



Pressure Variation in Porous Bearings under Second Order Rotation

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Abstract: The second order rotatory theory of hydrodynamic lubrication was founded on the expression obtained by retaining the terms containing first and second powers of rotation number in the extended generalized Reynolds equation. In the present paper, there are some new excellent fundamental solutions with the help of geometrical figures, expressions, calculated tables and graphs for the porous bearings in the second order rotator theory of hydrodynamic lubrication. The analysis of pressure equation, tables and graphs reveal that pressure is not independent of viscosity and varies inversely with viscosity. The pressure increases with increasing values of rotation number. In the absence of rotation, the equation of pressure gives the classical solutions of the classical theory of hydrodynamic lubrication. The relevant tables and graphs confirm these important investigations in the present paper.

Keywords: Continuity, Rotation number, Taylor's number, Reynolds equation, Density, Viscosity, Film thickness, Permeability

I. INTRODUCTION

The two dimensional classical theory [4], [10] of hydrodynamic lubrication was first given by Osborne Reynolds [11]. In 1886, in the wake of a classical experiment by Beauchamp Tower [12], he formulated a differential equation known as: Reynolds Equation [11]. The formation and basic mechanism of fluid film was analyzed by that experiment by taking some assumptions:

- [a] The film thickness is very small in comparison to the axial and longitudinal dimensions of fluid film.
- [b] If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Osborne Reynolds himself derived "Generalized Reynolds Equation" [7], [10], which depends on density viscosity, film thickness, surface and transverse velocities. The rotation [1] of fluid film about an axis that lies across the film gives some new results in lubrication problems. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation and the effects arising from it are predominant, for large Taylor's number, it results in the streamlines becoming confined to plane transverse to the direction of rotation. The extended version of "Generalized Reynolds Equation" [7], [10] is said to be "Extended Generalized Reynolds Equation" [1], [3], which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number M [1], i.e. the square root of the conventional Taylor's number. This generalization of the classical theory is known as the "Rotatory Theory of Hydrodynamic Lubrication" [1], [3]. The "Second Order Rotatory Theory of Hydrodynamic Lubrication" [3], [8] was given by retaining the terms containing up to second powers of M [1], and neglecting higher powers of M [1]. The present paper has the discussion about the variation of pressure in the porous bearings under the effect of second order rotation. These bearings are made of porous material and the lubricant flows out of the bearing surface with a definite velocity. These bearings are used in many devices, such as vacuum cleaners, extractor fans, motor car starters, hair dryers etc. The geometry [4] of bearing is described in figure [1], of solid shaft in sintered metal bush and figure [2], of shaft and bush opened up.

FIGURE [1] : SOLID SHAFT IN SINTERED METAL BUSH

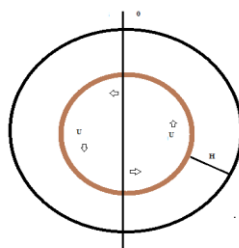
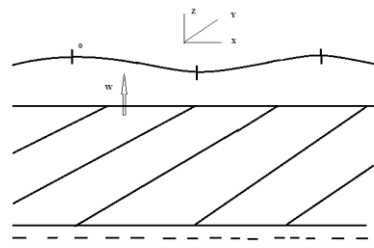


FIGURE [2] : SHAFT AND BUSH OPENED UP



If the bearing is infinitely short [2], then the pressure gradient in x-direction is much smaller than the pressure gradient in y-direction. In y-direction the gradient $\partial p/\partial y$ is of the order of (P/L) and in the x-direction, and is of order of (P/B) . If $L \ll B$ then

$$P/L \gg P/B, \text{ so } \partial P/\partial x \ll \partial P/\partial y \quad (1.1)$$

Then the terms containing $\partial P/\partial x$ can be neglected as compared to the terms containing $\partial P/\partial y$ in the expanded form of Generalized Reynolds Equation.

II. NOMENCLATURE

B : Total breadth of bearing parallel to the direction of motion
 C : Radial clearance
 e : Eccentricity ratio
 H : Wall thickness of porous bearing
 h : Film thickness
 K : Constant
 L : Bearing length normal to the direction of motion
 M : Rotation number
 P : Pressure
 R : Radius of bearing
 U : Sliding velocity
 W : Load capacity
 W^* : Fluid velocity in z-direction
 x : Co-ordinate along span of the bearing system
 y : Co-ordinate along length of the bearing system
 z : Co-ordinate across the fluid film
 ρ : Density of fluid
 μ : Absolute viscosity of fluid
 θ : Angular co-ordinate
 ϕ : Permeability

III. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation" [7] is given as:

$$\begin{aligned} & \frac{\partial}{\partial x} \left[-\sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[-\sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial y} \right] + \\ & \frac{\partial}{\partial x} \left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial y} \right] - \frac{\partial}{\partial y} \left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial x} \right] = \\ & -\frac{U}{2} \frac{\partial}{\partial x} \left[\rho \sqrt{\frac{2\mu}{M\rho}} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \right] - \frac{U}{2} \frac{\partial}{\partial y} \left[-\rho \sqrt{\frac{2\mu}{M\rho}} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \right] - \rho W^* \end{aligned} \quad (3.1)$$

The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M [1] and by retaining the terms containing up to second powers of M and neglecting higher powers of M , can be written as:

$$\begin{aligned} & \frac{\partial}{\partial x} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] - \\ & \frac{\partial}{\partial y} \left[-\frac{M\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial x} \right] = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] - \rho W^* \end{aligned} \quad (3.2)$$

Taking

$$h=h(x), U=-U, P=P(y) \text{ and } W^*=-\partial P/\partial z \Big|_{z=0} \phi/\mu \quad (3.3)$$

Where $\partial P/\partial z$ is the pressure gradient at the bearing surface and ϕ is the property called permeability, which varies with porosity and size of pores. From the requirements of continuity, we have for the porous matrix

$$\nabla W^* = (\phi/\mu) \nabla^2 P = 0 \text{ i.e., } \nabla^2 P = 0 \quad (3.4)$$

The problem then is to solve the governing equation (3.1) for the pressures in oil film simultaneously with that of Laplace for the porous matrix with a common pressure gradient $\partial P/\partial z$ at the boundary, we have

$$\partial^2 P/\partial x^2 + \partial^2 P/\partial y^2 + \partial^2 P/\partial z^2 = 0 \quad (3.5)$$

We have two assumptions to solving the equations (3.1) and (3.5) as follows:

(3.1.1) The bearing is infinitely short.

(3.1.2) $\partial P/\partial z$ is linear across the matrix and is zero at the outer surface of the porous bearing shell.

From (3.1.1), (3.1.2), we have

$$\partial^2 P / \partial x^2 = 0, \partial^2 P / \partial z^2 = K \text{ (constant)}, \partial^2 P / \partial y^2 = -K \quad (3.6)$$

From (3.3), we have

$$\partial P / \partial z \Big|_{z=0} = KH = -\partial^2 P / \partial y^2 \Big|_{z=0} H \quad (3.7)$$

The constant K can be taken as C/H in the porous bearings.

Now the equation (3.1) becomes

$$\left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2 \rho^2 h^4}{1680\mu^2} \right) \rho \right] \frac{d^2 P}{dy^2} + \left[\frac{M\rho^2}{120\mu^2} \frac{d}{dx} \left(h^5 - \frac{31M^2 \rho^2 h^9}{3024\mu^2} \right) \right] \frac{dP}{dy} = \frac{d}{dx} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2 \rho^2 h^5}{120\mu^2} \left(1 - \frac{31M^2 \rho^2 h^4}{3024\mu^2} \right) \right\} \right] - \rho \left(-\frac{dP}{dz} \Big|_{z=0} \frac{y}{\mu} \right) \quad (3.8)$$

The film thickness 'h' and 'y' can be taken as:

$$h = C (1 + e \cos \theta), x = R \theta, \quad (3.9)$$

Where θ being measured from x-direction.

For the determination of pressure the boundary conditions are as follows:

$$P=0, y = \pm L/2 \quad (3.10)$$

IV. DETERMINATION OF PRESSURE

The solution of the differential equation (3.8) under the boundary condition (3.10) gives the pressure for porous bearing as follows:

$$P = \frac{(3\mu CU\alpha + 12KH\phi R)(L^2 - 4y^2)}{4h^3 R} + \frac{\rho C\alpha(U\mu + 4KH\phi)(L^2 y - 4y^3)}{8\mu R h^2} M + \frac{(53U\mu\rho^2 C\alpha h - 68RKH\phi\rho^2 h)(L^2 - 4y^2)}{2240\mu^2 R} M^2 \quad (4.1)$$

Where $\alpha = e \sin \theta$, by taking the terms of equation (4.1) as:

$$A = \frac{(3\mu CU\alpha + 12KH\phi R)(L^2 - 4y^2)}{4h^3 R} \quad (4.2)$$

$$B = \frac{\rho C\alpha(U\mu + 4KH\phi)(L^2 y - 4y^3)}{8\mu R h^2} \quad (4.3)$$

$$C = \frac{(53U\mu\rho^2 C\alpha h - 68RKH\phi\rho^2 h)(L^2 - 4y^2)}{2240\mu^2 R} \quad (4.4)$$

The equation for pressure can be written as:

$$P = A + BM + CM^2$$

V. CALCULATION TABLES

By taking the values of different mathematical terms in C.G.S. system as follows:

$\alpha = 0.1$, $\theta = 30^\circ$, $\mu = 0.0002$, $C = 0.0067$, $e = 0.2$, $\rho = 1$, $U = 10^2$, $h = 0.00786$, $L = 2 \times 10^{-4}$, $y = 5 \times 10^{-3}$, $H = 0.05$, $\phi = 0.0025$, $R = 3.35$; the calculated tables are given by (5.1) and (5.2).

5.1 Variation table*of Pressure with respect to Viscosity by taking $M = \text{constant} = 0.1$

μ	0.0002	0.0003	0.0004	0.0005	0.0006
A	231.6825	231.6834	231.6843	231.6853	231.6862
B $\times 10^{11}$	622.17	617.11	614.58	613.06	612.02
C $\times 10^{11}$	-2.18115	-0.957	-0.5313	-0.33559	-0.22995
P $\times 10^{11}$	62.2	61.7	61.4	61.3	61.2

5.2 Variation table*of Pressure with respect to Rotation number by taking $\mu = \text{constant} = 0.0002$

M	0.1	0.2	0.3	0.4	0.5
A	231.6825	231.6834	231.6843	231.6853	231.6862
B $\times 10^{11}$	622.17	617.11	614.58	613.06	612.02
C $\times 10^{11}$	-2.18115	-0.957	-0.5313	-0.33559	-0.22995
P $\times 10^{11}$	62.2	124.4	186.6656	248.86	311.00

*All the calculations are in C.G.S. system.

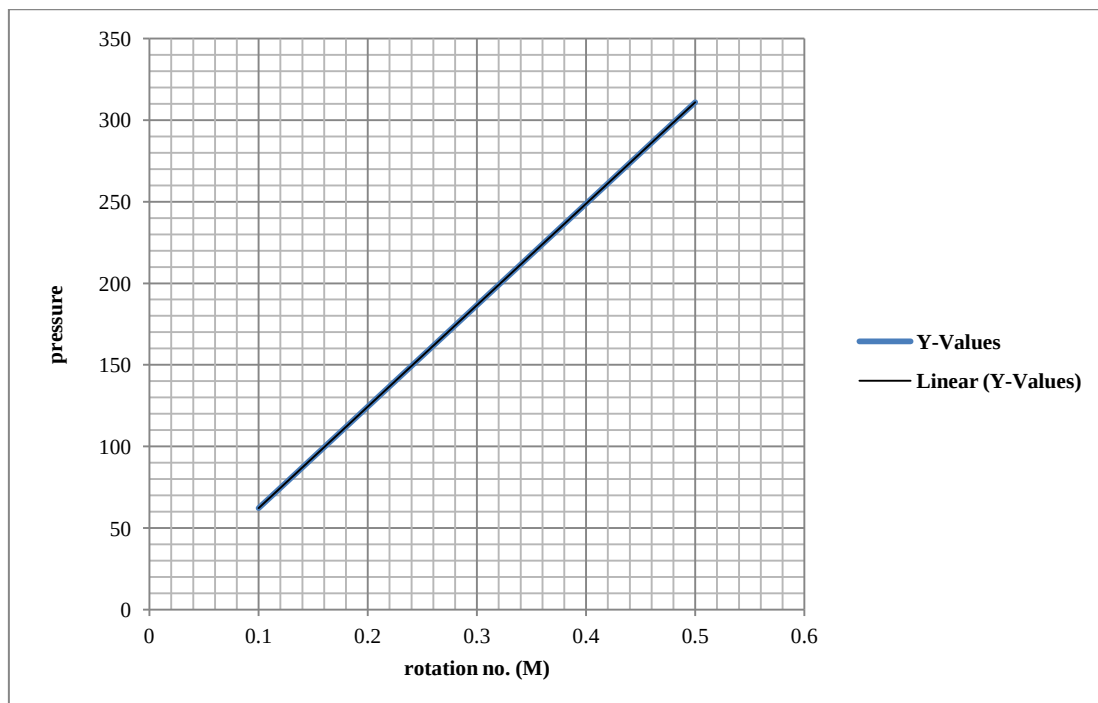
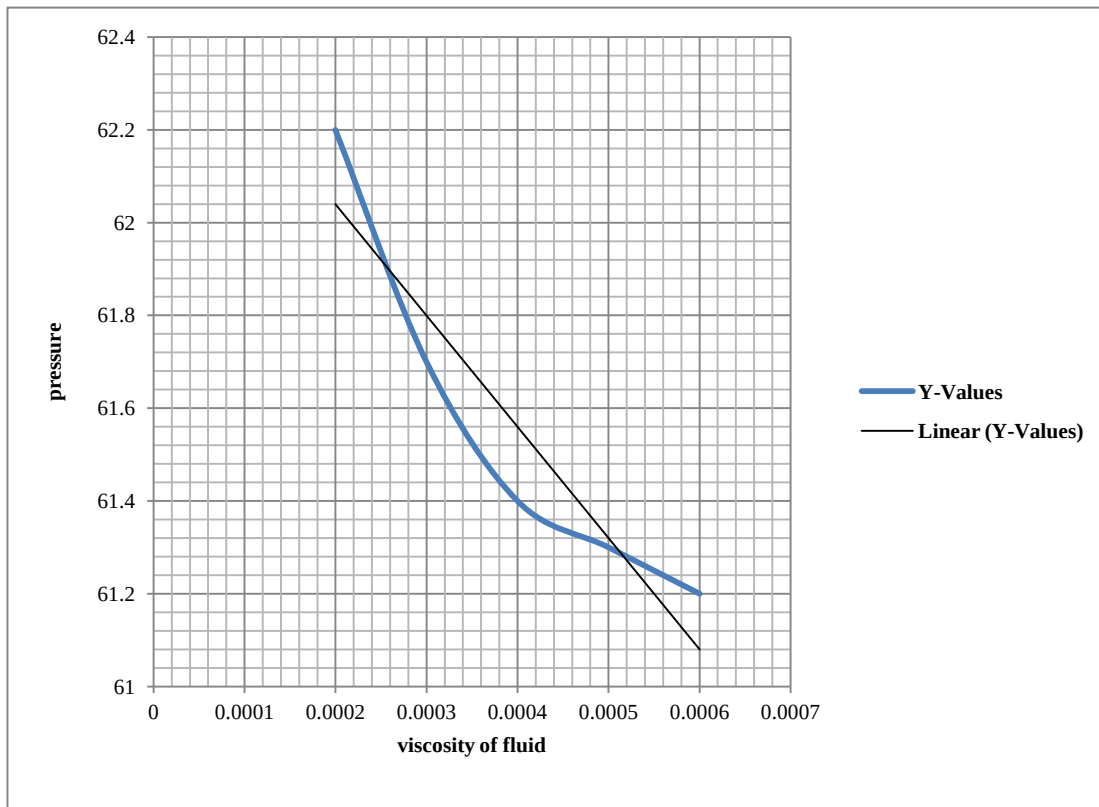
VI. GRAPHS

6.1 Graph for table (5.1):

The graph shown is between viscosity μ and pressure P by taking M as constant with the help of table 5.1.

6.2 Graph for table (5.2):

The graph shown is between pressure P and rotation number M by taking viscosity μ as constant with the help of table 5.1.



VII. CONCLUSIONS

The variation of pressure with respect to viscosity, when M is constant and with respect to rotation number M , when viscosity is constant, are shown by tables and graphs. Hence in the second order rotatory theory of hydrodynamic lubrication, the pressure is not independent of viscosity μ and varies inversely with μ , the

pressure increases with increasing values of the rotation number M . On taking ($M=0$) in the expression of pressure, we get the classical solution.

VIII. References

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