



Multi Objective Portfolio Optimization Based on Value-at-Risk by Non Constant Volatility and Using Greedy Search

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Abstract: In this paper we analyzed the multi-objective portfolio optimization problem, ie maximization of expected return and minimizing the risk of an investment portfolio. Portfolio risk in this matter was measured by using Value-at-Risk. Both expected return and Value-at-Risk portfolio estimated by using the time series approach, i.e. the mean and volatility are assumed to have non-constant. The non-constant mean is estimated using models of autoregressive moving average, while the non-constant volatility is estimated using models of autoregressive conditional heteroscedastic. To solve the multi objective portfolio optimization problem mentioned above done by using greedy search algorithm. As a numerical illustration in the multi-objective portfolio optimization problem was analyzed five stocks traded on the stock market in Indonesia. Portfolio optimization results obtained that the maximum expected return is 0.001204 and the minimum Value-at-Risk is 0.016006.

Keywords: ARMA, GARCH, optimization, Value-at-Risk, Greedy search

I. Introduction

"Do not put all your eggs in one basket!" That phrase is often heard in the field of investment. Because if the basket falls, all the eggs in it will break [16]. Similarly, investment, place the wealth in a variety of investment instruments such as on some stocks. The combination of various investment instruments referred to as a portfolio [5; 16]. Make an investment objective is to get the maximum return, in order to increase the value of wealth, but with the smallest possible of risk. Portfolio of stocks is an investment that consists of various different companies stocks the hope that if one stocks declined, while the other increases, the investment is not a loss [5; 6]. Risk of loss is often measured by the variance, but lately the risk is often measured using the quintile which is often referred to as Value-at-Risk (VaR) [2; 7; 8]. Value-at-Risk is widely used as a measure of risk using the variance often can not accommodate all risk of loss events [7].

Behavior of stock price movements in the stock market is usually related to "that does not irregular movements (volatility)" [15]. In statistics, volatility is usually measured by the variance or standard deviation. The level of volatility will describe the magnitude of the risk level of a stock. In time series analysis, volatility can be estimated using Autoregressive Conditional Heteroscedasticity (ARCH) models [1; 3; 15]. ARCH model developed by Engel in 1982, later developed by Bollerslev in 1986, and by Nelson in 1991, and by others can be used to demonstrate the volatility as a function of time. ARCH models are able to make many adjustments to the real model of the behavior of volatility. In the literature known to the various types of ARCH models, such as GARCH, GARCH-M, EGARCH, and TGARCH [8; 14]. Estimated volatility of stock returns with ARCH models, among others, the approach has been carried out by Deng [1] and Yu [15].

The strategy is often used in conditions in the stock investment is very volatile (risky) is forming a portfolio. The nature of the portfolio is to allocate the capital formation on a variety of investment alternatives; in order to obtain the maximum return with the risks can be minimized [4; 5; 16]. In this paper, risk is measured using the Value-at-Risk (VaR), thus the purpose of this research is to maximize the expected return of the portfolio and minimizing Value-at-Risk (VaR). To resolve the problem maximizing expected return and minimizing Value-at-Risk in this paper are done using by the *greedy search algorithms*.

Furthermore, a portfolio, categorized as efficient when the portfolio is located on the surface efficiently (efficient frontier). Efficient frontier is the curve that connects the efficient portfolio that has the lowest risk level with efficient portfolio that has the highest expected return [5; 12]. Among these efficient portfolios will be selected an optimum portfolio. In this paper also defined the set of efficient frontier and also determined the optimum portfolio pair. As a numerical experiment, the above methods used to analyze the five stocks traded on capital markets in Indonesia.

II. Methodologies

Let P_{it} and r_{it} respectively states price and stock return i ($i=1,...,N$ and N is the number the stock being analyzed), at time t ($t=1,...,T$ and T the period of observation data). Stock returns r_{it} are calculated using the formula $r_{it} = \ln(P_{it} / P_{it-1})$ [2; 14]. Return data are then made estimates of the mean and volatility models as follows.

2.1 Mean Model in Time Series

Models of *autoregressive moving average* (ARMA) are a combination of the AR and MA models. *Autoregressive moving average* model of degrees p and q are written as an ARMA(p, q), and has the equation as

$$r_{it} = \psi_{i0} + \sum_{g=1}^p \psi_{ig} r_{it-g} + a_{it} + \sum_{h=1}^q \theta_{ih} a_{it-h}, \quad (1)$$

with ψ_{i0} the constant, and ψ_{ig} ($g=1,...,p$) and θ_{ih} ($h=1,...,q$) coefficients of parameters of the mean model of stock return i ($i=1,...,N$ and N is the number the stock that were analyzed). It is assumed the $\{a_{jt}\}$ sequence of residual white noise with zero mean and variance $\sigma_{a_j}^2$ [14; 15].

Stages of the process of mean modeling include: (i) Identification of the model, (ii) Estimation of parameters, (iii) Test the diagnosis, and (iv) Prediction [14].

2.2 Volatility Model in Time Series

Stock return volatility modeling conducted using generalized models of autoregressive conditional heteroscedastic (GARCH). Suppose μ_{it} and σ_{it}^2 respectively mean and volatility of stock returns i ($i=1,...,N$ and N is the number the stock being analyzed), at time t ($t=1,...,T$ and T the period of observation data). Residual a_{it} mentioned above have the equation $a_{it} = r_{it} - \mu_{it}$ [1; 3; 14; 15]. Volatility will follow the GARCH model degrees m and n or written GARCH(m, n), if

$$a_{it} = \sigma_{it} \varepsilon_{it}, \quad \sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^m \alpha_{ik} a_{it-k}^2 + \sum_{l=1}^n \beta_{il} \sigma_{it-l}^2 + \varepsilon_{it}, \quad (2)$$

with α_{i0} constant, and α_{ik} ($k=1,...,m$) and β_{il} ($l=1,...,n$) parameters coefficient of volatility model of stock return i ($i=1,...,N$ and N is the number the stock that were analyzed). Assumed sequence $\{\varepsilon_{it}\}$ are independent random variables and identically distributed (iid) with mean 0 and variance 1, $\alpha_{i0} > 0$, $\alpha_{ik} \geq 0$, $\beta_{il} \geq 0$, dan $\sum_{k=1}^{\max(m,n)} (\alpha_{ik} + \beta_{ik}) < 1$ [1; 14].

Stages of the process of volatility modeling include: (i) estimate mean model, (ii) Test ARCH effects, (iii) Identification of the model, (iv) Estimated volatility models, (v) Test the diagnosis, and (vi) Prediction [14].

Using the mean model (1) and volatility (2), predictions made aims to calculate mean $\hat{\mu}_{it} = \hat{r}_{iT}(1)$ and $\hat{\sigma}_{it}^2 = \hat{\sigma}_{iT}^2(1)$ variance, ie, the prediction 1-step forward after a period of time to T [14].

2.3 Portfolio Model and Value-at-Risk

In the formation of an investment portfolio w , will relate to the proportion of funds allocated to each the stock analyzed. Suppose that w_i is the proportion of funds allocated to the stock i , where $\sum_{i=1}^N w_i = 1$, then the portfolio return can be expressed as

$$R_{wt} = \sum_{i=1}^N w_i R_{it}, \quad (3)$$

where R_{wt} portfolio return w at time t , and N is the number the stock in the portfolio formation [5; 13].

Based on (3), obtained mean (expected) of portfolio with weights w_i can be expressed as

$$\mu_w = \sum_{i=1}^N w_i \mu_i \quad (4)$$

While the variance of portfolio can be expressed as

$$\sigma_w^2 = \sum_{i=1}^N w_i^2 \sigma_i^2 + 2 \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} ; i \neq j \quad (5)$$

where $\sigma_{ij} = \text{Cov}(r_{it}, r_{jt})$ [5; 6; 12].

Suppose the amount of initial investment of one unit, and the significance level of risk a loss of α , and then the VaR for a portfolio with weights w_i is

$$\text{VaR}_w = -(\mu_w + z_\alpha \sigma_w) \quad (6)$$

where z_α is the percentile value of the standard normal distribution with a significance level α [2; 6; 7; 8; 14; 15].

2.4 Portfolio Optimizations

Portfolio of multi-objective optimization in this paper has two objective functions, the first objective function to maximize (expected return portfolio), and second to minimize the risk of Value-at-Risk. Two objective functions are restricted that portfolio weights should be numbered one, and if short selling is not allowed, the portfolio weights must be positive [16]. The formulation of a model of multi objective optimization problem is as follows:

The objective function:

$$\begin{aligned} \text{Max } \mu_w &= \max \sum_{i=1}^N w_i \mu_i \\ \text{Min } \text{VaR}_w &= \min \left\{ - \left(\sum_{i=1}^N w_i \mu_i - z_\alpha \left(\sum_{i=1}^N w_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} \right)^{1/2} \right) \right\} \\ \text{Restrictive: } &\sum_{i=1}^N w_i = 1 \text{ and } 0 < w_i < 1 \end{aligned} \quad (7)$$

where z_α is the α -quantile of the standard normal distribution. Equation objective is to minimize the total Value at Risk portfolio and maximize portfolio return expectations in this respect as the expected return on the investment portfolio.

To find the solution of multi-objective portfolio optimization problem mentioned above, in this paper are conducted using a greedy search algorithm approach. Compared with the dynamic algorithm, greedy search algorithm has the single decision of the circuit, while the dynamic algorithms have many series of decisions [11].

Greedy search algorithm is an algorithm that solves the problem step by step, at each step: (i) Take the best option which can be obtained at the time, and (ii) Hoping that by choosing a local optimum at each step will end with a global optimum. Greedy search algorithm assumes that the local optimum is part of the global optimum. The principle of greedy search algorithm is "take what you can get now!" This principle is also used in solving the optimization problem [9; 10; 11].

According to Munir [9], pseudo-code greedy search algorithm is as follows:

```

procedure greedy(input C: candidat_set;
                 output S : solution_set)
{determine the optimum solution of the optimization problem by a greedy algorithm
  Input: candidat_set C
  Output: solution_set S}

Declaration
  x : candidat;

Algorithm:
  S ← {}                                {initialization S with empty}
  while (not SOLUTION(S)) and (C ≠ {} ) do
    x ← SELECTION(C); {select a candidat from C}
    C ← C - {x}      {elements of the candidat set is reduced}

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if FEASIBLE( $S \cup \{x\}$ ) then
     $S \leftarrow S \cup \{x\}$ 
endif
endwhile
{SOLUTION(S) has been obtained or  $C = \{\}$ }

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In the portfolio optimization process, where there are many stocks that were analyzed, there are variations in the proportion of the portfolio process sequence determination using the greedy search algorithm. It thus makes inefficient in progress. The strategy is to sort according to overcome the difference between the return expectations and risk of each stock from largest to smallest.

III. Results and Analysis

3.1 Materials

The data analyzed here include stock data proofreaders, ISAT, TLKM, HMSP, BMRI and INDF, from the period October 1, 2009 through February 28, 2013. Henceforth, the names of the stock were given consecutive symbols S_1 to S_5 . The data is accessed through the website <http://www.finance.go.id/>. Data processing is done with the assistance software's: MS Excel 2007, Eviews-6 and Matlab-7.

3.2 Result

At first the specified closing stocks price data of each return, and then used for modeling the mean and volatility following.

Estimated mean model. Return data of each stock will be used to estimate the mean model using the software Eviews-4. Mean modeling was done with reference to equation (1). First, the identification and estimation of the mean model. Identification is done through the sample autocorrelation function (ACF) and partial autocorrelation function (PACF).

Based on the pattern ACF and PACF of each stock return S_1 to S_5 , determined tentative model. Estimation results, it can be shown that successive models ARMA(1,1), ARMA(1,1), AR(1), ARMA(1,1) and AR(2) is quite appropriate. Second, diagnostic testing of these models, using residual data correlogram and Ljung-Box test of the hypothesis. The test results showed residual models are white noise. Residual normality test results showed normal distribution. Based on the results of diagnostic testing these models are within their, so that models can be used for the modeling of volatility following.

Estimation of volatility models. First, detection of an element of autoregressive conditional heteroscedasticity (ARCH) to the residual a_{it} . The detection was made using the ARCH-LM method with the assistance of software Eviews-4. Detection results obtained value (obs * R-Square) for each stock return S_1 to S_5 , respectively, 132.9207; 62.8786; 15.1536; 27.7327, 11.8592 and 9.1534; with respective probabilities of each are 0.0000, or smaller than the significance level $\alpha = 0.05$. Means that each stock return S_1 to S_5 there is elements of ARCH. Second, the identification and estimation of volatility models. Here used generalized models of autoregressive conditional heteroscedasticity (GARCH) refer to equation (2). Based on the squared residuals correlogram, i.e., ACF and PACF graphs of each selected volatility models that may be tentative. Estimation model of return volatility of each stock conducted simultaneously with the mean model. Based on the estimation results, obtained the most within their model as given in Table-1.

Table-1. Estimations Result of Mean and Volatility Models

Stocks (S_i)	Model	Mean and Volatility Equations
S_1	ARMA(1,1)- ARCH(1)	$r_{1t} = 0.827877r_{1t-1} + a_{1t} - 0.895486a_{1t-1}$ $\sigma_{1t}^2 = 0.000530 + 0.401491\sigma_{1t-1}^2 + \varepsilon_{1t}$
S_2	ARMA(1,1)- ARCH(1)	$r_{2t} = 0.773159r_{2t-1} + a_{2t} - 0.840528a_{2t-1}$ $\sigma_{2t}^2 = 0.000416 + 0.241935\sigma_{2t-1}^2 + \varepsilon_{2t}$
S_3	AR(1)- ARCH(1)	$r_{3t} = -0.318502r_{3t-1} + a_{3t}$ $\sigma_{3t}^2 = 0.000404 + 0.307801\sigma_{3t-1}^2 + \varepsilon_{3t}$

S_4	ARMA(1,1)- ARCH(1)	$r_{4t} = 0.719495r_{4t-1} + a_{4t} - 0.755236a_{4t-1}$ $\sigma_{4t}^2 = 0.000629 + 0.395636\sigma_{4t-1}^2 + \varepsilon_{4t}$
S_5	AR(2)- TGARCH(1,1)	$r_{5t} = -0.013454r_{5t-1} - 0.004895r_{5t-2} + a_{5t}$ $\sigma_{5t}^2 = 0.0000735 - 0.025404a_{5t-1}^2 + 0.033608I_{5t-1} + 0.545633\sigma_{5t-1}^2 + \varepsilon_{5t}$

Testing is also done to ARCH element (the residuals of volatility model ε_{it} ($i=1,...,5$)). Based on the ARCH-LM test, the residuals of the models for stock returns S_1 to S_5 there was no element of ARCH, and also has white noise. Model mean and volatility will be used to predict the values of $\hat{\mu}_{it} = \hat{r}_{it}(1)$ and $\hat{\sigma}_{it}^2 = \hat{\sigma}_{it}^2(1)$, which is one step ahead prediction, recursively.

Predicted mean and volatility. Using the model mean and volatility of stock returns S_1 to S_5 in Table-1 above, then used to predict the values of $\hat{\mu}_{it} = \hat{r}_{it}(1)$ and $\hat{\sigma}_{it}^2 = \hat{\sigma}_{it}^2(1)$. Prediction conducted recursively, and the results are given in Table-2.

Table-2. Predictions Result of Mean and Volatility

Stocks	$\hat{\mu}_{it}$	$\hat{\sigma}_{it}^2$	VaR_{it}	$\hat{\mu}_{it} / VaR_{it}$
S_1	0.000786	0.000568	0.038415	0.020461
S_2	0.000618	0.000672	0.042019	0.014708
S_3	0.001204	0.000406	0.031942	0.037694
S_4	0.000612	0.000680	0.042297	0.014469
S_5	0.000158	0.000250	0.025857	0.006111

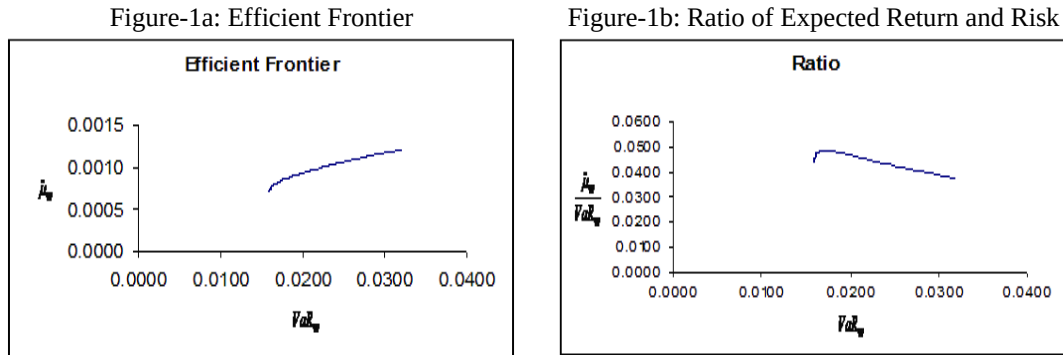
Suppose the risk of loss VaR is measured with a significance level $\alpha = 0.05$ or obtained $z_{0.05} = -1.645$, the value of VaR for of each stock looks are given in Table-2 column VaR_{it} . Comparison between mean $\hat{\mu}_{it}$ with a risk VaR_{it} the results given in column $\hat{\mu}_{it} / VaR_{it}$. Based on comparative sequence value $\hat{\mu}_{it} / VaR_{it}$ from

Table-3: Iterations Process of Greedy Search Algorithm

	S_3	S_1	S_2	S_4	S_5					
Iterations	w_3	w_1	w_2	w_4	w_5	$\sum w_i$	$\hat{\mu}_w$	$\hat{\sigma}_w^2$	VaR_w	$\frac{\hat{\mu}_w}{VaR_w}$
0	1	0	0	0	0	1	0.001204	0.000406	0.031942	0.037694
1	0.99	0.01	0	0	0	1	0.001200	0.000398	0.031617	0.037949
...	...	...	...	...	...	...	...	...	...	...
54	0.46	0.15	0.14	0.13	0.12	1	0.000857	0.000127	0.017678	0.048466
55	0.45	0.16	0.14	0.13	0.12	1	0.000853	0.000125	0.017540	0.048508
56	0.44	0.16	0.15	0.13	0.12	1	0.000847	0.000123	0.017423	0.048528
57	0.43	0.16	0.15	0.14	0.12	1	0.000841	0.000122	0.017303	0.048543
58	0.42	0.16	0.15	0.14	0.13	1	0.000830	0.000119	0.017102	0.048554
59	0.41	0.17	0.15	0.14	0.13	1	0.000826	0.000117	0.016993	0.048619
60	0.40	0.17	0.16	0.14	0.13	1	0.000820	0.000116	0.016907	0.048520
61	0.39	0.17	0.16	0.15	0.13	1	0.000814	0.000115	0.016818	0.048423
62	0.38	0.17	0.16	0.15	0.14	1	0.000804	0.000112	0.016640	0.048315
63	0.37	0.18	0.16	0.15	0.14	1	0.000800	0.000111	0.016562	0.048290
64	0.36	0.18	0.17	0.15	0.14	1	0.000794	0.000111	0.016509	0.048088
...	...	...	...	...	...	...	...	...	...	...
76	0.23	0.21	0.2	0.19	0.17	1	0.000709	0.000105	0.016016	0.043851
77	0.22	0.21	0.2	0.19	0.18	1	0.000698	0.000104	0.016006	0.043381

Table-2, the optimization process using a greedy search algorithm is done by placing the stock order as S_3 , S_1 , S_2 , S_4 and S_5 . Iteration is stopped until it can no longer achieve the condition $w_3 > w_1 > w_2 > w_4 > w_5$.

Because the covariance between the stock is very small, in this paper is assumed $\sigma_{ij} = 0$. Optimization is done by referring to equation (7). The results of some iteration greedy search algorithm are given in Table-3. Greedy search algorithm iteration process is performed using the change in weight of 0.01. Calculation of portfolio optimization process is stopped at iteration 77, as the iteration to 78 and so on are no longer meets the conditions of $w_3 > w_1 > w_2 > w_4 > w_5$. Furthermore, using the values in Table-3 column $\hat{\mu}_w$ and column VaR_w formed the efficient frontier that graph is given in Figure 1a. Similarly, the values of the ratio of mean against risk, namely $\hat{\mu}_w/VaR_w$ that graph is given in Figure 1b.



3.3 Analysis

Based on the results given in Table-3, when viewed as a partial can be explained as follows. For many couples the portfolio weights, the maximum value of the expected return of portfolio is equal to $\hat{\mu}_w = 0.001204$, is achieved when the entire capital invested in stocks. The maximum expected return of portfolio is also accompanied by the value of risk level $VaR_w = 0.031942$, which is the level of greatest risk. Meanwhile, the smallest value for the risk level is $VaR_w = 0.016006$ happens when the investment is made to the allocation of portfolio weighting in stocks S_1 , S_2 , S_3 , S_4 and S_5 magnitude respectively 0.21, 0.20, 0.22, 0.19 and 0.18. Minimum risk level is followed by the value of expected return portfolio $\hat{\mu}_w = 0.000698$, which is the smallest value of the expected return of portfolio. If you look, the rising value of the expected return of portfolio $\hat{\mu}_w$, also followed by increased levels of risk VaR_w .

Pairs between the values of expected return portfolio $\hat{\mu}_w$ with risk VaR_w , in various combinations of allocation of the portfolio weights can be established a set of efficient portfolios or efficient frontier, as shown in Figure 1a. Efficient Frontier is the set of feasible combinations of weighting the portfolio investment. Starting from a combination of portfolio weights that produce the level of risk VaR_w and expected return $\hat{\mu}_w$ smallest, until the combination of portfolio weights that produce the expected return $\hat{\mu}_w$ and risk VaR_w , the largest. Among the set of efficient portfolios is there an optimum portfolio, the portfolio that produce the ratio between the expected return on the risk $\hat{\mu}_w/VaR_w$ greatest. Portfolio optimum occurs when the value of the ratio $\hat{\mu}_w/VaR_w = 0.048619$ is the largest. The biggest ratio can be seen by naked eye in Figure 1b, which is the highest peak of the ratio chart. On optimum portfolio are generated the value of expected return $\hat{\mu}_w = 0.000826$ and risk level $VaR_w = 0.016993$, when capital allocation is done by a combination of portfolio weighting in stocks S_1 , S_2 , S_3 , S_4 and S_5 respectively the amount of 0.17, 0.15, 0.41, 0.14 and 0.13.

Based on this analysis, to make an investment portfolio consisting of five stocks S_1 to S_5 be given advice as follows. For investors who have a risk averse nature, do the investment by choosing the smallest level of risk, it will provide the expected return of portfolio is also small. For investors who have a brave nature of risk, do the investment by selecting the value of expectation return portfolio, the largest, would be followed by a large level of risk. For investors who are rational, do the investment by selecting the combination of portfolio weights are optimum.

IV. Conclusion

Greedy search algorithm has been used as an alternative to solving the portfolio optimization problem numerically. The principle of greedy search algorithm is "take what you can get now!", conducted iteratively. Disadvantages of greedy search algorithm do not provide criteria for the order of variables in the iteration process. In addition, do not allow the existence of shared values between the proportions with one another. The advantages, the greedy search algorithm have a single decision circuit, compared with dynamic algorithm which has a series of multiple decisions. Empirical research conducted on stocks data S_1 to S_5 , with assumed that the returns of each stock has a non-constant volatility. Stock returns data S_1 to S_5 model with the ARMA-GARCH models. The results estimate the mean and volatility using ARMA-GARCH models are used to develop a portfolio of multi-objective optimization problem, with risk measured using the Value-at-Risk at level of significance $\alpha = 0.05$. Optimization result, the maximum expected return portfolio $\hat{\mu}_w = 0.001204$ and minimum risk $Var_w = 0.016006$. While the optimum portfolio obtained with mean $\hat{\mu}_w = 0.000826$, and Value-at-Risk $Var_w = 0.016993$.

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