of nonlinear term. The results are consistent with the result obtained in analog system.

IV. Conclusion

The tracking ability of indirectly coupled response NDR has been studied in this paper. A response NDR is able to track the output of a driver NDR with some constant phase difference when both are in periodic oscillation state. The error signal strength is more dominating parameter to increase the frequency lock range than the nonlinear parameter of the response NDR system. Simulation results also confirms that a response NDR is capable to track the higher dynamical states (i.e. period-2,4, chaos etc.) of the driver NDR for critically chosen design parameter value. A reasonable zone of synchronization has been obtained using the auxiliary method. Thus, such system can be used as a synchronizer and can be used in digital communication system. In future the experimental studies are necessary for implementation; it may also be used in chaos synchronization communication.

V. References

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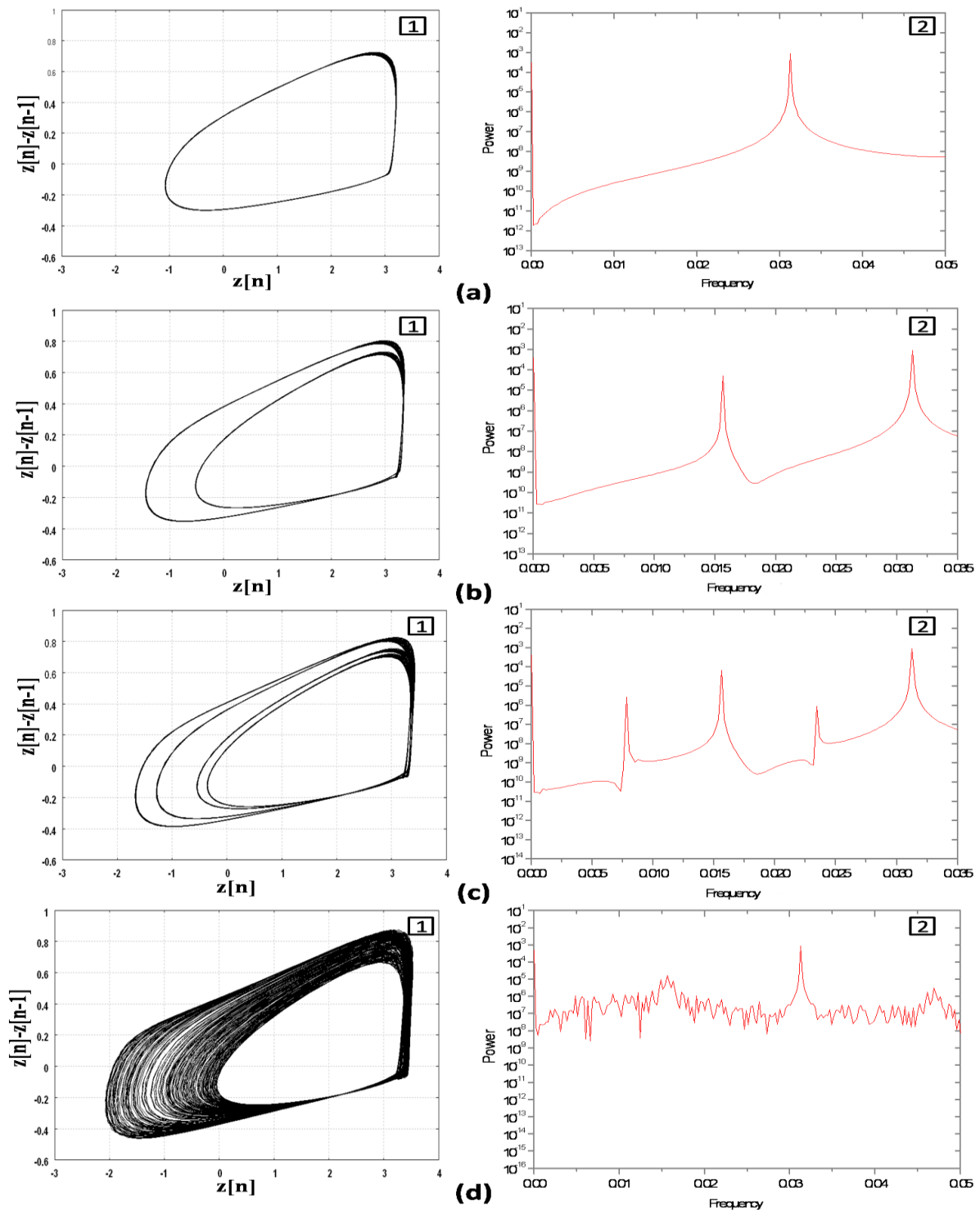


Figure 4: Numerical simulation results, showing the period doubling route to chaos of NDR under study, with system parameters $\varepsilon = 0.11$, $a = 2 + \varepsilon$, $b = 1.01 + \varepsilon$, $f = 2.28$ and strength of the external forcing signal is slowly increased; (a) $d = 0.09$, (b) $d = 0.1$, (c) $d = 0.103$, (d) $d = 0.11$. Here (1) is the phase plane plot of $z[n]$ and $z[n]-z[n-1]$ and (2) is the power frequency spectrum of $z[n]$.

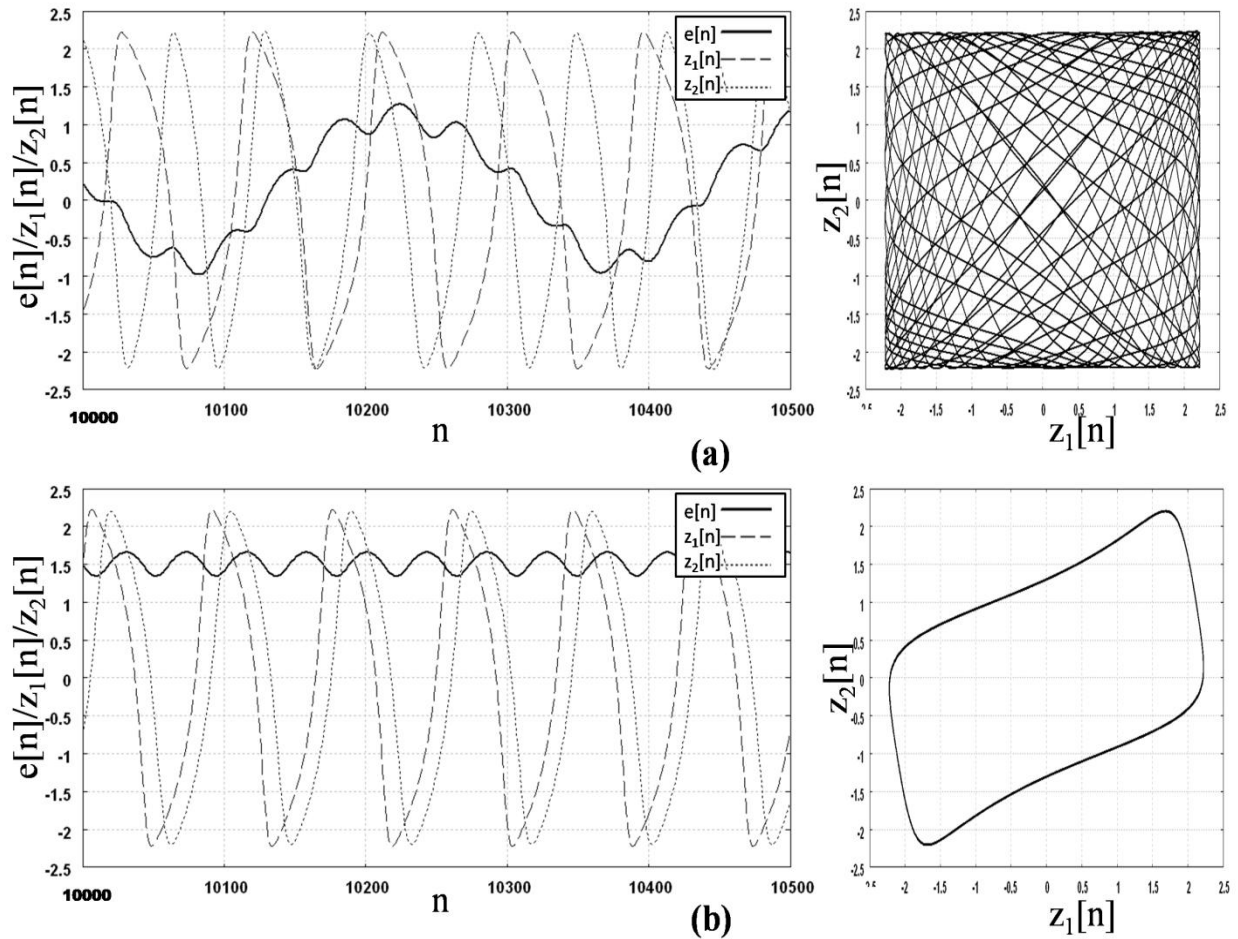


Figure 5: Numerical simulation results, showing the (a) unsynchronized and (b) synchronized state of response NDR (tracking ability) when driver NDR is in periodically oscillating state. Here (1) represents the time series plot of state variable $e[n], z_1[n], z_2[n]$ and (2) represents the x-y plot of $z_1[n] - z_2[n]$ with parameter values $\varepsilon_1 = 0.05, b_1 = 1.06, \varepsilon_2 = 0.05, a_2 = 2.05, b_2 = 1.06, p = 0.001, k = 0.99$ and in (a) $a_1 = 2.054$, in (b) $a_1 = 2.053$.

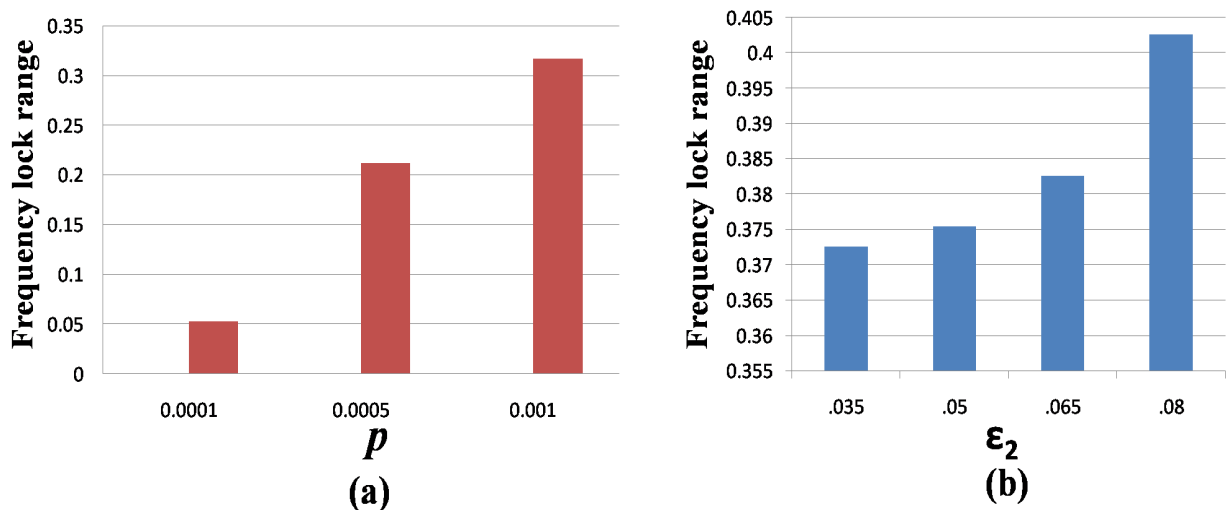


Figure 6: Numerically obtained frequency lock range (a) with respect to the strength of the error signal. The parameter values are $\varepsilon_1 = 0.05, b_1 = 1.06, \varepsilon_2 = 0.05, a_2 = 2.05, b_2 = 1.06, k = 0.99$ (b) with respect to the nonlinearity of the response NDR. The parameter values are $\varepsilon_1 = 0.05, b_1 = 1.06, a_2 = 2 + \varepsilon_2, b_2 = 1.01 + \varepsilon_2, k = 0.99, p = 0.001$.

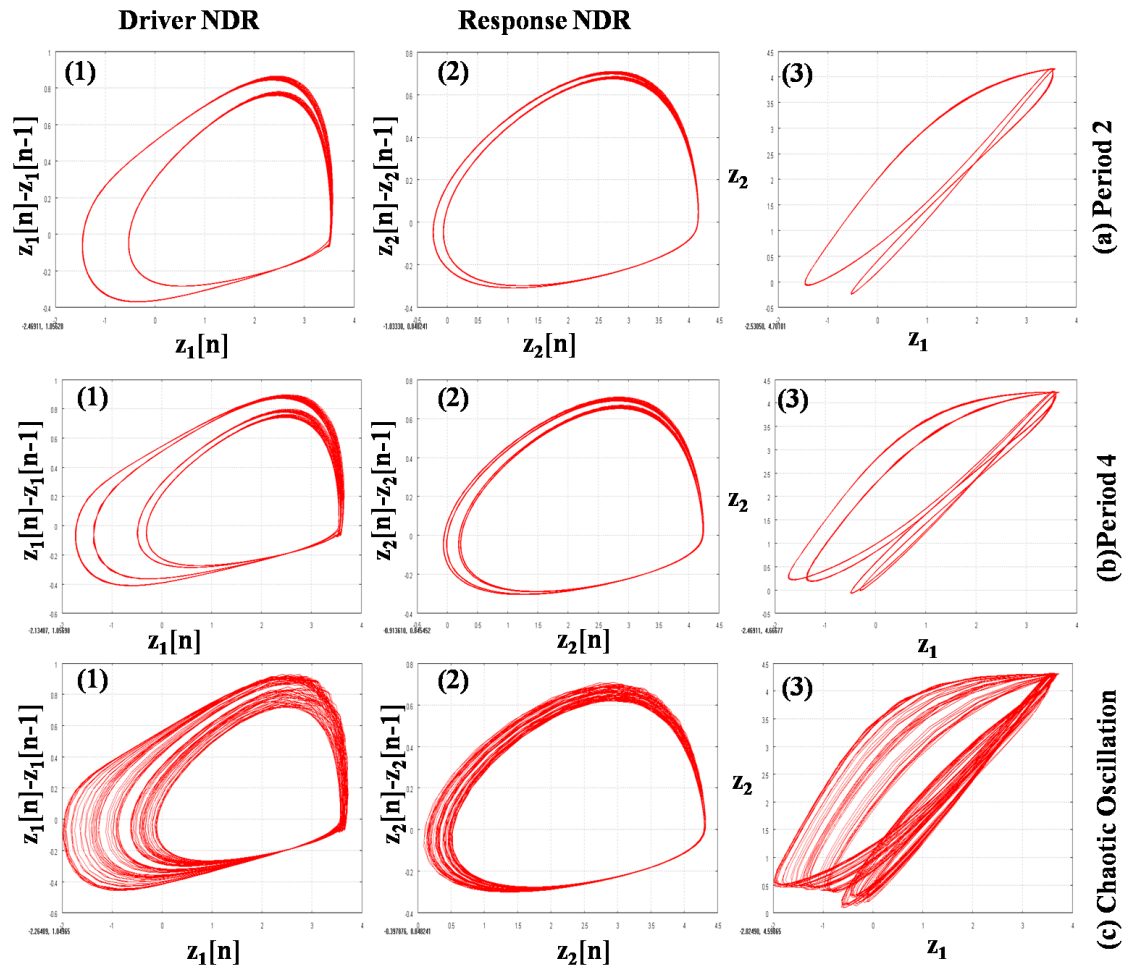


Figure 7: Numerical simulation observations, showing the tracking ability of the response NDR when the output of the driver NDR is in (a) period-2 (b)period-4 and (c) chaotic state. The system parameters of the driver NDR are: $a_1 = 2.1, \varepsilon_1 = 0.1, b_1 = 1.11$ and those of the response NDR are: $a_2 = 2.05, \varepsilon_2 = 0.05, b_2 = 1.06$. The other parameters are $k = 0.99, p = 0.0005$. The dynamics of the driver NDR is varied by increasing the external signal strength as (a) $d = 0.1$, (b) $d = 0.11$, (c) $d = 0.125$ with $f = 2.28$.

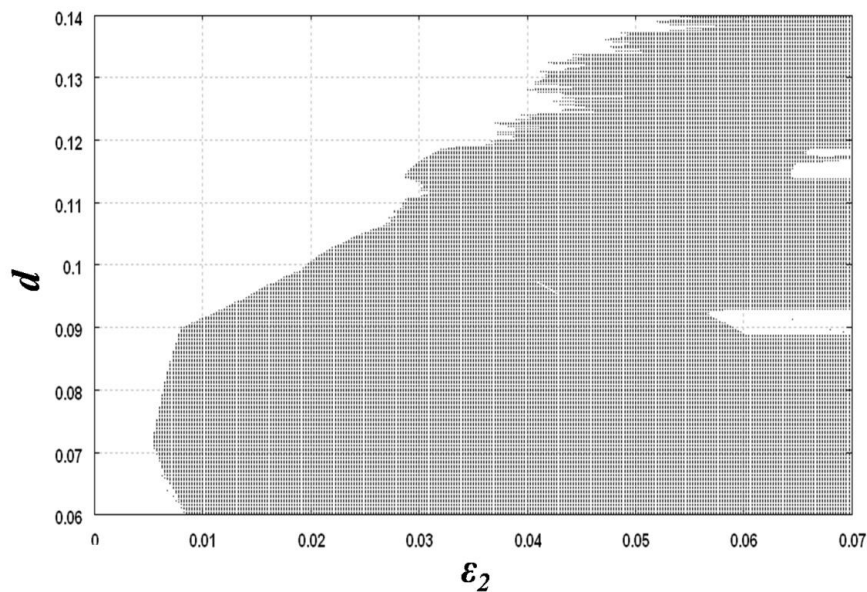


Figure 8: Numerically obtained solution points of Eq. (15) indicating the synchronized state of the response NDR to the output of the driver NDR. Here the driver NDR gradually goes into chaotic state through period-2, 4 with increasing d . The other parameters are $\varepsilon_1 = 0.11, a_1 = 2 + \varepsilon_1, b_1 = 1.01 + \varepsilon_1, a_2 = 2 + \varepsilon_2, b_2 = 1.01 + \varepsilon_2, k = 0.99, p = 0.001, f = 2.28$.