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The Crossed cube-Mesh: A New Fault-Tolerant Interconnection Network Topology for Parallel Systems

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***Abstract**: Recently, the Cube based networks have emerged as attractive interconnection structures in parallel computing systems. In this paper we propose a new interconnection network called crossed cube-mesh which is a product graph of crossed cube and mesh topology. The various topological properties of the new network are derived. The embedding properties, fault-tolerance, node disjoint paths, routing, cost and other performance aspects of the new network are discussed in detail. Based on the comparison, the proposed topologyis proved to be an attractive alternative to the existing Hyper-mesh topology.*

***Keywords**: Fault diameter, Node-disjoint path, Cost-effectiveness factor, Hyper-Mesh, Crossed Cube*

I. Introduction

Recent advances in parallel computing and the increasing need for massive parallelism have resulted in the emergence of many attractive interconnection networks. The Interconnection networks (IN) with a large number of processors have evolved to cope with the continued demand for more computing power. This requires the use of parallelism in many applications. In parallel processing, a large number of processors cooperates to solve a given problem. The cube type networks have received much attention over the past few years since they offer a rich interconnection structure with large bandwidth, logarithmic diameter and high degree of fault tolerance [1, 2]. The Hypercube (HQ) topology has attracted the attention of many researchers due to many of its attractive features suited for parallel computing [3]. One of the major drawbacks of the hypercube network is that for a large size network, the number of connections required per node is large which directly affects in the implementation phase of a parallel machine. For a network of size N, the number of connections required per node is $\log_2 N$. Thus, the number of connection per node is not practical for large systems. Many variations of hypercube topology have been proposed by the researchers over the years to improve some of its properties and to eliminate its drawbacks, either by making some modification in the link connectivity or by the cross product with other interconnection network [4][8][11][15-20]. The Crossed cube topology (CQ) has appeared as attractive alternative to the hypercube [4]. Further, in the mesh (M) topologies, the number of connections per node is fixed and does not increase as the network size increases. However, number of node-disjoint paths in the mesh is less as compared to the hypercube [6]. It has a higher diameter compared to the hypercube, especially for networks of large sizes. In general, as the node degree increases, the diameter increases linearly. The researcher in [5], have proposed a network called hyper-mesh (HM), which is the combination of two well-known interconnection networks i.e. hypercube [3] and mesh [1]. However, the hyper-mesh network has certain drawbacks which includes high diameter and high cost.

The current paper proposes a new interconnection topology called the Crossed cube-mesh (CQM) as an alternative to the Hyper-mesh [1]. The proposed topology is a product graph of Crossed cube and Mesh which has less diameter and less cost as compared to the hyper-mesh. The other important properties the proposed network are worked out. The paper is organized as follows: Section 2 provides the background with a brief description of the basic terminologies and topological features of the base networks. In Section 3 the details of the proposed network are presented. The Section 4 analyses the performance of the proposed network and compares the same with other networks. The Section 5 discusses the embedding of other networks in the proposed structure i.e. the CQM. The Sections 6, 7respectively discuss the distributed computing and routing aspects of the new network. The results and discussions represented in Section 8. The Section 9 presents the concluding remarks with indication for future scope.

II. Background

The basic terminologies of the Interconnection topology (IN) are discussed below in this section. Throughout the investigation, the Interconnection network is treated as an undirected graph, in which the vertices correspond to the processors and the edges correspond to the bidirectional communication links. In what follows we discuss the basics of the two base networks Crossed Cube and Mesh.

Definition 1: The IN is finite graph G = {V,E}, where V and E are a set of tuples, $v_1v_2\ldots v_n$ and $e_1e_2\ldots e_n$.

Definition 2: The degree of vertex in G is equal to the number of edges incident on v.

*Definition 3:*The diameter of a graph G denoted as D_G is defined to be max $\{d_G(u,v)\text{:}u,v\epsilon V\}$, where d_G is the distance between 2 nodes.

Definition 4: A graph is said to be regular if all its vertices have the same degree.

Definition 5: A graph G(V,E) is a vertex symmetric if for every pair of vertices, u and v, there exists an automorphism of the graph that maps u into v, u,vϵV.

Definition 6: A set of tuples is said to be node disjoint if no node except for the source and destination nodes appear in more than one path. The number of such path provides a measure of the fault tolerant and reliability of the network.

Definition 7: The Cartesian product $G=G_1 \oplus G_2$ of two graphs G_1=(v1,E1) and G_2=(V2,E2) is a graph G=(V,E), where V and E are given by:

i) V={<x,y>|xϵV1 and yϵV2}

ii) Let $u<x_u,y_u>$ and $v=(x_v,y_v)$ in V(u,v) is an edges in E iff either (x_u,y_u) ϵE1 and $y_u=y_v$ or (y_u,y_v) ϵE2 and $x_u=x_v$

The edge (u,v) is called a G_1-edge iff (x_u,x_v) is an edge in E1, and it is called a G2-edge if $(y_u,y_v)$is an edge in E2. We call x_u the G_1-component of u and y_u the G2-component of u.

Let n_1,d_1,D_1 be respectably the size(the number of nodes),degree, and diameter of G_1. n_2,d_2,D_2 be respectably the size, degree, and diameter of G_2.The expression for size and degree expression are fairly obvious. As for the diameter expression it can be justified by noticing that a path between any two vertices $u=<x_u,x_v>$ and $v=<x_v,y_v>$ of $G_1 \oplus G_2$ is composed of two types of edges: G_1 edges(affecting G_1-component)and G_2 edges(affecting G_2-component).If all the G_1-edges (resp. the G_2-edges) in the path from u to v are extracted and listed maintaining their relative order, we obtain a path from x_u to x_v in G_1(resp. from y_u to y_v in G_2). Therefore u would be at maximum distance from v in $G_1 \oplus G_2$ if, and only if, x_u is at the maximum distance from x_u in G_1 and y_u is at maximum distance from y_u in G_2. Next ,we discuss the structure of the two parent networks based on the CQ and M graph. The network structures of both CQ and M are discussed below.

A. Crossed Cube Topology

The Crossed cube denoted as CQ(m) is regular graph of 2^m nodes where m is the dimension. Every node in the CQ(m) is identified by a unique binary string of length m[6].

Definition1: Two binary strings, $X=X_1X_0$ and $Y=Y_1Y_2$, of length 2 are said to be pair related if and only if xyϵ{(00,00),(10,10),(01,11),(11,01)}.

Definition2: The CQ(m) is recursively defined as follows[4]:

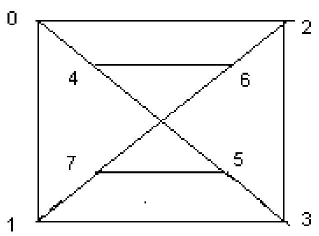
The CQ(m) contains CQ^0_{m-1} and CQ^1_{m-1} ,joined according to the following rule : the vertex $u=0u_{m-2}\ldots u_0$ from CQ^0_{m-1} and the vertex $v=1v_{m-1}\ldots v_0$ from CQ^1_{m-1} are adjacent in the CQ(m) if :

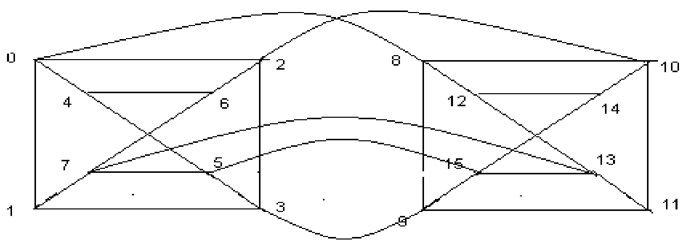
1.$u_{m-2}=v_{m-2}$ if m is even, and

2. For $0<=i< \lfloor n\text{-}1/2 \rfloor$,$u_{2i+1}u_{2i}\sim v_{2i+1}v_{2i}$

The Figs. 1 and 2 respectively show the CQ(m) topology for m=3 and m=4. Every vertex in the CQ(m) with a leading bit 0 bit has exactly 1 neighbor with a leading bit 1 and the vice versa. In CQ(m), when 2 adjacent vertices u and v have a leftmost differing bit at position d, then v is called d-neighbor of u and the edge (u,v) is called the edge of dimension d. For any 2 nodes u and v of the CQ(m), it is possible to reach v from u in at most $\lceil m+1/2 \rceil$ hopes.

Figure 1: The Crossed cube (for m=3) **Figure 2: The Crossed cube (m=4)**

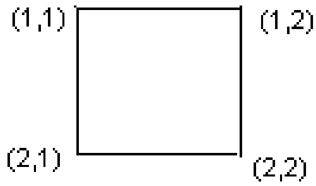




B. Mesh Topology

A mesh network denoted as M(r,c) is a two dimensional topology consisting of r rows and c columns with a total of r*c vertices 4 neighbors. The degree of M(r,c) is 4, diameter is r+c+2, and these are labeled(x,y), where $1\leq x\leq r$ and $1\leq y\leq c$.Each interior vertex has exactly exist at least n node-disjoint paths between any two nodes. Two nodes(x_u,x_v)a and (y_u,y_v) in mesh, M(k) are said to be adjacent if $x_u=x_v$ or $y_u=y_v$. For simplicity and without loss of generality, we will assume that r=c=k where k is an even number. The figure 3 shows a two dimensional mesh M(k), where the dimension k=2.

Figure 3: The Mesh topology (for dimension k=2)



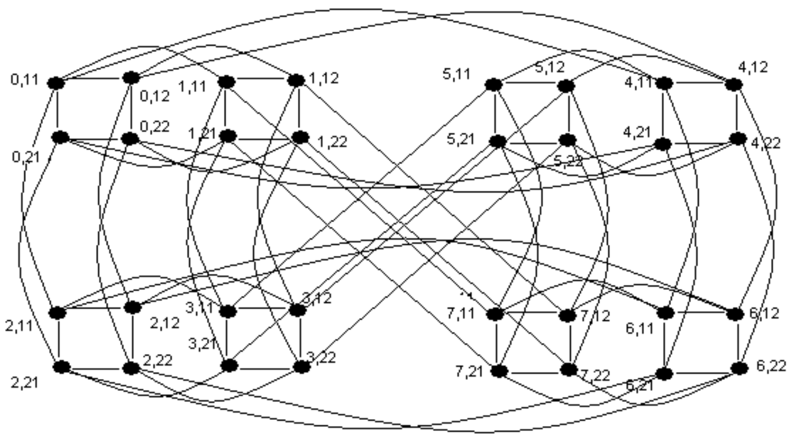
III. The Proposed Topology

The network details of the proposed topology: Crossed cube denoted as CQM(n,k) are described below.

A. Crossed cube-mesh topology

The proposed topology Crossed cube-Mesh denoted as CQM(n,k) is the product graph of the CQ(n) and M(k).That is an n dimensional crossed cube, where each vertex is replaced with k dimension mesh. Next, the node address of each vertex in the resulting graph will have two parts$<x_{n-1},x_{n-2},\ldots x_0,y_1,y_{2,\ldots,}y_{k-1}>$, where the x_is represent the CQ and the y_is represent the mesh part. Each vertex will have two types of neighbors, namely the CQ-part and mesh-part neighbor with node address which have the same mesh labeling and CQ labeling respectably. The CQM (3,2) is shown in Fig. 4.

Figure 4: Crossed cube-Mesh topology (3,2)



Given a node<x,y,z> of the CQM (n,k), x is called the Crossed cube(n) part label and <y,z> the Mesh(k) part label. It is noteworthy that the node with the same Crossedcube-graph part label forms a k-Mesh whereas the nodes with the same Mesh-graph part label form a Crossedcube of order n. It therefore follows that there are k^2 Crossed Cube sub graphCQ(n) in CQM(n,k), where the nodes in each CQ(m) have the same Mesh-graph part .TheCQM (n,k) can be thought of having 2^m Mesh-graph M(k), where the nodes in each M(k) have the same Crossed Cube part label.

Theorem 1: The total number of nodes in CQM (n,k) graph is $2^n k^2$.

Proof: Each node of CQ(n) is replaced with k dimension mesh. Which implies that there are k^2CQ(n) whose nodes have the same mesh-graph part. It also has the k^2M(k), which in turn has the same crossed cube-part label. Which gives that the total number of nodes in CQM(n,k) graph is $2^n k^2$.

Theorem 2: The degree of CQM (n,k) is n+2 and diameter is $(2k-2)+\lfloor n-1/2 \rfloor$

Proof: From the description given above for CQM(n,k) it is obvious that the degree of CQM(n,k) is n+2. To see that the diameter is $(2k-2)+\lfloor n-1/2 \rfloor$, consider two nodes <u,v> and $<u^{/},v^{/}>$.Now proceeding from the node <u,v>one can reach the node $<u^{/},v^{/}>$ in at most $\lfloor n-1/2 \rfloor$ hopes. Then proceeding from the node$<u^{/},v^{/}>$ one can reach the node $<u^{/},v^{/}>$ in at most 2k-2 hopes. This again follows from the description given above that node with the same Crossed Cube part label form a Mesh graph of order k.It is known that the diameter of Crossed Cube graph is $\lfloor n-1/2 \rfloor$. Hence in at most $\lfloor n-1/2 \rfloor$ hopes, we can reach $<u^{/},v^{/}>$ from<u,v> i.e. the diameter of CQM(n,k) is $(2k-2)+\lfloor n-1/2 \rfloor$.

Theorem 3: The total number of links in CQM(n,k) is $k^2n2^{n-1}+2^nk^2$.

Proof: From the above description this can be treated as k^2 Crossed cube of order n connected in Mesh-graph fashion. Similarly 2^n Meshes of order k connected in Crossed cube-graph fashion. Hence the total number of links in CQM (n,k) is $k^2n2^{n-1}+2^nk$.

Theorem 4: The bisection width of CQM (n,k) is $2^{n-1}k^2$.

Proof: It is the minimum number of edges whose removal renders the graph in two equal halves. From the above description it is clear that there are 2^n mesh –graph have the same Crossed cube part label and hence $2^n/2$

number of edges require to cut half the Mesh-graph part. It is also clear that there are k^2 Crossed cube graph have the same Mesh-part label. So the total no of edges require to cut the CQM (n,k) to half will be

$$2^n/2 *k^2=2^{n-1}k^2$$

Theorem 5: The connectivity of the CQM (n,k)is n+2k-2.
Proof: The connectivity of Crossed Cube graph is n and the connectivity of Mesh graph is 2k-2 and hence the connectivity of CQM(n,k) is n+2k-2.
Theorem 6: The cost of CQM(n,k) is (n+2)(2k-2)+⌊n-1/2⌋
Proof: The Cost of network is defined by the definition below:
Cost=Degree*Diameter. Hence the Cost of CQM(n,k) is (n+2)(2k-2)+⌊n-1/2⌋

Table 1:Comparision of the Topological Properties of Crossed cube,-Mesh CQM(n,k)

Parameters	Mesh	CQ	CQM(n,k)	HQ	HM
Nodes	k^2	2^n	$2^n k^2$	2^n	$2^n k^2$
Degree	k	N	n+2	n	n+2
Diameter	2k-2	⌊n-1/2⌋	(2k-2)+⌊n-1/2⌋	n	n+2k-2
Bisection width	K	2^{n-1}	$2^{n-1}k^2$	2^{n-1}	$2^{n-1}k^2$
Connectivity	2	n	n+2k-2	n	n+2k-2
Cost	$2k^2$-2k	n+⌊n-1/2⌋	(n+2){(2k-2)+⌊n-1/2⌋ }	n^2	(n+2)(n+2k-2)
Fault Tolerant	k-1	n-1	n+1	n-1	n+1
Node-Disjoint Paths	2	N	n+2	0n	n+2

IV. Performance Analysis

It is very essential to do analysis the performance of a parallel interconnection network as it reflects important aspects of a multiprocessor. To make the parallel interconnection network more attractive, more emphasis if given to fault tolerance and reliability analysis. All these factors are derived in the following section.

A. *Fault Tolerance*

In parallel computing environment the fault tolerance of a network is an important characteristic. For a graph, it is defined as the maximum number of vertices that can be removed from it provided that the graph is still connected. Hence the fault tolerance of a graph is defined to be one less than to its connectivity. As discussed in [9], a system is said to be k-fault tolerant if it can sustain upto k number of edge fault without disturbing the network. For CQM(n ,k) the node degree is n+2, hence CQM tolerate up to n+1 faults.

B. *Fault Diameter*

In parallel computing environment the fault tolerance of a network is an important characteristic. For a graph, it is defined as the maximum number of vertices that can be removed from it provided that the graph is still connected. Hence the fault tolerance of a graph is defined to be one less than to its connectivity. As discussed in [9], a system is said to be k-fault tolerant if it can sustain upto k number of edge fault without disturbing the network. For CQM(n ,k) the node degree is n+2, hence CQM tolerate up to n+1 faults.

C. *Cost Effectivenes Factor*

While calculating the cost of a multiprocessor network, along with the cost of the processing elements, the cost of the communication link is also considered [12].In cube based network the number of links is a function of the number of processors. The cost effectiveness factor takes this into account and gives more insight to the performance of the multiprocessor system**.**
Theorem 8: The cost effectiveness factor of CQM(n ,k) is $1/1+\delta(2^{n-1}+1)$. Where δ is the ratio of the link cost to the processor cost.
Proof: The total number of processor in CQM is $2^n k^2=p$.
The total number of edges is $k^2 n2^{n-1}+2^n k^2=2^n k^2(2^{n-1}+1)=p(2^{n-1}+1)=f(p)$.
So $g(p)=f(p)/p=(2^{n-1}+1)$
Hence, $CEF(p)=1/1+\delta g(p)= 1/1+\delta(2^{n-1}+1)$.

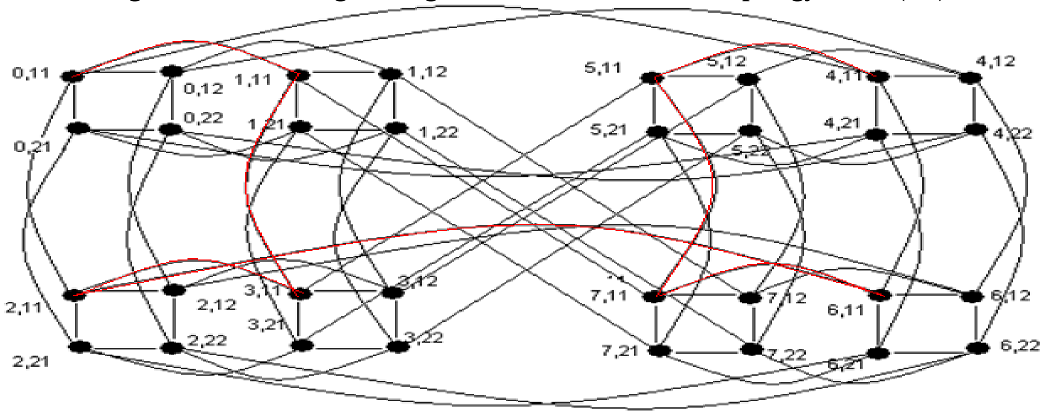
V. Embedding of Ring Network in Crossed Cube-Mesh

The embedding of a guest graph G(V,E) into another host graph S is a mapping of the set of vertices V(G) into that of S, that V(S) and E(G) into E(S).
The mapping is denoted by R. Thus, any vertex x of G is mapped through R(x) of s uniquely. That means R(x) ≠R(y) for x≠y and x,y belongs to V(G).However the embedding to exist:

|S|≥|G| and S to be connected
The ratio |S|/|G| is called expansion.

The dilation is defined as follows:
Dilation(D_L)= max(length of shortest path from R(x) to R(y))
A ring topology can be embedded into the CQM(n,k) using the Gray code. A Gray code is a well-known sequence of binary bits, where 2 consecutive codes differ by only 1 bit, G(0,1)=(0,1). From G_1 , G_2 can be defined as (00,01,11,10).
For n>2, $G_n=(0G_{n-1}, 1G^r_{n-1})$, Where G^r_n is the reverse string of G_n.
$G_3=(0G_1, 1G^r_1)$=(000,010,110,101,100)
Next $G_4=(0G_3, 1G^r_3)$ and $G_5=(00G_3, 01G^r_3\, 11G^r_3, 10G_3)$ the first and last labels differ by only 1 bit.
A Ring can be successfully embedded in CQM(n,k).In figure bellow arrow heads show the sequence of embedding. A ring with 8 nodes bearing the code as in (0,1,3,2,6,7,5,4) can be easily embedded in CQM(3, 2).
R(0)=(000,11),R(1)=(001,11),R(3)=(011,11),…..,R(4)=(100,11).
For a 16 node ring with codes as in G_4=(0,1,3,2,6,7,5,4,12,13,15,,14,10,11,9,8), the embedding extended to nodes with mesh level 12 as follows:
R(0)=(000,11),R(1)=(001,11),R(3)=(011,11),…..,R(4)=(100,11).
R(12)=(100,12),R(1)=(101,12),R(3)=(111,11),…..,R(4)=(000,11).
Obviously, the dilation and expansion for this embedding are both 1.
In the following diagram of CQM(3,2), A ring of 8 nodes is embedded. The red outline shows the embedding.

Figure 5: Embedding of Ring in Crossed Cube-Mesh Topology CQM (3,2)



VI. Crossed Cube-Mesh For Distributed Computing

In this section, the suitability of the proposed Crossed cube-Mesh network for distributed computing is discussed through construction of node disjoint paths and shortest path self-routing algorithms in fault-free environment

A. Node-Disjoint Paths

Lemma 1:
Let u,v be any two nodes of n order crossed cube CQ(n).Then there exist n node disjoint paths of length at most n+2 between u and v.
Lemma 2:
Let u,v be any two nodes in K-order Mesh M(k).Then there are 2 node disjoint path of length at most K.
Theorem 9: A Crossed cube-Mesh CQM (n,k) has n+2 node-disjoint path between any two nodes.
Proof: Let (u, v) and (u′v′) be any two nodes of Crossed cube-Mesh CQM(n, k).There are three cases to be considered here.

*Case1***:** u≠ u′ and v= v′

Consider the nodes with Mesh-graph part label v. They form a n-order Crossed cube. Both <u, v> and <u′,v′> nodes are present in this Crossed cube and the n node-disjoint paths within the Crossed cube can be found. Clearly, all the nodes in these n paths have n paths have v as their Mesh-graph-part label. Consider the n-1 nodes <u, $v^{(1)}$><u, $v^{(2)}$>…..<u, $v^{(n-1)}$> adjacent to <u, v>. The presence of such nodes follows from the fact that any node in a Mesh-graph of order k has k-1neighbors. Since the node with Mesh-graph part label $v^{(1)}$ form a Crossed cube, proceeding from <u, $v^{(1)}$>,<u, $v^{(1)}$> can be reached through a path within this Crossed cube and it does not intersect any of previously formed path. Since, <u′,$v^{(1)}$>is adjacent to <u′,v′>, the above function yield a node-disjoint path between <u, v> and (u′v′).

Case2: u=u′ and v≠ v′

Consider the node with Crossed cube-part label u. They form an k-order Mesh graph. Both <u, v> and<u′,v′> are present in n-order Mesh graph and 2 node-disjoint path can be found within the Mesh graph.

Clearly all the nodes in these paths have u as their Crossed-cube label. Now, consider the k nodes $<u^{(1)},v>,<u^{(2)},v>,\ldots,<u^{(k)},v>$ adjacent to <u, v>. The presence of such nodes follows from the fact that any node in Crossed cube of order k has n neighbors. Since the nodes with the Crossed cube-graph part label $u^{(1)}$ form a Mesh graph, proceeding from$<u^{(1)},v><u^{(1)},v'>$,can be reached through a path within this Mesh graph. This path does not intersect with any of the previously formed 2 paths. Since $<u^{(1)},v'>$ is adjacent to $<u',v'>$, this construction yields a node-disjoint path between (u, v)and $(u'v')$.By similar construction one can find n-1 more node-disjoint paths, Since this n paths have different Crossed cube –part labels, they do not intersect each other . By this we can say that there are n+2 node-disjoint paths between (u, v) and $(u'v')$.

Case3: $u\neq v$ and $u'\neq v'$

Consider n-order Crossed cube. There are n node-disjoint paths between u and u'. These path will be addressed as $C_{p1},C_{p2},\ldots C_{pn}$. Clearly, the first node of these paths is u. Let the second node in these paths be $u^{(1)},u^{(2)},\ldots,u^{(n)}$, ie, the n neighbors adjacent to u. At most, one of these nodes can be u'. Now, let us consider an n-order Mesh graph. There are 2 node-disjoint paths between v and v'. These path will be addressed as M_{p1}, M_{p2}. Clearly, the first node of these path is v. Let the second node of these paths be $v^{(1)},v^{(2)},\ldots,v^{(k)}$.The n+2 node-disjoint paths between (u, v)and $(u'v')$ are shown below.

$<u, v><u^{(1)},v>\ldots\ldots v'$s are switched as in $S_{p1}\ldots<u^{(1)}, v>\ldots v'$s are switched as in $C_{p1}\ldots<u',v'>$. $<u, v><u^{(2)}, v>\ldots v'$s are switched as in $s_{p1}\ldots ,u^{(2)}, v'>\ldots c'$s are switched as in $c_{p2}\ldots <u', v'>$

$<u, v><u^{k}, v>\ldots v'$s are switched as in $s_{p1}\ldots<u^{k}, v'>\ldots u'$s are switched as in $c_{pk}..<u', v'>$
$<u, v><u^{(n)}, v>\ldots u'$s are switched as in $c_{pk}\ldots<u', v>\ldots v'$s are switched as in $s_{p1}\ldots <u' , v'>$.
$<u,v>\ldots v'$s are switched as in $s_{p1}\ldots <u,v'>\ldots u'$s are switched as in $c_{pn}\ldots <u', v'>$.
$<u, v><u, v^{(2)}>\ldots u'$s are switched as in $s_{p2}\ldots <u', v'>$
$<u, v><u, v^{(n-1)}>\ldots u'$s are switched as in any one of the $c_{p1}, c_{p2},\ldots c_{pk},\ldots<u',v^{(n)}>\ldots v'$s are switched as in $s_{pm}\ldots<u',v/>$.

To see that these n+2 paths are node-disjoint, consider the first 2 paths. Since, c_{p1}, c_{p2} are node-disjoint, it follows that any Crossed cube-part label can occur in only one of these 2 paths this ensures that these paths do not intersect each other except at nodes <u,v> and $<u', v'>$. Also, none of the nodes repeat in any of these paths. The nodes in these paths have u' as their Crossed cube-part label only at $<u', v'>$. Now, consider the last n paths. Since the nodes in these paths have different star-graph-part labels when their Crossed cube-part labels are the same and vice versa, these paths are node-disjoint. Also they do not intersect with any of the 2 paths constructed earlier. Thus it gives n+2 node-disjoint paths to reach $<u',v'>$ proceeding from <u, v>

B. Shortest Path length

Next we find the shortest path between any two distinct pair of nodes in CQM(n,k). Before finding the path length between two distinct nodes, we introduce some notation.
Change Bit (C_B):
Let us consider two nodes in CQM(n,k) as (i,j) and (p,q), then the C_B will be the number of change bit between these two nodes.
Example:
Let (000,11)and (111,22) are two nodes in CQM(3,2) .Then the C_B of these two nodes will be 5.
First Bit 0 is changed to bit 1,second bit 0 is changed to 1, third bit 0 is changed to 1 of i.
First bit 1 is changed to 2 and second bit is changed to 2 of j. Hence number of bits changed is 5.
Proposition:
The length of the shortest path from a source node(i,j) to a destination node(p,q) in CQM(n,k)is as follows:
If $i=p$ & $C_B(j,q)=1$
Then path length=1
If $i=p$ & $C_B(j,q)=2$
Then path length=2
If $j=q$ & $C_B(i,p)=1$
Then path length=1
If $j=q$ & $C_B(i,p)=2$
Then path length=2
If $i\neq p$ *and* $j\neq q$
Then path length$=C_B(i,j)+C_B(p,q)$
If $C_B(i,p)=3$
Then path length$=C_B(i,p)+C_B(j,q)-1$

To explain the proposition, we illustrate it through an example. Suppose (i,p)=(000,11) and (j,q)=(100,12).Then the path length between these two nodes will be:
$C_B(000,100)=1$ &$C_B(11,12)=1$
Path Length = $C_B(000,100)+C_B(11,12)$
=1+1=2

VII. Routing in Crossed Cube-Mesh Topology

The problem of finding a path from a source node S to a destination node D and forwarding the message along the path is called routing. The routing between source node S, (i,j) and destination node D, (p,q) is described below.

A node which has the labeling as(i,j), i is the Crossed cube graph label and j is the Mesh graph label. In Routing in CQM(n,k) we have to consider following cases:

Case 1:when i=j and $p\neq q$ and $C_B(j,q)=1$,
i.Find the neighbor of j i.e. $N_j(x)$ for x=1,2
ii. Compare the $N_j(x)$ for x=1,2 with D(q)
If $D(q)=N_j(x)$
Then message is consumed and routing stops.

Case2: when i=j and $p\neq q$ and $C_B(j,q)=2$,
i.Find the neighbor of j i.e. $N_j(x)$ for x=1,2
ii.Compare the $N_j(x)$ for x=1,2 with D(q)
If $D(q)=N_j(x)$ for x=1,2
Then message is consumed and routing stops.
Else
Update the value of j by $N_j(1)$
Route the msg. to new (i,j)
iii. Find the $C_B(j,q)$
If $C_B=1$
Then Case1.

Case 3: When $i \neq p$ and j=q and $C_B(i,p)=1$
i. Find the neighbors of i,i.e, $N_i(y)$ for y=1,2,..,n, where n is the size of CQ.
ii. Compare the $N_i(y)$ for y= 1, 2 ,..,n with D(p)
If $N_i(y)=D(p)$
Then message is consumed and routing stops.

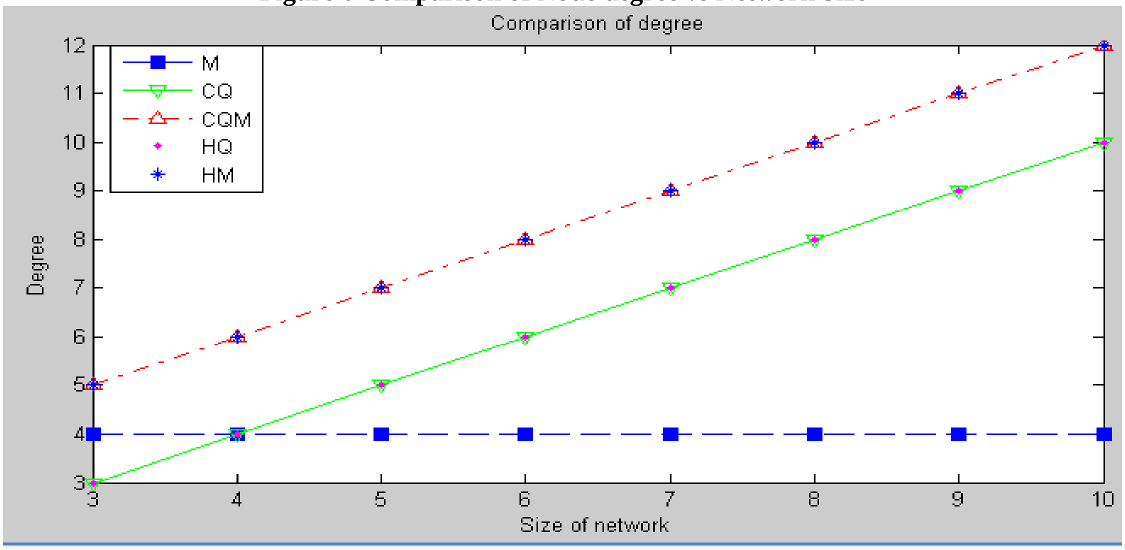
Case 4: When $i \neq p$ and j=q and $C_B(i,p)=2$
i. Find the neighbors of i, i.e, $N_i(y)$ for y=1,2,..,n, where n is the size of CQ.
ii. Arrange the neighbors in ascending order.
iii. Compare the $N_i(y)$ for y= 1, 2 ,..,n with D(p)
If $N_i(y)=D(p)$
Then message is consumed and routing stops.
Else
go to step iv.
iv. Update the value of i by $N_i(1)$
v. Check $C_B(i,p)$
If $C_B(i,p)=1$
Then Case 3.

Case 5: $i \neq p$, $j \neq q$, and $C_B(i,p)=3$
i. Find the neighbors of i, i.e, $N_i(y)$ for y=1,2,..,n, where n is the size of CQ.
ii. Arrange the neighbors in ascending order.
iii. Compare the $N_i(y)$ for y= 1, 2 ,..,n with D(p)
If $N_i(y)=D(p)$
Then message is consumed and routing stops.
Else
go to step iv.
iv. Update the value of I by $N_i(1)$.
v. Find the neighbors of Ni(1), for all I where i=1,2,…,n.
vi. Compare with $N_i(1)$ ith D(p)
If $N_i(1)=D(p)$
Then message is consumed and routing stops.

VIII. Results and Discussion

This section presents the results obtained followed by a brief discussion. We compare the performance of the proposed topology with the existing topology to extract its better features. The node degree of the hyper-mesh and the crossed cube-mesh is same but it is higher than the mesh and crossed cube and as well as traditional hypercube due to the hybrid structure. Figure 6 shows the result.

Figure 6 Comparison of Node degree vs Network Size



While comparing the diameter hyper-mesh possesses the highest value as its size goes at faster rate. The hypercube and crossed cube possess smaller value as they are smaller network. Crossed cube-mesh stands lowest among the hybrid network, as shown in Figure 7.

Figure 7 Comparison of diameter Vs. Network Size

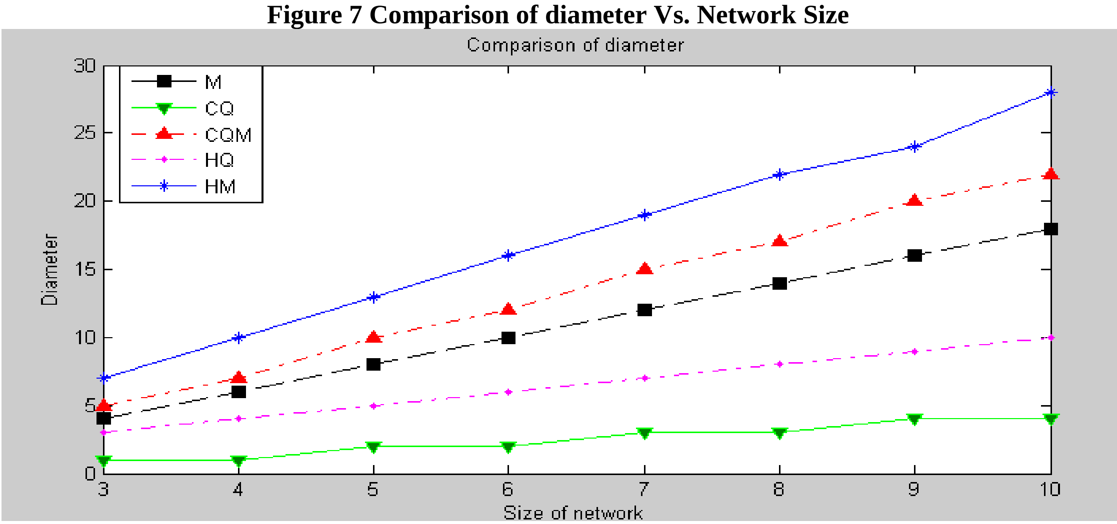
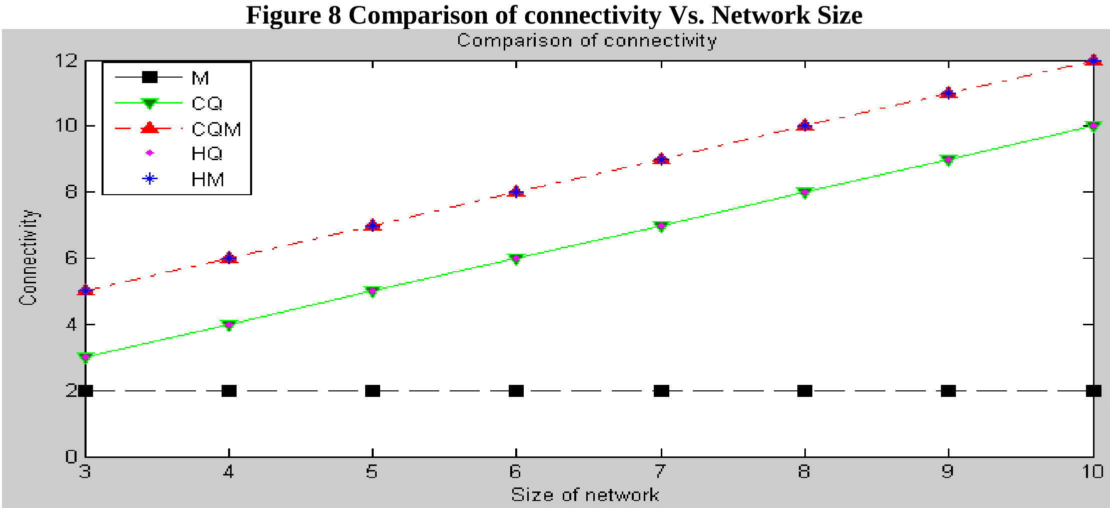


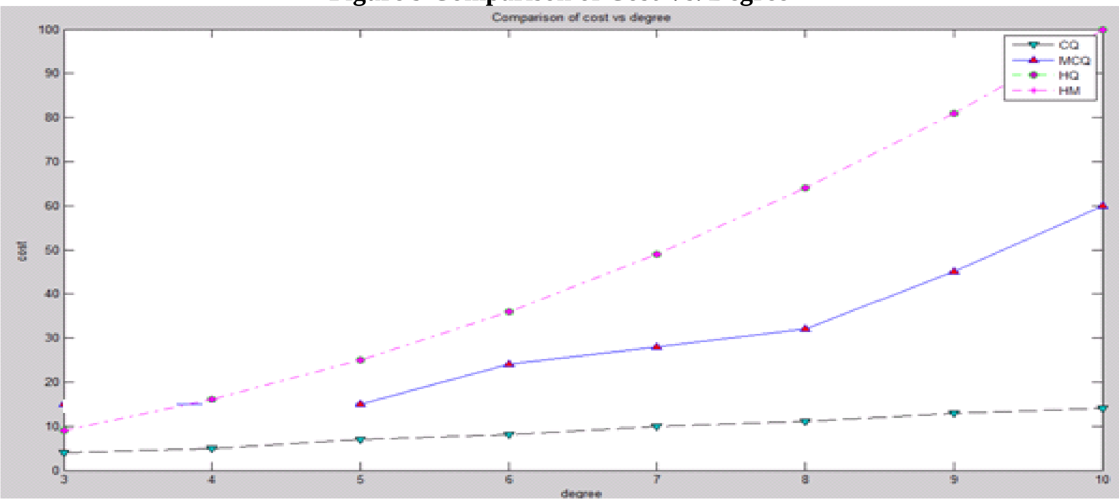
Figure 8 shows the connectivity among the network. From figure it clear that mesh has a constant connectivity and on the other hand crossed cube and hyper cube has the lower value than hyper-mesh and crossed cube mesh. Both the hybrid net possesses the same connectivity as the size increases.

Figure 8 Comparison of connectivity Vs. Network Size



However the cost is found to be less than that of hypercube and hyper-mesh as shown in Figure 9.The cost is compared against the node degree. With node degree 5 and beyond, The CQM bears the lowest value.

Figure 9 Comparison of Cost Vs. Degree



IX. Conclusion

This paper proposes a new parallel interconnection topology called Crossed cube Mesh (CQM) that is suitable for large-scale parallel systems. Being a of hybrid structure, the CQM bears the advantages of both the parent networks i.e., the Crossed cube and Mesh. Our proposed network when compared to the existing hyper-mesh is found to possess better characteristics in terms of diameter, cost, fault diameter and cost effectiveness factor. The new network offers high degree of embedding property and is proved to be a very promising and suitable candidate for parallel systems.

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