



ABILITY ASSESSMENT OF E10 B EXCHANGE SYSTEM UNDER PERFECT SWITCHING USING BOOLEAN FUNCTION TECHNIQUE

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Abstract: The paper presents a multivariable Boolean function technique that supports the class function of Reliability of synchronous Transport module subjected to specific operating conditions. The whole system consists of five main parts. All five parts connected in series. The author has been used the Boolean functions and the algebra of logic to evaluate ability measures for considered system. Reliability of the whole system has obtained in two different cases as the failures follows exponential time distribution and the failures follow weibull time distribution.

KeyWords: Operational Availability, Reliability Algebra logics, Boolean function technique, exponential Distribution, parallel redundancy, weibull distribution

I. Introduction

Gupta P.P, Agarwal S.C has considered a Boolean Algebra Method for Reliability calculations and again Gupta P.P Kumar Arvind, Reliability and M.T.T.F Analysis of Power Plant. Sharma, Deepankar Sharma, Neelam, Evaluation of some Reliability Parameters for Tele-communication system by Boolean Function Technique and Sharma Deepankar. Sharma. Neelam, Evaluate the Reliability and M.T.T.F of solar voltaic cell. In this paper, the author has considered the ability of data transfer in network management system. The system configuration has shown in figure 1. The whole system consists of seven main parts. First part of the system is a container and it has four sections in standby redundancy we can use four types of container C_4, C_3, C_2, C_1 . Second part of the system is a Virtual container. Third part of the system is a tributary unit. Fourth part of the system is a tributary unit Group. Fifth part is a virtual concatenation. Sixth part is known as administrative unit group. Seventh part is known as synchronous Transport Module. All these seven parts connected in series. Therefore, the failure of any of the four parts causes the total failure of the system.

To avoid the tedious calculations the author has used the Boolean functions and algebra of logic to evaluate ability measures for considered system. Reliability of the whole system has obtained in two different cases as the failures follow Weibull time distribution. One numerical example together with its graphical illustration has also been appended at the end to highlight the important results of the study.

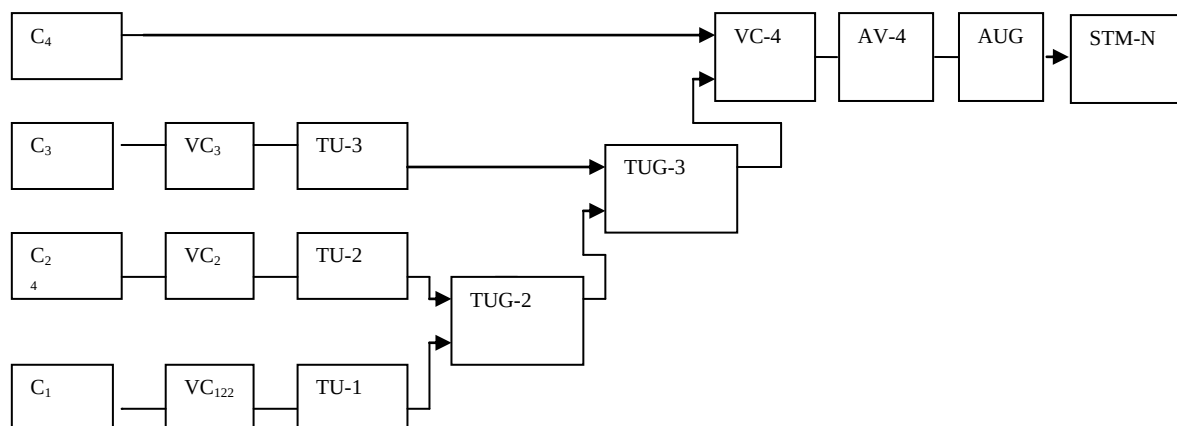


Fig. 1: System Configuration

II. Assumptions

The following assumptions have been associated with this model:

- (1) Initially, all parts of the system are in good state.
- (2) The reliability of the each part moreover each section is in advance.
- (3) The transaction of fuel from one part to other is hundred percent reliable.
- (4) There is no repair facility to repair a failed part.
- (5) Failures are s-independent.
- (6) Every part of the system remains either in good or in bad state

III. Nomenclature

$\wedge / \vee / '$	:	Conjunction / Disjunction / Complement.
X_1, X_2, X_3, X_4	:	States of Container C_1, C_2, C_3, C_4
X_5, X_6, X_7	:	States of virtual Container
X_8, X_9, X_{10}	:	States of Tributary unit
X_{11}, X_{12}	:	States of Tributary unit Group
X_{13}	:	State of virtual Container.
X_{14}	:	State of administrative unit
X_{15}	:	State of administrative unit group
X_{16}	:	State of synchronous Transport Module.
$X_i (i = 1, 2, \dots, 16)$	:	1 in good state; 0 is bad state.
$\left \begin{array}{c} \\ \\ \end{array} \right $	:	This notation has used to represent logical matrix.
R_i	:	Reliability of the i^{th} part of the system, $\forall i = 1, 2, \dots, 16$
R_s	:	Reliability of the whole system.
R_{SW}/R_{SE}	:	Reliability of the system as a whole when failures follow weibull/ Exponential distribution.

IV. Formulation of Mathematical Model

By making use of Boolean function technique, the conditions of capability of successful operations of the system in terms of logical matrix are expressed as shown below:

$$F(x_1, x_2, \dots, x_{16}) = \left| \begin{array}{cccccc} x_1 & x_{13} & x_{14} & x_{15} & x_{16} \\ x_2 & x_5 & x_8 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_3 & x_6 & x_9 & x_{11} & x_{12} & x_{14} & x_{15} & x_{16} \\ x_4 & x_7 & x_{10} & x_{11} & x_{12} & x_{14} & x_{15} & x_{16} \end{array} \right| \quad (1)$$

V. Solution of Model

By using algebra of logics, we may write equations (1) again as follows

$$F(x_1, x_2, \dots, x_{16}) = \left| (x_{13}, x_{14}, x_{15}, x_{16}) \wedge f(x_1, x_2, \dots, x_{16}) \right| \quad (2)$$

$$\text{Where, } f(x_1, x_2, \dots, x_{16}) = \left| \begin{array}{cccc} x_1 & & & \\ x_2 & x_5 & x_8 & x_{12} \\ x_3 & x_6 & x_9 & x_{11} \\ x_4 & x_7 & x_{10} & x_{11} \end{array} \right| = \left| \begin{array}{c} M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} \right| \quad (3)$$

$$\text{Where } M_1 = |x_1| \quad (4)$$

$$M_2 = |x_2 x_5 x_8 x_{12}| \quad (5)$$

$$M_3 = \begin{vmatrix} x_3 & x_6 & x_9 & x_{11} \end{vmatrix} \quad (6)$$

$$M_3 = \begin{vmatrix} x_4 & x_7 & x_{10} & x_{11} \end{vmatrix} \quad (7)$$

By orthogonalization, algorithm (3) may be written as

$$f(x_1, x_2, \dots, x_{16}) = \begin{vmatrix} M_1 & & & \\ M'_1 & M_2 & & \\ M'_1 & M'_2 & M_3 & \\ M'_1 & M'_2 & M'_3 & M_4 \end{vmatrix} \quad (8)$$

$$\begin{aligned} \text{Now, } M'_1 M'_2 &= \begin{vmatrix} x'_1 & x_2 & x_5 & x_8 & x_{12} \end{vmatrix} \\ &= \begin{vmatrix} x'_1 & x_2 & x_5 & x_8 & x_{12} \end{vmatrix} \end{aligned} \quad (9)$$

$$\text{Similarly, } M'_1 M'_2 M'_3 = \begin{vmatrix} x_1 & x_3 & x_6 & x'_7 & x'_8 & x_9 \\ x_1 & x_3 & x_6 & x_7 & x'_8 & x_9 \end{vmatrix} \quad (10)$$

$$M'_1 M'_2 M'_3 M'_4 = \begin{vmatrix} x'_1 & x'_2 & x'_3 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x'_5 & x'_3 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x_5 & x'_8 & x'_3 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x_5 & x_8 & x'_{12} & x'_3 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x'_2 & x_3 & x'_6 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x'_5 & x_3 & x'_6 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x_5 & x'_8 & x_3 & x'_6 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x_5 & x_8 & x'_{12} & x_3 & x'_6 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x'_2 & x_3 & x_6 & x'_9 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x'_5 & x_3 & x_6 & x'_9 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x_5 & x'_8 & x_3 & x_6 & x'_9 & x_4 & x_7 & x_{10} & x_{11} \\ x'_1 & x_2 & x_5 & x_8 & x'_{12} & x_3 & x_6 & x'_9 & x_4 & x_7 & x_{10} & x_{11} \end{vmatrix} \quad (11)$$

Putting all these values in equation (8), and again using this result in (2),

$$\begin{array}{l}
 x_1 \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_5 \quad x_8 \quad x_{12} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2' \quad x_3 \quad x_6 \quad x_9 \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_3 \quad x_5' \quad x_6 \quad x_9 \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_3 \quad x_5 \quad x_6 \quad x_8' \quad x_9 \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_3 \quad x_5 \quad x_6 \quad x_8 \quad x_9 \quad x_{11} \quad x_{12}' \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2' \quad x_3' \quad x_4 \quad x_7 \quad x_{10} \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_3' \quad x_4 \quad x_5' \quad x_7 \quad x_{10} \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_3' \quad x_4 \quad x_5 \quad x_7 \quad x_8' \quad x_{10} \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_3' \quad x_4 \quad x_5 \quad x_7 \quad x_8 \quad x_{10} \quad x_{11} \quad x_{12}' \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2' \quad x_3 \quad x_4 \quad x_6' \quad x_7 \quad x_{10} \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
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 x_1' \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6' \quad x_7 \quad x_8 \quad x_{10} \quad x_{11} \quad x_{12}' \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2' \quad x_3 \quad x_4 \quad x_6 \quad x_7 \quad x_9' \quad x_{10} \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
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 x_1' \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8' \quad x_9' \quad x_{10} \quad x_{11} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16} \\
 x_1' \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9' \quad x_{10} \quad x_{11} \quad x_{12}' \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16}
 \end{array} \tag{12}$$

Since equation (12) is the disjunction of pair wise disjoint conjunctions, therefore, the reliability of the whole system is given by:

$$\begin{aligned}
 R_s = & R_{13}R_{14}R_{15}R_{16} [R_1 + R_2R_5R_8R_{12} - R_1R_2R_5R_8R_{12} + R_3R_6R_9R_{11} - R_1R_3R_6R_9R_{11} - R_{12}R_2R_3R_5R_6R_8R_9R_{11} \\
 & + R_1R_{12}R_2R_3R_5R_6R_8R_9R_{11} + R_4R_7R_{10}R_{11} - R_1R_4R_7R_{10}R_{11} \\
 & + R_1R_2R_3R_4R_5R_7R_{10}R_{11} - R_2R_4R_5R_7R_8R_{10}R_{11}R_{12} - R_1R_2R_4R_5R_7R_8R_{10}R_{11}R_{12} \\
 & + R_2R_3R_4R_5R_6R_7R_8R_9R_{10}R_{11}R_{12} - R_1R_2R_3R_4R_5R_6R_7R_8R_9R_{10}R_{11}R_{12} \\
 & + R_1R_8R_2R_3R_4R_5R_7R_{10}R_{11} - R_9R_3R_4R_6R_7R_{10}R_{11} - R_1R_3R_4R_6R_7R_9R_{10}R_{11})
 \end{aligned} \tag{13}$$

Where, R_i ($i = 1, 2, \dots, 16$) is the reliability of the section state x_i ($i = 1, 2, \dots, 16$) respectively.

VI. Some Particular Cases

Case i: If the reliability of every section of the system is R:

Then equation yields:

$$R_s = R^5 + 3R^8 - 3R^9 - R^{11} - 2R^{12} + 3R^{13} + R^{15} - R^{16} \tag{14}$$

Case ii: when failure rates follow Weibull time distribution:

Let a_i be the failure rate corresponding to section state x_i , $\forall i = 1, 2, \dots, 16$, then reliability of diesel engine at an instant 's', is given by

$$\begin{aligned}
 R_{sw}(t) = & \exp\{-5at^p\} + 3\exp\{-8at^p\} - 3\exp\{-9at^p\} - \exp\{-11at^p\} \\
 & - 2\exp\{-12at^p\} + 3\exp\{-13at^p\} + \exp\{-15at^p\} - \exp\{-16at^p\}
 \end{aligned} \tag{15}$$

Where, p is a positive parameter;

Case iii: When failure rates follow exponential time distribution:

Exponential distribution is nothing but a particular case of Weibull distribution for $p=1$ and is very useful for practical problems purpose. Therefore, the reliability of considered system as a whole at an instant's' is expressed as:

$$R_{SE}(t) = e^{-5at} + 3e^{-8at} - 3e^{-9at} - e^{-11at} - 2e^{-12at} + 3e^{-13at} + e^{-15at} - e^{-16at} \quad (16)$$

The expression for M.T.T.F. in this case is given by

$$\begin{aligned} \text{M.T.T.F.} &= \int_0^{\infty} R_{SE}(t) dt \\ &= \frac{0.55236014}{a} \end{aligned} \quad (17)$$

VII. Numerical Computation

Setting $a = 0.1$, $p = 2$, $t = 0, 1, 2, \dots$ in equations (15) and (16) and $a = 0.01, 0.02, 0.03, \dots$ in equation (17), one can sketch the graphs as given in figures 2,3 respectively.

t	Rse(t)	Rsw(t)
0	1	1
1	0.63837809	0.638378087
2	0.41727886	0.164280782
3	0.26574185	0.012372858
4	0.16428078	0.000342044
5	0.09938859	3.73233E-06
6	0.05935791	1.52309E-08
7	0.03522836	2.28974E-11
8	0.02086904	1.26642E-14
9	0.01237286	2.57676E-18
10	0.00735209	1.92875E-22

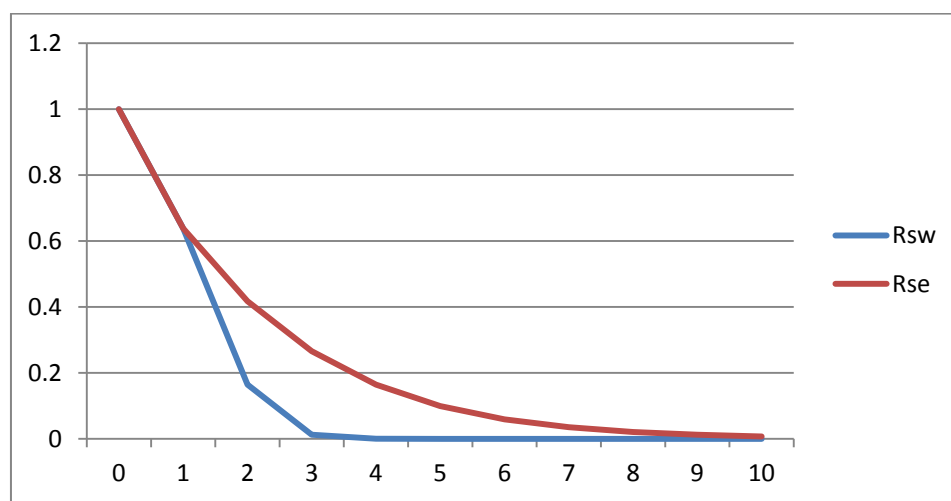


Fig.1 (Graph between Rse and Rsw)

Time(t)	M.T.T.F
0.01	55.236014
0.02	27.618007
0.03	18.41200467
0.04	13.8090035
0.05	11.0472028
0.06	9.206002333
0.07	7.890859143
0.08	6.90450175
0.09	6.137334889
0.1	5.5236014

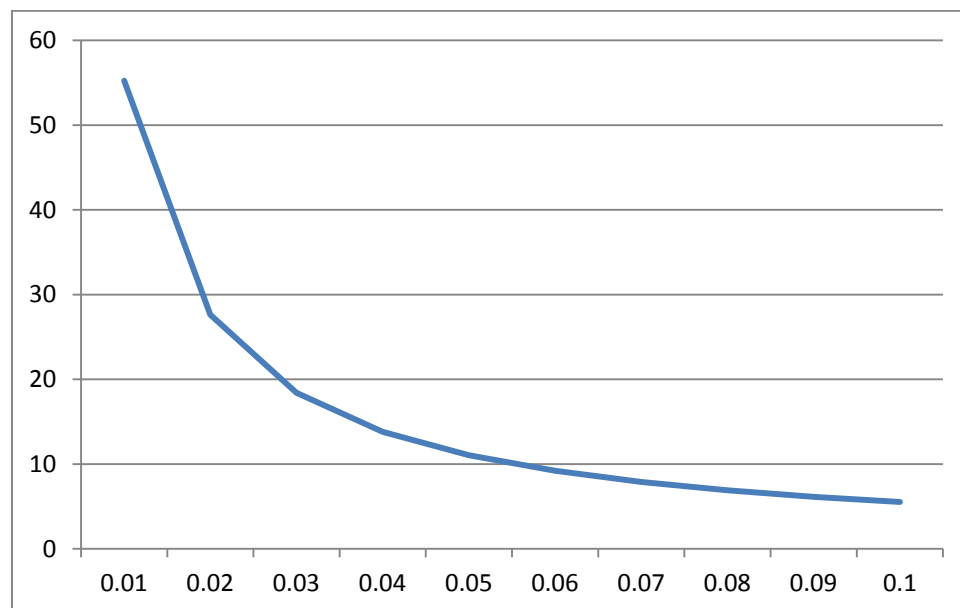


Fig.2 (Graph between Time and M.T.T.F)

VIII. Results and Discussion

The reliability of the whole system has been obtained in two different cases as the failure follows exponential and weibull time distributions.

A critical examination of the graph, reliability Vs time, indicates that the reliability of this complex system decreases approximately at a uniform rate in case of exponential distribution, whereas it decreases very rapidly when failure follows Weibull distribution.

Fig.2 indicates the mean time to failure of the system for different values of failure rates. An inspection of the graph M.T.T.F. Vs failure rate, shows that in the beginning M.T.T.F decreases catastrophically but latter on it decreases approximately at a uniform rate

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