



Effect of Controller Parameters on Pantograph-Catenary System

Rachana Garg¹, Priya Mahajan², Parmod Kumar³

¹Associate Prof., ² Assistant Prof, ³Dean (IRD)

^{1,2}Electrical Engg. Department,

Delhi Technological University, Shahbad Diary, Bawana Road,
Delhi 110042, India

³Maharaja Agrasen Institute of Technology, Sector 22, New Delhi 110085
INDIA

Abstract: Contact force fluctuations is one of the major problem encountered in today's high speed train. These fluctuations can be minimized by an active control of pantograph-catenary system. In this paper, the authors have presented the modeling of active PID controller for pantograph-catenary system and studied the effect of controller parameters on the system by developing the sensitivity functions of the controller wrt parameters of interest, first time in the literature. The effect of train speed on these sensitivity functions has also been studied.

Keywords: Pantograph-catenary system, PID controller, Controller Parameters, Sensitivity functions, Normalized sensitivity

I. Introduction

Railway traction model is a complex mix of electrical and mechanical parameters. Pantograph acts as an active link between the catenary and electric drive used for driving the electric loco. It collects the current from the catenary and supplies it to loco transformer for feeding to the drive. Stable current collection is one of the foremost requirements for the efficient and reliable operation of locomotives. This becomes even more important for high speed locos. Normally speed of the train is less than the wave speed of the contact wire. As train speed approaches wave speed, chances of loss of contact are increased which results in electric arc between contact wire and pantograph further aggravating the problem[1]. It is, therefore, necessary to study the effect of controller parameters on dynamic behavior of pantograph- catenary system to improve the performance of the pantograph. The authors in this paper have first developed the mathematical model of active pantograph - catenary system with PID controller. The Zeigler-Nicholas method has been used for computing the parameters of the controller. Then the effect of controller parameters on the system has been studied using sensitivity analysis. The sensitivity functions of the active control system model wrt parameters of interest have been developed first time in the literature in this paper. The effect of train speed on sensitivity functions have been observed and analyzed. The effect of parameters of open loop pantograph- catenary system has already been studied by the authors[2].

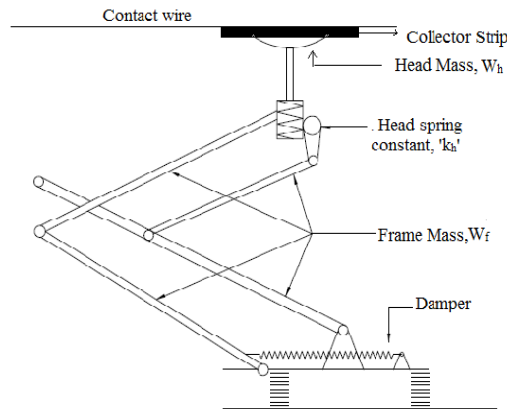
The mechanical and dynamic model of pantograph-catenary system has been presented by various researchers[3]-[9]. T.J. Park et al [10] have studied the dynamic interaction between catenary and pantograph using the sensitivity analysis method. It was also observed that span length and plunger spring constant are some of the important design variables of the PAC system[11].

II. Modeling Of Pantograph-Catenary System

Pantograph is an important link to transfer the electric energy from the power conductor to loco transformer. It is a mechanical system composed of an articulated frame, carrying a collector head on which the collector strips for current collection are mounted. Constant pressure is applied at the head of the pantograph through pneumatic control mechanism to ensure that it always remain in contact with the contact wire. When the electric train moves, a dynamic force is applied on the contact wire by the head of the pantograph. The contact wire after the passage of pantograph suddenly sags and starts vibrating due to the elasticity of the system. The quality of current collection by the pantograph depends upon the contact pressure between the pantograph head and speed of the train[12,13]. If the pressure is kept too low, it may lead to unsatisfactory current collection and

sparking whereas very high pressure increases the wear and tear of the contact wire. For 25kV ac system, the pressure between 4-6 bars is considered optimum. The supply from catenary to the power conductor to loco transformer through pantograph form a dash pot mass spring system. Fig. 1 shows the physical representation of pantograph[14].

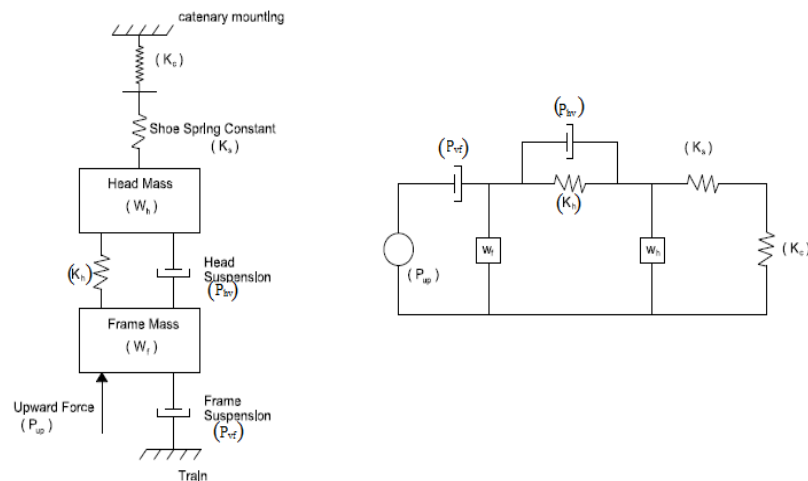
Figure 1 Physical Representation of Pantograph



The pantograph is modeled as a set of masses, springs and shock absorbers. For a symmetric pantograph, it is assumed that complete pantograph frame moves as a rigid body[14]-[15] and the pantograph-catenary system can be represented by the two degrees of freedom of mechanical system as shown in fig.2. The upper mass represents the head mass while mass of frame is represented by lower mass. The contact between the head of the pantograph and the catenary is represented by a spring with a constant stiffness. The displacement of catenary is the function of force on the catenary due to pantograph.

Fig.2. Representation of Pantograph- Catenary System

2(a) Pantograph- Catenary System as dash pot spring mechanism 2(b) Series analog of Pantograph- Catenary System



A. Open Loop Model[2]:

Pantograph -catenary system has been modeled as dash pot mass spring system as shown in fig. 2(a). Applying Newton's law of motion, the dynamic equations can be formed. Taking the laplace transform of these equations and solving them, we obtain the open loop transfer function of the pantograph-catenary system, as given by (1),

$$G(s) = \frac{D_{hp}-D_{ct}}{P_{up}} = \frac{[P_{hv}s+k_h]}{k[C_4s^4+C_3s^3+C_2s^2+C_1s+C_0]} \quad (1)$$

$$\text{Here, } C_4 = W_f W_h, \quad C_3 = (W_h P_{vf} + W_h P_{hv} + P_{hv} W_f), \\ C_2 = (K_2 W_f + P_{vf} P_{hv} + K_h M_h), \quad C_1 = (K_2 P_{vf} + K_2 P_{hv} + 4K_h P_{hv}), \quad C_0 = (K_2 K_h - K_h^2)$$

where D_{hp} = displacement of pantograph head, D_{ct} = displacement of catenary,

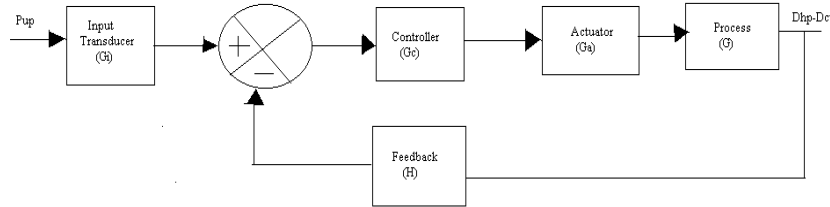
P_{up} = upward force applied to the pantograph, W_f = Mass of frame, W_h = Head Mass,

k_h = Head Spring Constant, k_c = Catenary represented by spring, k_s = Shoe Spring Constant,
 P_{hv} = Viscous damping of pan-head suspension, P_{vf} = Viscous co-efficient of frame suspension.

B. Closed Loop Model:

Force applied to the pantograph as derived above is quite sensitive to external disturbances and internal variations in the system parameters. To stabilize the oscillation of contact wire, PID controller and actuator are added in forward path of the pantograph and feedback mechanism is provided as shown in fig. 3 to reduce the sensitivity of the pantograph system. The transient response and steady state error can thus be controlled in closed loop system by adjustment of gain in the loop or by redesigning the controller. The simple models of the catenary are used to develop the control strategy of the active pantograph.

Figure 3 Block Diagram for Closed Loop Model



The transfer function of closed loop system shown in fig. 3 is given by (2),

$$M(s) = \frac{D_{hp}-D_{ct}}{P_{up}} = \frac{G_i G_c G_a G(s)}{1 + H_o G_i G_c G_a G(s)} \quad (2)$$

where, G_i , G_c , G_a , H_o are gain of input transducer, controller, actuator and feedback, respectively.

The transfer function of PID controller designed is given by (3).

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (3)$$

where, K_p = proportional gain, T_i = integral time constant, T_d = derivative time constant.

From (2) and (3), the closed loop transfer function with PID controller is given by (4),

$$M(s) = \frac{D_{hp}-D_{ct}}{P_{up}} = \frac{G_i G_a K_p [P_{hv} T_i T_d s^3 + (P_{hv} T_i + T_i T_d k_h) s^2 + (P_{hv} + T_i k_h) s + k_h]}{[a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0]} \quad (4)$$

where, $a_5 = C_4 T_i$, $a_4 = C_3 T_i$, $a_3 = (C_2 T_i + H G_i G_a K_p P_{hv} T_i T_d)$,

$a_2 = (C_1 T_i + H G_i G_a K_p P_{hv} T_i + H G_i G_a T_i T_d K_p k_h)$, $a_1 = (C_0 T_i + H G_i G_a K_p T_i k_h)$, $a_0 = H G_i G_a K_p k_h$

PID controller is designed using Ziegler-Nichols frequency response tuning method and then fine tuning is done depending on the response.

III. Effect of Parameters on PAC System

The effect of parameters on open loop system and closed loop system has been studied using sensitivity analysis. Sensitivity analysis is an effective method to predict the effect of a parameter on the response of the system [16]-[18]. Mathematically, it is given by (5):

$$\hat{S}_p^F = \frac{\partial F/F}{\partial p/p} = \left(\frac{p}{F} \right) \left(\frac{\partial F}{\partial p} \right) = \frac{\partial(\log F)}{\partial(\log P)} \quad (5)$$

A. Open Loop System [2]

The sensitivity function of open loop transfer function, $G(s)$, with respect to the generic parameter x_i is given by (6),

$$[S_{x_i}^{G(s)}] = \frac{\partial G(s)}{\partial x_i} \times \frac{x_i}{G(s)} \quad (6)$$

Thus the sensitivity functions of open loop transfer function, $G(s)$ wrt parameters of interest are given by (7)-(11),

The sensitivity function of $G(s)$ wrt mass of frame, W_f , is given by,

$$[S_{W_f}^{G(s)}] = \frac{-k_1 [W_f W_h s^4 + W_f P_{hv} s^3 + W_f k_2 s^2]}{[P_{hv} s + k_h]} \times G(s) \quad (7)$$

The sensitivity function of $G(s)$ wrt head mass, W_h , is given by,

$$[S_{W_h}^{G(s)}] = \frac{-k_1[W_f W_h s^4 + W_h(P_{hv} + P_{vf})s^3 + W_h k_h s^2]}{[P_{hv}s + k_h]} \times G(s) \quad (8)$$

The sensitivity function of $G(s)$ wrt head spring constant, k_h , is given by,

$$[S_{k_h}^{G(s)}] = \frac{k_h}{[P_{hv}s + k_h]} - \frac{k_1[(W_f + W_h)s^3 + (4P_{hv} + P_{vf})s + \frac{k_s}{k_1}(1 - \frac{1}{k_1})]}{[P_{hv}s + k_h]} \times G(s) \quad (9)$$

The sensitivity function of $G(s)$ wrt viscous friction, P_{vf} , is given by,

$$[S_{P_{vf}}^{G(s)}] = \frac{-k_1 P_{vf}[W_h s^3 + P_{hv} s^2 + k_2 s]}{[P_{hv}s + k_h]} \times G(s) \quad (10)$$

The sensitivity function of $G(s)$ wrt damping co eff, P_{hv} , is given by,

$$[S_{P_{hv}}^{G(s)}] = \frac{s P_{hv}}{[P_{hv}s + k_h]} - \frac{k_1 P_{hv}[(W_f + W_h)s^3 + P_{vf} s^2 + (k_2 + 3k_h)s]}{[P_{hv}s + k_h]} \times G(s) \quad (11)$$

B. Closed Loop System:

In the closed loop system, the error is fed to PID controller, G_c , and the closed loop model is developed in section II(B). In order to study the effect of controller parameters variations on the closed loop model of pantograph, the sensitivity functions wrt controller parameters i.e. K_p , T_i , T_d are developed as given by (12)-(14).

$$[S_{K_p}^{M(s)}] = \frac{T_i s \Delta}{[T_i s \Delta + (G_i G_a K_p H)(T_i T_d s^2 + T_i s + 1)(P_{hv}s + k_h)]} \quad (12)$$

$$[S_{T_i}^{M(s)}] = \frac{-T_i s \Delta}{(T_i T_d s^2 + T_i s + 1)[T_i s \Delta + (G_i G_a K_p H)(T_i T_d s^2 + T_i s + 1)(P_{hv}s + k_h)]} \quad (13)$$

$$[S_{T_d}^{M(s)}] = \frac{T_i^2 T_d s \Delta}{(T_i T_d s^2 + T_i s + 1)[T_i s \Delta + (G_i G_a K_p H)(T_i T_d s^2 + T_i s + 1)(P_{hv}s + k_h)]} \quad (14)$$

Where, $\Delta = [C_4 s^4 + C_3 s^3 + C_2 s^2 + C_1 s + C_0]$

IV. Numerical Results

In order to show the utility of the developed model, simulation studies using Matlab version 7.0 are carried out. The data for the parameters of the pantograph, catenary, and gain of the input transducer, actuator, proportional controller and feedback device are given in Appendix I. Substituting these values of parameters of the pantograph and catenary in (1), the open loop transfer function is obtained as given by (15),

$$G(s) = \frac{s + 53.8}{1.27s^4 + 29.92s^3 + 12412.67s^2 + 132628.28s + 4.43 \times 10^6} \quad (15)$$

Using Ziegler-Nichols frequency response tuning method and fine tuning the response, we get, $K_p = 1.02 \times 10^5$, $T_i = 0.002$, $T_d = 8 \times 10^{-3}$

Putting these values in (3), we get,

$$G_c(s) = \frac{312.5}{s} (s^2 + 125s + 6.25 \times 10^4) \quad (16)$$

Substituting in (2),

$$M(s) = \frac{16.2s^3 + 2860s^2 + 1.109 \times 10^6 s + 5.38 \times 10^7}{2.54s^5 + 59.84s^3 + (2.5 \times 10^4)s^3 + (2.6 \times 10^5)s^2 s + 9.97 \times 10^6 s + (5.38 \times 10^7)} \quad (17)$$

The step response of open loop transfer function and transfer function with PID controller is plotted as shown in fig. 4(a) and 4(b) respectively. It has been observed that the overshoot decreases from 45% to just 4.8% using PID controller. Fig.5 shows the bode plot of the system with PID controller. It is observed from the fig. that the system is stable with given parameters.

Figure 4(a) Response of the open loop model

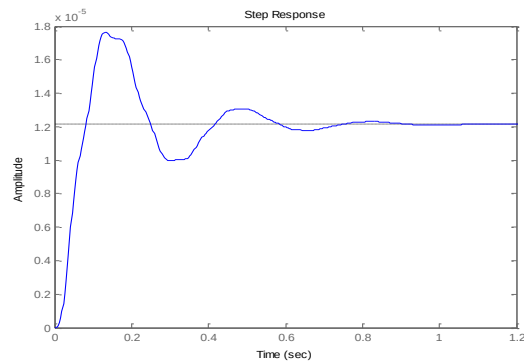


Figure 4(b) Response of the close loop model with PID Controller

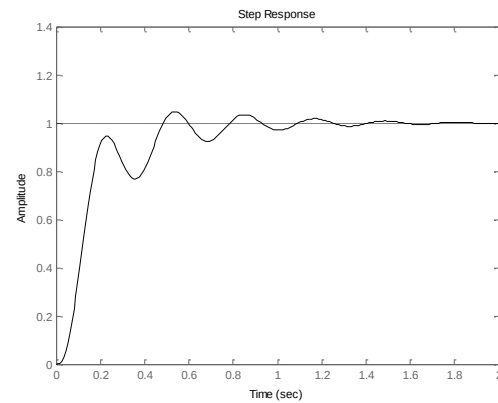


Figure 5 Bode Plot of the system with PID controller

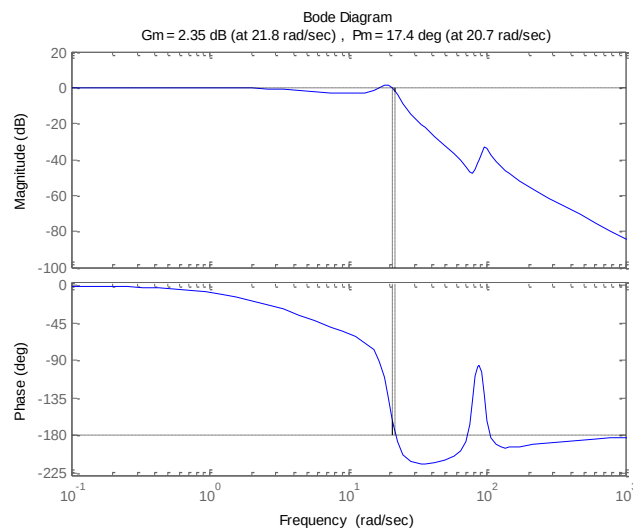


Table I shows the variations in frequency of oscillations of contact wire for different span length and train speed. It is seen that the frequency of oscillations increases as train speed increases.

Table I Variations in frequency of oscillations of contact wire for different span length and train speed

Train Speed	Frequency of Oscillations($\omega = \frac{2\pi v}{l}$)		
	Span length, l, 54m	Span length, l, 63m	Span length, l, 72m
100km/h	3.23	2.76	2.42
200km/h	6.46	5.52	4.84
300km/h	9.69	8.28	7.26

For different train speeds, the normalized sensitivity of open loop system and closed loop system with PID controller has been calculated, with respect to parameters of interest. The results are presented in table II and table III respectively. It has been observed that as the frequency of oscillations increase, the system becomes more sensitive wrt all the parameters of interest for both open loop and closed loop system. Further, for closed loop system, the system is more sensitive to proportional gain and time integral constant whereas it is least sensitive wrt time derivative constant.

Table II Computed values of normalized sensitivity of open loop system from sensitivity functions developed

Parameter, p $\rightarrow$	Normalised sensitivity ($\omega=2.42$)	Normalised sensitivity ($\omega=4.84$)	Normalised sensitivity ($\omega=7.26$)
$\hat{S}_{w_f}^{G(s)}$	0.0150 - 0.0010i	0.0621 - 0.0090i	0.1473 - 0.0348i
$\hat{S}_{w_h}^{G(s)}$	$-1.1862 \times 10^{-4} + 4.5086 \times 10^{-5}i$	$-2.2586 \times 10^{-4} + 3.4139 \times 10^{-4}i$	$-0.0007 + 0.0013i$
$\hat{S}_{p_{vf}}^{G(s)}$	-0.0008 - 0.0110i	-0.0033 - 0.0226i	-0.0084 - 0.0357i
$\hat{S}_{p_{rw}}^{G(s)}$	-0.0803 - 1.1021i	-0.3474 - 2.2721i	-0.8932 - 3.5780i
$\hat{S}_{k_h}^{G(s)}$	0.9471 - 0.0576i	0.9413 - 0.1161i	0.9312 - 0.1766i

Table III Normalized sensitivity of closed loop system from sensitivity functions developed

Parameter, $p \rightarrow$	Normalised sensitivity ($\omega=2.42$)	Normalised sensitivity ($\omega=4.84$)	Normalised sensitivity ($\omega=7.26$)
$\hat{S}_{k_p}^{M(s)}$	$0.9432 + 0.2435i$	$0.9895 + 0.1322i$	$1.0002 + 0.0957i$
$\hat{S}_{T_i}^{M(s)}$	$-0.9452 - 0.2392i$	$-0.9945 - 0.1225i$	$-1.0098 - 0.0798i$
$\hat{S}_{T_d}^{M(s)}$	$-8.7109 \times 10^{-4} - 2.2046 \times 10^{-4}i$	$-0.0037 - 0.0005i$	$-0.0085 - 0.0007i$

In order to verify the correctness of sensitivity functions, the sensitivity values of closed loop system are also computed by using difference equations at $\omega=2.42$ as shown in table IV. The results conform to each other.

Table IV Validation of Results for closed loop system

Parameter, $p \rightarrow$	From sensitivity functions developed	From Difference Equations
$\hat{S}_{k_p}^{M(s)}$	$0.9432 + 0.2435i$	$0.93+0.20i$
$\hat{S}_{T_i}^{M(s)}$	$-0.9452 - 0.2392i$	$-0.959-0.1852i$
$\hat{S}_{T_d}^{M(s)}$	$-8.7109 \times 10^{-4} - 2.2046 \times 10^{-4}i$	$0.00-0.00i$

V. Conclusions

The mathematical model of active pantograph-catenary system using PID controller has been developed and the effect of controller parameters on the system has been studied using sensitivity analysis has been studied first time in the literature. The system is found to be most sensitive wrt proportional gain and time integral of the PID controller.

The sensitivity of the pantograph-catenary system wrt parameters of interest is also dependent on the frequency of oscillations of contact wire, which depend on the train speed. The system becomes more sensitive at higher frequency of oscillations.

The numerical results are validated using difference equations. These studies shall be useful in improving the performance of the active controller for pantograph-catenary system.

Appendix I

Data for pantograph-catenary system,

Mass of frame, $W_f = 17.2\text{kg}$, Head Mass, $W_h = 9.1\text{kg}$, Head Spring Constant, $k_h = 7 \times 10^3\text{N/m}$,
Catenary represented by spring, $k_c = 1.535 \times 10^6\text{N/m}$, Shoe Spring Constant, $k_s = 82.3 \times 10^3\text{N/m}$
Damping co-efficient, $P_{hv} = 130\text{Ns/m}$, Viscous co-efficient, $P_{vf} = 30\text{Ns/m}$, $P_{hv} = 130\text{Ns/m}$
Gain of input transducer, $G_i = 1/10$, Gain of actuator, $G_a = 1/10$, Feedback, $H=1$

References

- [1] J.H. Lee et al "Performance evaluation and design optimization using differential evolutionary algorithm of the pantograph for the high speed train", Journal of Mechanical Science and Technology, 26(10), 2012, pp. 3253-3260.
- [2] P. Mahajan, R.Garg and P. Kumar, "Sensitivity Analysis of Pantograph Catenary System Model," Presented at IEEE India International Conference on Power Electronics, IICPE 2012. [Online]. Available: IEEE Xplore, DOI: 10.1109/IICPE.2012.6450475.
- [3] Boissonnade P., "Catenary Design for High Speeds," Rail International, Mar. 1975.
- [4] Communications of the O.R.E., "Behavior of Pantograph and Overhead Equipment at Speeds above 160km/hr," Rail International, Jan. 1972.
- [5] Coxon D.J., Gostling R.J., and Whiteland K.M., "Evolution of a Simple High Performance Pantograph," Railway Gazette International, Jan. 1980.
- [6] F. Resta et al., "Actively Controlled pantograph: An application," in Proc. IEEE/ASME Int. Conf. Adv. Intell. Mechatron., Como Italy 2001, pp.243-248.
- [7] A. Pisano and E. Usai, "Output feedback regulation of the contact force in high speed train pantographs," Proc. IEEE Conf., Decision Control, Orlando, FL, 2001, pp.1831-1836.
- [8] Lin Yu-Chen et al., "Robust active vibration control for rail vehicle pantograph," IEEE Transactions on Vehicular Technology, Vol.56, No. 4, July 2007, pp. 1994-2004.
- [9] A.Alberto and J. Benet, "A high performance tool for the simulation of the dynamic pantograph-catenary interaction", Mathematics and Computers in Simulation, 79(2008), 652-667.
- [10] T.J. Park, C.s. Han, and J.H. Jang, "Dynamic sensitivity analysis for the pantograph of a high speed rail vehicle", Journal of Sound and Vibration, 266(2003), 235-260.
- [11] J.W.kim et al, "State sensitivity analysis of the pantograph system for a high speed rail vehicle considering span length and static uplift force," Journal of Sound and Vibration, 303(2007), 405-427.
- [12] S. Midya et al., "Pantograph arcing in electrified railways mechanism and influence of various parameters part I: with dc traction power supply," IEEE Transactions on Power Delivery, vol. 24, no. 4, 2009, pp. 1931-1939.

- [13] S. Midya et al., "Pantograph arcing in electrified railways mechanism and influence of various parameters part II: with ac traction power supply," IEEE Transactions on Power Delivery, vol. 24, no. 4, 2009, pp. 1940-1950.
- [14] S.D. Eppinger, et al, "Modeling and Experimental Evaluation of Asymmetric pantograph Dynamics", Journal of Dynamic System, Measurement and Control, June 1988, Vol. 110/169, pp 168-174.
- [15] O.Conoor, et al, "Active Control of High Speed Pantograph", Journal of Dynamic System, Measurement and Control, March 1997.
- [16] Cristina, S. and D'Amore, M. "Sensitivity analysis of modal quantities for horizontal transmission lines", Proc. IEEE, Canadian communications and Power conference, October 18-20, 1978, pp 172-175.
- [17] C.S.Indulkar, Parmod Kumar and D.P. Kothari, "Sensitivity Analysis of multi conductor transmission line", *Proceeding IEEE*, Vol. 70, No.3, March 1982, pp 289 – 300.
- [18] P. Mahajan, R. Garg, and P. Kumar, (2011, Jan.) "Sensitivity analysis of railway electric traction system", Presented at IEEE India International Conference on Power Electronics, IICPE 2010. [Online]. Available: IEEE Xplore, DOI: 10.1109/IICPE.2011.5728107