



Modified lumped model for Transient heat conduction in spherical shape

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Abstract: Classical lumped model is valid for lower biot number and lower temperature gradient so an effort to implement the modified lumped model which is use for higher biot number and higher temperature gradient. The article deals with the improved lumped model for spherical shape, analysis of temperature variation with time for heat generated in spherical shape and natural convection cooling in spherical shape by using polynomial approximation method. The transient temperature is found which is a function of time, heat source parameter and biot number. The modified biot number is also obtained which is an important role in transient heat conduction.

Keywords: *modified biot number; modified lumped model; polynomial approximation method; biot number.*

I. Introduction

When a solid object placed at a fluid of lower temperature, the heat transfer takes place by the heat conduction within the solid and heat convection between the hot surface and the fluid. The conductive resistance in solid is sufficiently lower than convective resistance then consider a lumped model. Classical lumped model is applicable for only lower biot number and in addition to lower temperature gradient. In many engineering application as the analysis of thermo hydraulic nuclear reactor, boiling water reactor involve higher biot number so classical lumped model is not valid. Many efforts have to be done to improve the lumped model for several years.

P. Keshavarz and M.Taheri [1] developed improved lumped parameter model for long hot object which is placed in ambient by use of polynomial Approximation Technique. The obtained results of the presented model are comparing to the simple lumped model as well as the Sadat model.

However, a few efforts have been made to formulate a modified lumped parameter model for cylinder and slab (specified heat flux and heat generation), S.K.Sahu and P. Behera [2] developed improved lumped model by employing polynomial approximation method. The results obtained by polynomial approximation method are better accuracy than finite difference method and perturbation method. Classical lumped model is limited to values of Biot numbers less than 0.1075. Jian Su [3] analyzed the unsteady cooling of a long slab by heat convection and developed improved lumped models that can be applied in transient heat conduction with larger values of Blot numbers. The improved lumped models are obtained through two point hermit approximations for integrals.

H. Sadat [4] developed improved lumped model by using perturbation technique to develop first order conduction model for the slab, infinite cylinder and the spheres problem. The method developed by this improved lumped model evaluated against the exact solution in cylindrical geometry.

Jian Su and M. Cotta [5] developed improved lumped model for nuclear fuel rod by using hermite approximation for integrals. Hermit integration method is use to obtain the average temperature of cladding and fuel rod in radial direction. Transient analysis of fuel, cladding and coolant can be analyzed.

A classical lumped parameter model for transient cooling of a spherical body has been applicable for radiation conduction parameter (Nrc) less than 0.7. Jian Su [6] proposed a improved lumped parameter model for transient cooling of a spherical body which is applicable for larger value of the radiation conduction parameter Nrc. Proposed lumped parameter model developed by using of two point hermite approximation for integral.

For predicting the temperature variation and analysis of structure in the nuclear fuel rod classical lumped approach is not applicable due to higher temperature gradient thus K.M. Pandey and Amrit Sarkar [7] using Finite Element method in ANSYS prediction of temperature distribution from a nuclear fuel rod. .

R.M Cotta [8] formalizes the ideas in this so-called coupled integral equations approach (CIEA), interpreted as a formulation simplification tool. Therefore we intend to demonstrate that under certain boundary conditions and other characteristics of the diffusion process the fully differential formulations can be markedly simplified through a reduction of the number of independent variables involved. Wolfgang Wulff [9] presents an alternative formulation for thermal conduction in reactor components which is based on the integral method. Integral methods lead from the partial differential equation of heat conduction to only one (or in the case of fuel elements with pellet and cladding possibly to two) ordinary differential equation(s) for the mean temperature(s), and they account fully for the difference between mean and surface temperatures. Transient heat conduction models based on integral methods are simple. They are amenable to evaluation on programmable pocket calculators, as has been shown with the successful analysis of temperature transients in reactor fuel elements under conditions of a large break loss of coolant accident.

In the present work an attempt to develop a modified biot number for spherical shape and to analyze a temperature versus time for heat generated in spherical shape and natural convection cooling in spherical shape.

Nomenclature

Θ Dimensionless temp.	k thermal conductivity	α constant	Q dimensionless heat source parameter
$\bar{\Theta}$ Average temperature	$\bar{q}$ internal heat generation W/m^3	a_0, a_1, a_2 constant	defined in equation (5)
τ Dimensionless time	x dimensionless length	X, Y defined parameter in (27)	B Biot number
T Object temperature	r, R length coordinates	Bi parameter defined in (30)	h heat transfer coeff.
T_∞ ambient temperature	t time		
T_0 initial temp. of hot obj.			

II. Theoretical analysis

II.1. Analysis of temperature variation with time and develop a modified lumped model for heat generated in spherical shape: Consider a heat generated spherical long shape object (lower thermal conductivity, higher biot number) which is at very higher temperature (T) placed in a fluid of lower temperature. The heat transfer takes place by the heat conduction within the solid and heat convection between the hot surface and the fluid. The conductive resistance in solid is sufficiently lower than convective resistance then consider a lumped mode. classical lumped model is not valid due to long solid object, lower thermal conductivity, higher biot number and higher temperature gradient so an effort will be made to obtain the modified lumped parameter model by using polynomial approximation method. (Thermal conductivity, specific heat and density are assume to be constant)

Generalized transient heat conduction equation for heat generated spherical shape is given as

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{\bar{q}}{k} \quad (1)$$

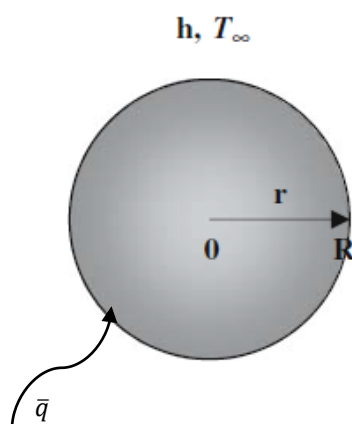


Fig. II.1: Schematic of spherical shape with heat generation

Apply Boundary conditions

$$\frac{\partial T}{\partial r} = 0 \text{ at } r = 0 \quad (2)$$

$$-k \frac{\partial T}{\partial r} = h(T - T_\infty) \text{ At } r = R \quad (3)$$

$$\text{And initial cond. } T = T_0 \text{ at } t = 0 \quad (4)$$

Dimensionless parameter defined as:

$$\Theta = \frac{T-T_\infty}{T_0-T_\infty}, B = \frac{hR}{k}, \tau = \frac{\alpha t}{R^2}, x = \frac{r}{R}, Q = \frac{\bar{q}}{K(T-T_\infty)} R^2 \quad (5)$$

Using equation (5): equation (1) to (4) can be written as

$$\frac{\partial \Theta}{\partial \tau} = \frac{1}{x^2} \frac{\partial}{\partial x} \left(x^2 \frac{\partial \Theta}{\partial x} \right) + Q \quad (6)$$

$$\frac{\partial \Theta}{\partial x} = 0 \text{ at } x = 0 \quad (7)$$

$$\frac{\partial \Theta}{\partial x} = -B\Theta \text{ at } x = 1 \quad (8)$$

$$\Theta = 1 \text{ at } \tau = 0 \quad (9)$$

Solution procedure by polynomial approximation method

Polynomial approximation method:

The method involve two steps first selection of the proper polynomial with time dependent coefficient, and second to convert a partial differential equation into an integral equation. This integral can be converted into an ordinary differential equation.

Assume polynomial temperature profile:

$$\Theta = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2 \quad (10)$$

$$\frac{\partial \Theta}{\partial x} = a_1 + 2a_2 x \quad (11)$$

Apply first Boundary cond.

$$\frac{\partial \Theta}{\partial x} = a_1 + 2a_2 x = 0 \text{ At } x = 0 \quad (12)$$

$$a_1 = 0$$

Applying second boundary condition so that

$$a_1 + 2a_2 = -B\Theta \quad (13)$$

$$a_2 = -\frac{B\Theta}{2} \quad (14)$$

Second boundary condition can written as

$$\frac{\partial \Theta}{\partial x} = -B\Theta = -B(a_0 + a_1 + a_2) \quad (15)$$

$$a_0 = \Theta \left(1 + \frac{B}{2} \right) \quad (16)$$

Average temperature for long spherical shape can be written as [1]

$$\bar{\Theta} = 3 \int_0^1 x^2 \Theta dx \quad (17)$$

$$\begin{aligned} &= 3 \int_0^1 (x^2 a_0 + a_1 x^3 + a_2 x^4) dx \\ &= 3 \left[\frac{a_0}{3} + \frac{a_1}{4} + \frac{a_2}{5} \right] \end{aligned} \quad (18)$$

Put the value of a_0, a_1 & a_2 in equation (18)

$$\bar{\Theta} = \Theta \left[1 + \frac{B}{5} \right] \quad (19)$$

$$\frac{\partial \bar{\Theta}}{\partial \tau} = \frac{\partial \Theta}{\partial \tau} \left(1 + \frac{B}{5} \right) \quad (20)$$

Integrating the governing equation with respect to x us has

$$\int_0^1 x^2 \frac{\partial \Theta}{\partial \tau} dx = \int_0^1 \frac{\partial}{\partial x} \left(x^2 \frac{\partial \Theta}{\partial x} \right) dx + \int_0^1 Q x^2 dx \quad (21)$$

$$\int_0^1 x^2 \frac{\partial \theta}{\partial \tau} dx = \int_0^1 6a_2 x^2 dx + \int_0^1 Q x^2 dx$$

$$= 2a_2 + \frac{Q}{3}$$

$$3 \int_0^1 x^2 \frac{\partial \theta}{\partial \tau} dx = 6a_2 + Q \quad (22)$$

$$\frac{\partial \theta}{\partial \tau} = -3B\theta + Q \quad (23)$$

$$\frac{\partial \theta}{\partial \tau} = \frac{-3B\theta}{1+\frac{B}{5}} + \frac{Q}{1+\frac{B}{5}} \quad (24)$$

Or we may write as

$$\frac{\partial \theta}{\partial \tau} = -X\theta + Y \quad (25)$$

Integrate (25)

$$\frac{1}{X} \ln(X\theta - Y) = -\tau$$

$$\theta = \frac{\exp(X\tau) + Y}{X} \quad (26)$$

$$\text{where } X = \frac{B}{\frac{1}{3} + \frac{B}{15}}, \text{ \& } Y = \frac{Q}{1 + \frac{B}{5}} \quad (27)$$

For natural convection cooling in spherical shape put $Q = 0$ in equation (6) and solved by the same procedure than $\theta = \exp\left(\frac{-15B\tau}{B+5}\right)$ (28)

For simple lumped model can be written as $\theta = \exp(-Bi\tau)$ (29)

Compare (28) and (29) Thus modified biot number $Bi = \frac{15B}{B+5}$ (30)

III. Result and discussion

We have tried to analyze the dimensionless temperature versus dimensionless time for heat generated in spherical and cylindrical shape by using the modified lumped model.

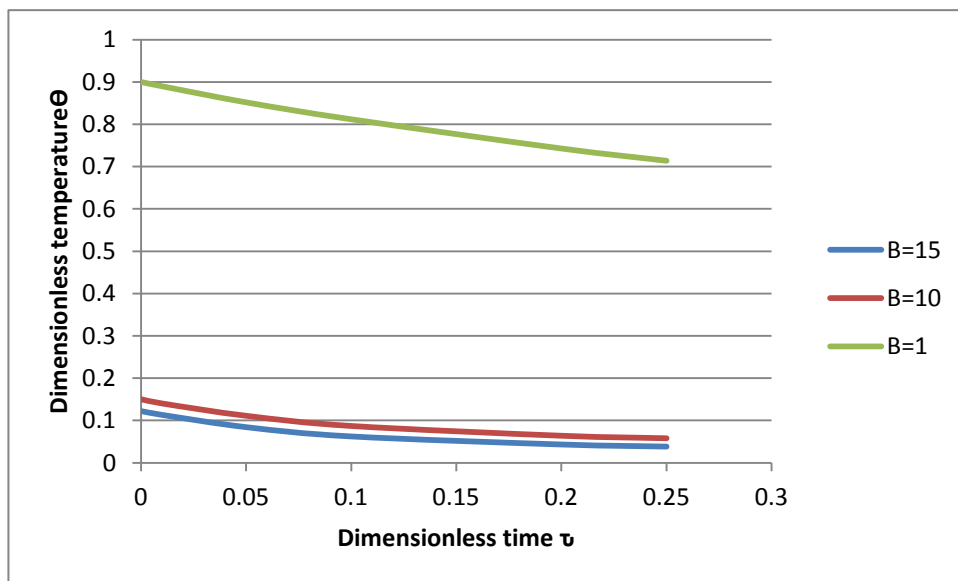


Fig.III.1 Θ versus τ for heat generated in spherical shape ($Q=1$)

From equation (27) X increases and Y decreases by increasing the value of Biot number (B) than Θ decreases so Fig.III.1 shows biot number increases than Θ decreases at a particular time.

Biot no increases so convective resistance decreases temperature drop is increases rapidly with respect to time at higher biot no. Fig.III.2 shows that at the higher biot number temperature drop with respect to particular interval is more than lower biot number for heat generated in spherical shape

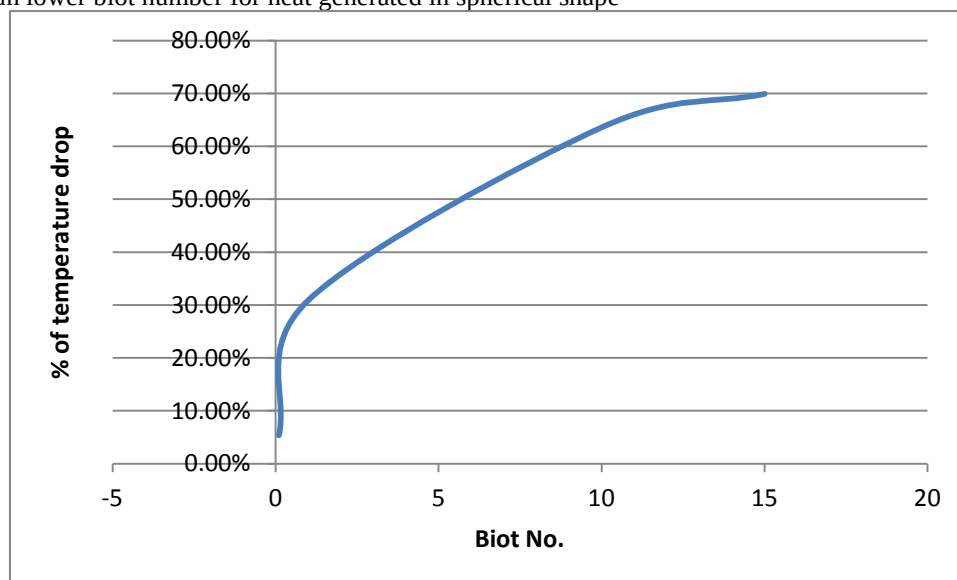


Fig.III.2 % of temperature drop versus Biot no. for heat generated in spherical shape

IV. Conclusion

A modified lumped model can be calculated the temperature at any higher biot number. A temperature versus time relationship for heat generated in spherical shape and natural convection cooling in spherical shape is obtained by the Polynomial approximation method. A unique modified biot number is obtained which play an important role in transient heat conduction.

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