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# Star-Mobius Cube: A New Interconnection Topology for Large Scale Parallel Processing

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***Abstract**: The interconnection topology plays a vital role in parallel computing systems. In this paper a new interconnection network topology named as Star-mobius cube (SMQ) is introduced. The various topological and performance parameters such as diameter, cost, average distance, and message traffic density are discussed. The embedding and broadcasting aspects of the new network are also presented. Based on the performance analysis, the proposed topology SMQ is proved to be a better alternative to its contemporary networks.*
***Key words**: Interconnection networks, topological parameters, broadcasting, routing, embedding.*

## I. Introduction

The rapid progress in VLSI technology has lead to the development of multiprocessor systems that constitutes large scale parallel processing. In such systems numerous processors work together to execute tasks in parallel. A parallel computing system is of two types: Loosely coupled system and tightly coupled system. In a tightly coupled system, the processors are interconnected through a shared memory. However in loosely coupled system, each processor has its own private memory. To execute a task the processors in a multiprocessor system should exchange data among themselves and the interconnection network undertakes the active role in this data exchange.

The various network topologies for interconnection proposed in the literature include star[4], hypercube[8], tree, mesh and the ring[3]. Extensive research has been done on cube based networks as it has lower diameter, scalability and high embeddability. The different derivatives of hypercube have been proposed in literature. The prominent candidates among them are crossed cube[2] ,dual cube[10] , meta cube[9] , mobius cube[1] and the star cube[5] . The Star network for the long time has been in research due to its attractive properties like maximum fault tolerance and optimal broadcasting. Different performance parameters of a good network topology are small diameter, low degree, low cost, low average distance, low message density, efficient routing and broadcasting. Recently more research works are being done on product graphs. The main objective of a product graph network is to derive a new topology which comprises of positive features of both the base graphs. Many product networks such as Starcube[5], Star crossed cube[6], have been investigated

The principal objective here is to design a new topology which can be better than the star graph, mobius cube, Starcube and Star crossed cube. Our proposed topology in this work is the Star Mobius cube(n, k) which is a product graph of n- star[4] and k- mobius cube[1]. The proposed network Star mobius cube inherits properties of both the star graph and the mobius cube and possesses many attractive properties as compared to its parent graph and other product graphs. Next, we compare the performance parameters of the said networks to substantiate the merits of the new network.

This paper is organized as follows .The Section II describes the related work. In section III, we present the proposed topology and derive its properties. We propose algorithms for broadcasting and routing in Sections IV-V respectively. In Section VI, we discuss about embedding of other networks in the proposed topology. We analyse the performance and illustrate the merits of the proposed topology in section VII. The Section VIII concludes the paper.

## II. Related work

The hypercube [8] is considered to be the best among the various loosely coupled topologies. The most improved form of hypercube is the crossed cube [2]. The Crossed cube is better than the hypercube in terms of its diameter i.e.$\lceil (n+1)/2 \rceil$. It is also edge pancyclic for $n \geq 2$. But the crossed cube has higher message traffic density than the hypercube. The Dual cube[10] is an another variant of hypercube. The Meta cube [9] proposed by Li, Peng and W.Chu has the least no. of links. But the embedding of metacube and dualcube in other networks is quite difficult. Paul Cull and M. Larson showed that the mobius cube[1] has effectively smaller diameter than hypercube. The n-star graph [4] is a permutation graph as it has n! nodes. The Star graph is a better alternative to the cubic networks. But the factorial increase of nodes is the main drawback of the star graph. An interconnection Star Cube topology [5] has better diameter and average distance than the hypercube and the star network. It inherits the advantages of both the star and the hypercube. Subsequently, Star Crossed Cube[6] has been proposed with better performance measures than the Starcube. Here, the main challenge that remains to be addressed is the development of a new topology that overcomes the drawbacks of the above said topologies.

## III. Proposed Topology

In this section, our main objective is to propose a new topology taking into account the best features of the star graph and the mobius cube. Before proceeding further, a brief discussion on the parent networks follows.

### 3.1 Star graph

The m-dimensional Star denoted as S(m) has m! no. of nodes $(x_0,x_1,x_3.....x_{m-1})$[4]. Each node of S(m) is represented by permutation of address bits. The address bits of each node are arranged with $SWAP_i$ operation in routing. There exists an edge between two nodes when the address bits differ by their position e.g. 123 is the first node; then $SWAP_2$ i.e. swap between initial and second position, so the next node is 213. A Star graph has m!(m-1)/2 number of edges. But when we go for higher dimensional star graph then the number of nodes increases in a higher rate. For example, S(3) has 3!=6 no. of nodes where as S(4) has 4!=24 no. of nodes i.e.4x differ in S(3).The m-star graph is vertex and edge symmetric and has a diameter $\lfloor 3/2(m-1) \rfloor$ and is a m! permutation network. Because it has a set of m-1 generators, so the degree of star graph is m-1[4].The Fig.1 illustrates a 3-dimensional Star graph.

**Fig. 1: Star network(dimension = 3)**

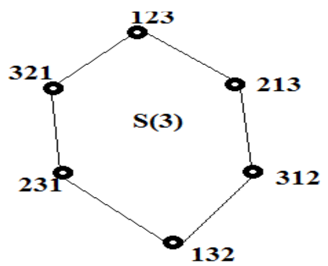



**Fig. 2(a): Mobius cube $MQ_3^0$**

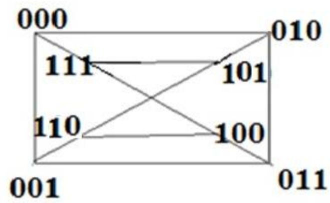
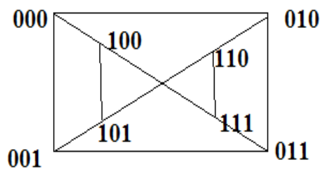



**Fig. 2(b): Mobius cube $MQ_3^1$**




### 3.2 Mobius cube

An n-dimensional mobius cube $MQ_n$ has $2^n$ nodes [1]. An edge joins node $X=x_1x_2x_3....x_n$ to the node $Y=y_1y_2y_3...y_n$ if $Y_i$ satisfies one of the following condition:

$$Y_i = x_1.....x_{i-1}\bar{x}_i\, x_{i+1}......x_n \quad if\, x_{i-1}=0 \tag{1}$$

$$Y_i = x_1 ..... x_{i-1}\bar{x}_i\bar{x}_{i+1}.....x_n \quad if\, x_{i-1}=1 \tag{2}$$

Where $\bar{x}_i$ is the complement of $x_i$ in (0,1). When the address bits differ only in the leftmost bit then the mobius cube known as 0-type mobius cube $MQ_3^0$ and when address bits differ in all bits then it is known as 1-type mobius cube $MQ_3^1$. According to the condition (1) an edge between a node and its $j^{th}$ node can be established if its $j^{th}$ value is 1. Similarly, based on second rule if $j^{th}$ through $n^{th}$ components equal to 1.The Figs. 2(a) and 2(b) show the 0-type mobius cube and 1-type mobius cube respectively. In the n-dimensional mobius cube the diameter is $\lceil (n+1)/2 \rceil$. In mobius cube, the routing algorithm takes O(n) runtime. On comparison of topological properties the mobius cube is found better than other variants of hypercube. Mobius cube has also efficient broadcast algorithm. However 1-type mobius cube is better than 0-type mobius cube. We can construct a 4-dimensional mobius cube out of figure 2(a) and 2(b) by following the above conditions 1and 2.

### 3.3 Proposed topology: Star mobius cube

In this section, we propose the new topology Star-mobius cube. The Star mobius cube denoted as SMQ(n,k) is the product graph of m-Star S(m) and K-mobius cube $MQ_k$. Here, each node of star graph is substituted by mobius cube. The address of each node in SMQ has two parts Xi that represents star part and $Y_i$ represents MQ part $\{x_0x_1....x_ny_{k-1}y_{k-2}......y_0\}$. In simple logic, mobius cubes are placed on star platform. The Fig. 3 illustrates the proposed Star-Mobius cube topology SMQ(3,3) for dimension 3.

#### 3.3.1 Topological features of Star mobius cube(n,k)

This subsection derives the expressions for various topological parameters of the proposed network.

**a) Node (N)**

The total number of nodes of the network shows the network size.

*Theorem 1:* The total number of nodes in the SMQ(n,k) graph is n! $2^k$.

*Proof:* An n-star has n! number of nodes and the MQ(k) has $2^k$ nodes. So the SMQ(n.k) which is the product graph of $MQ_k$ and n- Star network shall have n! $2^k$ nodes in all.

**b) Edges or Links (E):**

The edges connect the nodes and communication between nodes takes place through the edges.

*Theorem 2:* The total number of edges in the SMQ(n, k) is n! $2^{k-1}$ (k + n- 1).

*Proof:* The SMQ(n,k) has n! $MQ_k$ nodes connected to n-1 neighbours and each MQ has k$2^{k-1}$ edges. Hence, to connect n! MQs in star requiers n! $2^{k-1}$ (k + n- 1) number of edges.

**Illustration:**

In SMQ(3,3) the star dimension n=3 and the mobius cube dimension k=3.There can be 6 no. of mobius cube and each mobius cube has k $2^{k-1}$ i.e. 12 no. of links. All 6 mobius cube need to be connected in star platform. So the total no. of edges :3! *$2^{3-1}$(3+3-1)= 6*4*5= 120 no. of edges.

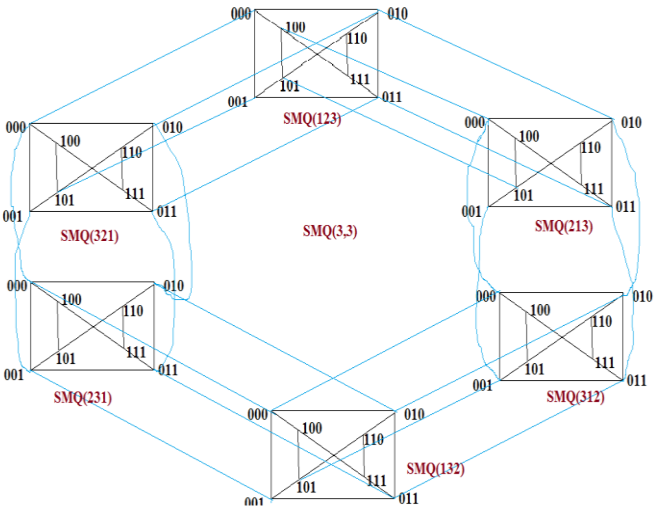



**Fig. 3: Star Mobius cube**

**c) Degree**

The degree is defined as the total no. of edges come out from each node. In symmetric network like Starmobius cube, each node has equal node degree. If the degree of network is high then it can connect more no. of nodes. So the requisite is higher degree network topology.

*Theorem 3:* The degree of SMQ(n,k) is (n+k-1) **.**

*Proof:* In $MQ_k$ , each node's degree is k and (n-1) edges are incident from each node of $MQ_k$ . So, the degree of SMQ is k+n-1.

**d) Diameter**

The diameter of an interconnection network is defined as the maximum distance between any two nodes in the network. The distance between two nodes is the shortest path between the nodes. Obviously the diameter network topology should be low so that we can have optimal number routing steps with higher degree, otherwise the complexity may rise.

*Theorem 4:* The diameter of the Star- mobius cube topology SMQ(n, k) is

$\lfloor(3(n-1)/2)\rfloor + \lceil(k+2)/2\rceil$ k>=4 in 0-type $SMQ_k$ .

$\lfloor 3(n-1)/2\rfloor + \lceil(k+1)/2\rceil$ k>=4 in 1-type $SMQ_k$ .

*Proof:* In an n-star travelling from one node to other takes $\lfloor(3(n-1)/2)\rfloor$ steps. For $MQ_k^0$ the diameter is $\lceil(k+2)/2\rceil$ k>=4 . For $MQ_k^1$ the diameter is $\lceil(k+1)/2\rceil$ k>=4. This lower , the diameter, the better is the network. In $\lfloor(3(n-1)/2)\rfloor + \lceil(k+2)/2\rceil$ k>=4 number of hops for $SMQ_k^0$ or $\lfloor 3(n-1)/2\rfloor + \lceil(k+1)/2\rceil$ k>=4 number of hops for $SMQ_k^1$ ,we can move from any node to any other node in SMQ.

**e) Cost**

The cost is a significant performance measure of any network topology. The cost deals with the communication links. The cost is the product of network degree and diameter. It should be less for a network.

*Theorem 5:* The cost of the Star-Mobius cube network SMQ(n, k) is

$(n+k-1)[\lfloor 3(n-1)/2\rfloor + \lceil(k+2)/2\rceil]$ k>=4 $SMQ_k^0$ .

$(n+k-1)[\lfloor 3(n-1)/2\rfloor + \lceil(k+1)/2\rceil]$ k>=4 $SMQ_k^1$.

*Proof:* In SMQ the cost= Degree∗Diameter; we know from Theorem 3 that the degree of (SMQ) is (n+k-1) and from Theorem 4 the diameter of 0-type and 1-type mobius cube. Using both theorems we can get the expression of cost for SMQ.

**f) Average Distance (d)**

*Theorem 6:* The average distance (d) of the SMQ(n,k) is given by

$k/3+[1-(-1/2)^k]/9 + n-4+2/n+\sum_{i=1}^{n} 1/i \leq (d) \leq k/3+[1-(-1/2)^k]/9+1+n-4+2/n+\sum_{i=1}^{n} 1/i$

*Proof:* The average distance of the n-star graph is $n-4+2/n+\sum_{i=1}^{n} 1/i$ and for $MQ_k^0$ average distance is $k/3+[1-(-1/2)^k]/9$ and for $MQ_k^1$, $k/3+[1-(-1/2)^k]/9+1$. Hence, the average distance of SMQ is the sum of average distance of the both star and mobius cube.

**g) Message Density(ρ):**

The message density is the next important measure of any network topology. It relates with the number of message sent from the source to the destination. The message density of a network should be less so that the message traffic will be minimum as the message traffic affects the communication efficiency.

*Theorem 7:* The message density of SMQ(n, k) is represented as $\rho= 2d/(k+n-1)$.

*Proof:* Message density is defined as $\rho=(d*N)/E$ ,i.e. (the average distance * total no. of nodes) / total no. of edges.

From the Theorem 6, we can get the value of average distance d.

Here, $N=n!*2^k$ and $E=n!*2^{k-1}(k+n-1)$

Hence, $\rho= d*n!2^k/ n!2^{k-1}(k+n-1) = 2d/(k+n-1)$.

A comparative and brief account of the various topological parameters of the proposed network is worked out in Table1.

**Table1: Comparison of Topological Parameters**

| **Parameters** | **Hypercube[8]** | **Mobius cube[1]** | **Star graph[4]** | **Star Cube[5]** | **SCQ[6]** | **SMQ [proposed]** |
|---|---|---|---|---|---|---|
| **Nodes** | $2^m$ | $2^m$ | k! | $k!2^m$ | $k!2^m$ | $k!2^m$ |
| **Edges** | $m\ 2^{m-1}$ | $m\ 2^{m-1}$ | k!(k-1/2) | $k!2^{m-1}(m+k-1)$ | $k!2^{m-1}(m+k-1)$ | $k!2^{m-1}(m+k-1)$ |
| **Degree** | m | M | k-1 | (m+k-1) | (m+k-1) | (m+k-1) |
| **Diameter** | m | $\lceil(m+2)/2\rceil$ m>=4 $MQ_m^0$<br>$\lceil(m+1)/2\rceil$ m>=1 $MQ_m^1$ | $\lfloor 3/2(k-1)\rfloor$ | m+ $\lfloor 3/2(k-1)\rfloor$ | $\lceil m+1/2\rceil$ + $\lfloor 3/2(k-1)\rfloor$ | $\lfloor 3(k-1)/2\rfloor$+ $\lceil(m+2)/2\rceil$ m>=4 $SMQ_m^0$<br>$\lfloor 3(k-1)/2\rfloor$+ $\lceil(m+1)/2\rceil$ m>=4 $SMQ_m^1$ . |
| **Cost** | $m^2$ | m$\lceil(m+2)/2\rceil$ m>=4 $MQ_m^0$<br>m$\lceil(m+1)/2\rceil$ m>=1 $MQ_m^1$ | (k-1) $\lfloor 3/2(k-1)\rfloor$ | (m+k-1) (m+ $\lfloor 3/2(k-1)\rfloor$) | (m+k-1) ($\lceil m+1/2\rceil$ + $\lfloor 3/2(k-1)\rfloor$) | (m+k-1)($\lfloor 3(k-1)/2\rfloor$ + $\lceil(m+2)/2\rceil$) m>=4 $SMQ_m^0$<br>(m+k-1)($\lfloor 3(k-1)/2\rfloor$ +$\lceil(m+1)/2\rceil$) m>=4 $SMQ_m^1$ |
| **Average Distance(d)** | m/2 | $d=m/3+[1-(-1/2)^m]/9$ for $MQ_m^0$<br>$d=m/3+[1-(-1/2)^m]/9 +1$ for $MQ_m^1$ | $k-4+2/k+\sum_{i=1}^{k} 1/i$ | $m/2+k-4+2/k+\sum_{i=1}^{k} 1/i$ | $(11x+4y/8)+ k-4+2/k+\sum_{i=1}^{k} 1/i$ | $d = m/3+[1-(-1/2)^m]/9 +k-4+2/k+\sum_{i=1}^{k} 1/i \leq (d) \leq m/3+[1-(-1/2)^m]/9 +1+k-4+2/k+\sum_{i=1}^{k} 1/i$ |
| **Message Density** | 1 | 2d/m | 2d/(k-1) | 2d/(m+k-1) | 2d/(m+k-1) | 2d/(m+k-1) |

## IV. Broadcasting

The present section is devoted for illustrating the process of Broadcasting in the proposed SMQ network topology. The parallel algorithms often require that a processor should send data to all other processesors, this is known as braodcasting. For an interconnection network, it is essential that it must broadcast messages efficiently to other nodes. There are two main situations of broadcasting: one-to-all broadcast and all-to-all broadcast. In one-to-all broadcast a single node transfers its data to all other nodes and in all-to-all broadcast every node broadcasts data to every other nodes.

*Theorem 7:* The one-to-all broadcast algorithm for the SMQ(n, k) takes O(k + nlogn) time.

*Proof:* In One to all broadcasting message transmits from the source node(s) of mobius cube to destination node(v) another mobius cube inside the star i.e. s=<0,123> and v=<0,213>. So message is broadcast in both the mobius cube and the star. One to all broadcasting in mobius cube takes k communication steps and star graph takes nlogn steps. As broadcast of message is done in both ways so in all the SMQ takes (k+nlogn) comuunication steps for broadcasting.

*Algorithm: One-to-All Broadcasting*

Broadcast (u, v, msg) /* u= source node v= destination node msg=message */

*Step 1:* Send message (msg) to neighbour node along the z-axis of u in mobius cube and one node of star (i.e. to a node of another mobius cube in star base)

*Step 2*: Send the msg from the received node to nodes along y-axis of the sender node and repeat step 1 for the other mobius cube.
*Step 3:* Send the msg from the received node to nodes along x-axis and continue step 2 for other mobius cube.
*Step 4:* Continue the step 3 for rest of nodes till all nodes of all mobius cubes receive the msg of source node.
*Step 5:* end

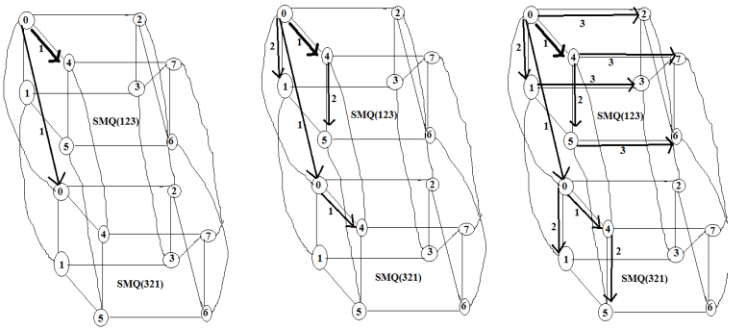



**Fig.4: Broadcasting in SMQ(123) and SMQ(321)**

**Illustration:**
In Fig.4, we have taken two cubes of star mobius cube. The message is broadcast from the source node SMQ(123,0). In step 1the message will broadcast from (123,0) to node along z-axis i.e (123,4) and one node of star i.e. SMQ(321,0). In step 2 the message is broadcast from (123,0) and (123,4) to nodes along y-axis i.e. 0 to 2 and 4 to5 and in next mobius cube the message is broadcast from SMQ(321,0) to along z-axis SMQ(321,4) and SMQ(231,0). Similarly, step 3 is repeated. The steps are marked in bold arrow.
*Theorem 8:* The all-to-all broadcast algorithm for the SMQ(n,k) takes O(M + nlogn) time, where M no. of k mobius cubes take part in the broadcasting.
*Proof:* In all-to-all broadcast each node of every mobius cube transmits message to other nodes in star. i.e. each node transmits its data to all other nodes and also receives data from all nodes. 0-type mobius cube takes atmost $\lceil (n+2)/2 \rceil$ communication steps and 1-type mobius cube executes in atmost $\lceil (n+1)/2 \rceil$. As all the cubes take part in broadcast at a time and if there is M number of mobius cubes, then all-to-all broadcast will take (M + nlogn) communication steps. Hence, the theorem is proved.

## V. Routing

This section explains the process of routing in the proposed SMQ(n, k) topology. The routing is a mechanism in which the path to forward message from source to destination is determined. In routing, it needs not to visit all the nodes unlike broadcast. However it should determine the shortest path from the source to the destination. The routing algorithms can be applied to both the star graph and the mobius cube. In the SMQ(n, k) routing algorithm works in two steps:

a) Routing for mobius cube
b) Routing for star graph

*Algorithm: SMQ Routing(s, d, m)*
*Step 1:* Perform E-cube routing from the source node to other nodes inside the mobius cube.
*Step 2:* From node of one mobius cube to node of other cube in star platform send message in the shortest path to the destination.
*Step 3:* For each intermediate node concatenate the path from the source to the destination.
**Illustration:**
Let us assume that the message will be forwarded from the source <000,123> to the destination <001,213>. In point to point routing the is path: <000,123> → <100,123> → <000,213> → <001,213> and the distance = 3.
Hence length of the path is the sum of length inside the mobius cube and length from mobius cube of source star node to next destination star node i.e. u, v belong to mobius cube and w is at another mobius cube. Then the routing path from u to w is the sum of length of u to v and length of v to w.

## VI. Embedding of Networks

The embedding of a network in another network is an interesting area of research. Here, we consider the embedding of Ring network and binomial tree in the propose SMQ ( n, k) topology.

### a) Embedding Ring

A ring can be efficiently embedded in the Star mobius cube. Using gray code a network can be embedded in another network. The Figure 5 shows embedding of ring with $SMQ_{123}$ and $SMQ_{213}$. The function value of gray code is G (i, d) where i= node index of ring and d= dimension of SMQ.

G(0,1) = 0 , G(1,1) = 1

$$G(i,x+1) = \begin{cases} G(i,x) & i < 2^x \\ 2^x + G(2^{x+1} - 1 - i, x) & i \geq 2^x \end{cases}$$

The gray code of a ring with 8 nodes is G= (0,1,2,3,6,7,5,4). The 8 node ring can be embedded to 8 node i.e. 3-dimensional mobius cube in this way: R(0)= <123,0> R(1)= <123,1> R(2)= <123,2> R(3)= <123,3> R(7)= <123,4> R(6)= <123,5> R(5)= <123,7>
R(4)= <123,6>

### b) Embedding Binomial tree

The Binomial tree follows the principle of divide and conquer. We can successfully map Star mobius cube into the binomial tree. The figure 6 is the partial binomial tree representation of Star mobius cube. The Binomial tree has a regular structure. The nodes of the binomial tree are notified by SMQ(3,3) addresses. Every vertex of Star-mobius cube is the root of atleast one binomial tree.

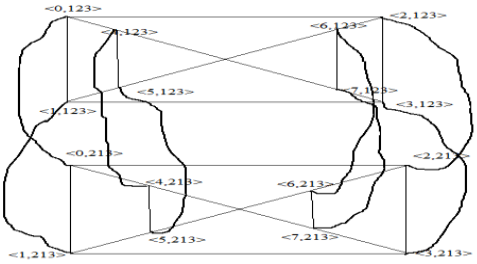


**Fig.5 Ring embedding in $SMQ_{123}$ and $SMQ_{213}$**

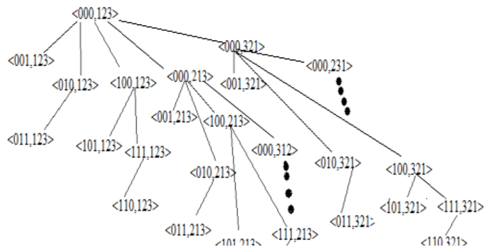


**Fig.6 Binomial tree of SMQ(3,3)**

## VII. Results and Discussions

In this section, the results of comparison of various performance parameters of the Star-Mobius cube (SMQ) with other networks are presented. The Table 1 presents the various topological parameters of mobius cube and five related network topologies. The various candidate networks considered here for the purpose of comparison are Hypercube(HC), Star, Starcube(SC), Star crossed cube(SCC), Mobius cube(MQ).

In Fig. 7, the network degree is compared with respect to other networks. As it is a product graph of the MQ and star, it is quite obvious for the Star mobius cube to have higher degree than Hypercube and Star. In other words, the proposed network Star mobius cube can connect more number of nodes than other networks. The comparison of diameter is shown in Fig 8. The Star mobius cube is observed to have lower diameter than the Star cube network. This adds to the low communication cost of SMQ and is therefore considered to be an advantage of the proposed topology. The cost of Star mobius cube is lower than Star cube as per the comparison shown in Fig 9. The Fig 10 shows that the Star mobius cube has a lower average distance than the Star cube and Star Crossed cube. The message density versus dimension is shown in Fig 11. Overally, the proposed topology Star- Mobius cube (SMQ) is observed to perform better when compared with other networks.

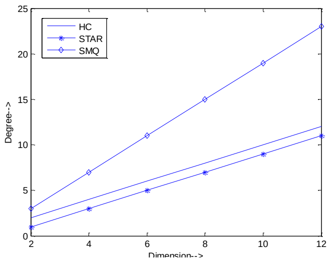



**Fig.7 Comparison of Degree**

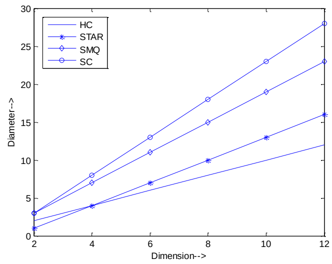



**Fig.8 Comparison of Diameter**

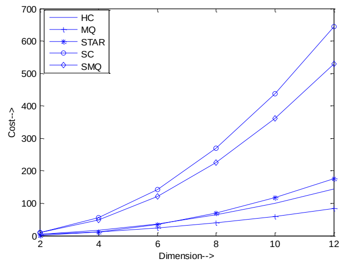



**Fig.9 Comparison of Cost**

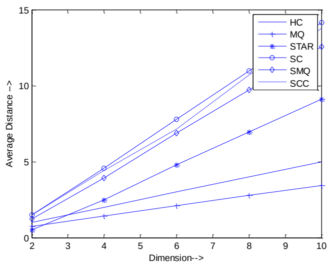



**Fig.10 Comparison of Average distance**

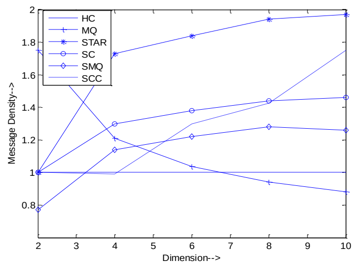



**Fig.11 Comparison of Message density**

## VIII. Conclusions

In this paper, we proposed a new network topology called Star- mobius cube for large scale parallel processing. The different topological parameters of the new topology are discussed. Two algorithms one for the broadcasting and the other for routing are proposed. Embedding of the new topology with the ring and binomial tree is described The various performance features of the proposed network are analysed and compared with other cube based product graphs in terms of average distance, message density , diameter and cost. Based on comparison and analysis the Star mobius cube is found to be a better network topology in comparison to other networks.

## IX. References


[1] P. Cull and S.M. Larson, "The Möbius cubes", *IEEE Trans. on Computers* 44 (5) (1995) 647–659.

[2] K. Efe, "The Crossed cube architecture for parallel computing",*IEEE Trans. Parallel Distributed Systems* 3 (5) (1992), 513–524.

[3] T.Y. Feng,"A survey of interconnection networks", *IEEE Computer,vol.14(1981)12-27*.

[4] S.B. Akers, D. Harel and B. Krishnamurthy, "The star graph: an attractive alternative to the n-cube", *International Conference on Parallel Processing* (1987), 1249-126.

[5] C.R. Tripathy, "Star-cube: a new fault tolerant interconnection topology for massively parallel systems", *Journal of the Institution of Engineers (India)* 84(2004), 83-92.

[6] N.Adhikari and C.R.Tripathy,"Star-crossed cube:an alternative to star graph", *Turkish Journal of Electrical Engineering & Computer Sciences* (To appear)

[7] Jun- Ming XU, Meijie MA, "Survey on path and cycle embedding in some networks", Front. Math,China 2009, 4(2): 217–252

[8] Y. Saad and M.H. Schultz,"Topological properties of hypercubes", IEEE Trans. on Computers 37(1988), 867-872 .

[9] Yamin Li, Shietung Peng and Wanming Chu,"Metacube– A New Interconnection Network for Large Scale Parallel Systems", *ACSAC02, Australian Computer Science Communications*, 24 (4) ( 2001) 29–36.

[10] Yamin Li and S Peng," Dual-cubes: A new interconnection network for high-performance computer clusters", *International Computer Symposium, Workshop on Computer Architecture,* (2000).

[11] Hilbers, Koopman, van de Snepscheut, " The twisted cubes*", Lecture Notes in Computer Science,Springer,* (1987) 258-259.

[12] Chang Hsiung Tsai, "Embedding of meshes in Möbius cubes", *Theoretical Computer Science, Elsevier*,(2008) 181-190