Numerical Analysis of the Chaotic Circuit under the influence of One Component Change

Abo-Talib Abbas, Forat Ahmed, Noori H.N. Al-Hashimi
Department of Physics, College of Education for pure science, University of Basra, Basra, Iraq

Abstract: In this paper, we study one of the circles of chaos with the non-linear type of Memoductance, Memristor flux-controlled, the effect of the capacitance on dynamics behavior has been analysis in particular the area of chaos. It is found that the dynamical behavior existing in all cases when the capacitance is change and that the area of chaos is sensitive to the value of the capacitance.

Keywords: Chua’s circuit; Lagrange stability; Attraction-repulsion function; chaos; bifurcation

I. Introduction

For long time the question was raised about the phenomenon of chaos that appear in a solutions of some mathematical modeling and the differentials equations which are related to this modeling and this question read as follow "Is there any experiments reflects this phenomenon of chaos". This question encourage the scientist to search for some practical applications that show the chaos in their behavior and it was the most important of these applications had been focused on the electrical circuit. Vander pol and Vander mark [1] succeed of getting building oscillatory electrical circuit with some dynamical behavior. Ueda and Akamatsu [2] have managed to get the phenomenon of chaos in the resistance negative oscillator. Chua in 1983 [3] design an electronic circuit showed some dynamical behavior, this circuit has been known as Chua circuit. Chua was trying to get circuits containing three elements can storage the energy, with three-point balance. Chua circuits contain also a non-linear part of type piecewise linear. Chua circuit has been applied successfully by Matsumoto [4], where he was observed the following phenomena: period-doubling route; period-adding Sequence; quasi-periodic route; and chaos.

After Chua circuit successfully expressed a verity of dynamical behavior, researcher focused on the benefit of this circuit in practical applications. Many researchers has succeeded in using this circuit in a lot of applications in many important areas, see for example, [4, 5, 6]. These applications are as follow:; hiding many of the information to be concealed and not leakage to other parties, the use of the circuit in the control of information systems and control mechanism reorient, and the use of machines in musical mechanics in addition of many other applications. Chua model has been change several times by adding many electronic parts or add an external source [7] or transform the original circuit of the circle from three-dimensional to the four-dimensional [8] or five-dimensional [9] by adding or deleting one of the basic elements constituting the circuit. It's worth to say that all of these changes have shown many of variety in dynamical behavior. In this paper we will consider a chaos circuits shown in the figure (1). Where R represents the non-linear resistance, G_N represent the impedance, C_1 and C_2 capacitance of first and second capacitor, L self-inductance of the coil used, \( \phi \) is the magnetic flux, q represents the electric charge, \( v_{c1} \) and \( v_{c2} \) voltage across the capacitor respectively, \( i_l \) the current across the coil. In order to study the stability of or instability of this circuit a non-linear part has been added to this circuit represent by the symbol M, which is a type of Memristor flux-controlled memductance [10] as shown in figure (2). Some computer simulations on the modified Chua’s circuit with the suitable function are presented together with the effect of the capacitor C_1 on the dynamical behavior of the chaotic circuit is analysis.

**Figure (1): Chaotic circuit under study**

**Figure (2): Non-Leniar part of the caaotic circuit**
II. Theory

Usually, any dynamical system can be described in differential equations; the center manifold theory [11] is first applied to obtain a locally invariant small-dimensional manifold—a center manifold. Then additional nonlinear transformations are introduced to further simplify the center manifold to a normal form. To find the “form” of a normal form, first a homogeneous polynomial vector field of degree k is found in a space complementary to the range of the so-called “homological operator”. Then the original vector field is decomposed into two parts: one of them, called the non-resonant terms, is eliminated and the other, called the resonant terms, is kept in the normal form. This simple form can be used conveniently for analyzing the local dynamic behavior of the original system. For a practical system, not only the possible qualitative dynamical behavior of the system of concern but also the quantitative relationship between the normal forms and the equations of the original system needs to be established. Normal forms are, in general, not uniquely defined and finding a normal form for a given system of differential equations is not a simple task. In particular, finding the explicit formulas for normal forms in terms of the coefficients of the original nonlinear system is not easy. Therefore, the crucial part in computing a normal form is the computational efficiency in finding the coefficients of the normal forms and the corresponding nonlinear transformations. Furthermore, the algebraic manipulation becomes very involved as the order of approximation increases. The idea of normal form theory is to use successive nonlinear transformations to derive a new set of differential equations by removing as many nonlinear terms from the system as possible. The terms remaining in the normal form are called the resonant terms. If the Jacobian matrix of the linearized system evaluated at equilibrium can be transformed into diagonal form, then the bases of the nonlinear transformations are decoupled from each other. However, for a general singular vector field such as a system with non-semi simple double-zero or triple-zero Eigen values, these bases are coupled. Such a coupling makes computation of the normal forms complicated. First, we introduce a general approach for computing normal forms based on the work of [11]. Consider a system described by the general nonlinear ordinary differential equation, produced by applying the Kirchhoff’s law around the first and second capacitor and around the coil and the resistance, this system can be describe by the following differential equations:

\[
\frac{dv_{c1}}{dt} = v_{c1} - \frac{1}{c_1}(i_1 - w(\varphi)v_{c1}) \tag{1}
\]

\[
\frac{dv_{c2}}{dt} = -\frac{1}{c_2}(i_1 - G_N v_{c2}) \tag{2}
\]

\[
\frac{di_1}{dt} = \frac{1}{L}(v_{c2} - Ri_1 - v_{c1}) \tag{3}
\]

Where \(w(\varphi)\) is the function of the non-linear parts shown in figure (2) which one can assume is follow the behavior of the differential equation of the form:

\[
w(\varphi) = \frac{d\varphi(\varphi)}{\varphi} = -\alpha + 3\beta\varphi^2 \tag{5}
\]

Where \(\alpha, \beta\) are some constant. It is worth noticing that the equations governing the circuit are symmetrical with respect to the origin, i.e., they are invariant under the transformation \((v_{c1}, v_{c2}, i_1) \rightarrow (-v_{c1}, -v_{c2}, -i_1)\), this means that the system is symmetric with respect to the origin which can be proved via the above transformation. Since the nonlinearity of the Chua’s circuit is a piecewise-linear function, i.e. has the form:

\[
f(x) = \max\{l_1(x), \ldots, l_k(x)\} \tag{6}
\]

Where \(l_k\) are linear functions, then the set of points on or above the graph is an intersection of closed half spaces and hence it is convex, which means that the function \(f(x)\) is convex. Based on this property the circuit can be divided into a set of separate regions. Analyzing the behavior of the system in each of these regions is helpful to understand the global behavior of the circuit. Now if one assumes a study state condition, i.e.

\[
\frac{dv_{c1}}{dt} = \frac{dv_{c2}}{dt} = \frac{di_1}{dt} = 0, \tag{7}
\]

And

\[
v_{c1} = 0, \tag{8}
\]

Equations (2-4) reflect a unique equilibrium point at the origin which is \(S_0 = (v_{c1}, v_{c2}, i_1, \varphi) = (0, 0, 0, \varphi_0)\). If these conditions do not satisfy, on can expected some of dynamical behavior will arises on both side of the equilibrium point and a chaotic properties will be clear. Now the above system can be rewritten in a matrix form as \(V = AV + cont. w(\varphi)\) where:

\[
V = \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & \frac{1}{c_1} \\ 0 & G_N & -\frac{1}{c_2} \\ -\frac{1}{L} & \frac{1}{L} & -\frac{\beta}{L} \end{bmatrix}, \quad cont. = \begin{bmatrix} -\frac{\beta}{c_1} \\ \frac{\beta}{c_1} \\ 0 \end{bmatrix} \tag{9}
\]

And \(w(\varphi) = v_{c1}\exp(\gamma v_{c1}^2)\), where \(w(\varphi)\) is bounded when \(\gamma < 0\).
III. Result and Discussion

It is easy to solve the systems of equations (9) mention above for difference situation; but our focus in this study on the dynamical behavior of the chaotic circuit and the corresponding performance curve of the $v_{c_2}$ Vs $c_2$ under the condition when $c_1$ kept constant for each case. Our calculation is carry out for the chaotic circuit shown in figure (1) under some more assumptions as follow:

1) The resistance is fixed to the value of $R=400\Omega$.
2) The inductance fixed to the value of $L=100\ mH$.
3) The values of $C_i$ for each case and $C_2$ are variable.

The difference cases when the capacitor $C_1$ is change are shown in figures.

Case (1): $C_1 = 20\ nf$, figure (3): the existence of multiplicity in dynamical behavior in terms of cyclical behavior varied with the presence of one area of the chaotic. For the values of the capacitance $C_2 \geq 40\ nF$ the figure shows, the behavior of $P_1$ per session until reach what is called Hopf - bifurcation which means that the circuit behave as a DC circuit which appears in the phase space as a fixed point. This bifurcation can be controlled experimentally by either linear or nonlinear delayed state-feedback by using a washout filter.

Case (2): $C_1 = 24\ nf$, $28\ nf$, $32\ nf$, $36\ nf$; figures (4-7): almost same explanation can be say with the presence of two areas of the chaotic. Again for the values of the capacitance $C_2 \geq 40\ nF$ the figure shows, the behavior of $P_1$ per session until is reach Hopf - bifurcations. The actions of periodic one, periodic two, and periodic four are observe when the value of $C_1 = 36\ nF$.

Case (3): $C_1 = 40\ nf$, $44\ nf$, $46\ nf$; figure (8-10): For the value of the capacitance $C_1 > 36\ nF$ the electronic circuit become purely periodically (i.e, the chaotic is disappear) as observe the actions of periodic one, and periodic two only is observe, $P_4$ when the value of $C_1 = 38\ nF$ and when $C_1$ increase the dumping of the electronic circuit will be limited to $P_1$ only.

It is worth to say that the area of chaos exists within the range of values $20.\ nF \leq C_1 \leq 35.\ nF$. The area of chaos at the begging contain the behavior of periodically with wide ranges of values of $C_2$ but as $C_1$ increase the values of these ranges diminish and become so small that overcoming the chaotic behavior but returns accommodate those ranges for the actions of the periodically again, but when the values of $C_1 > 24.\ nF$ the chaotic region disappear completely.

![Figure (3): Dynamical bifurcation for $C_1=20\ nf$](image3)

![Figure (4): Dynamical bifurcation for $C_1=24\ nf$](image4)
Figure (5): Dynamical bifurcation for $C_1=28$ nf

Figure (6): Dynamical bifurcation for $C_1=32$ nf

Figure (7): Dynamical bifurcation for $C_1=26$ nf

Figure (8): Dynamical bifurcation for $C_1=40$ nf
REFERENCES