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Optimal Selection of Alternatives: Application of Grey Theory to Value Engineering

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Abstract: *Value engineering (VE) is a systematic methodology based on a standard job plan. It has been applied in industrial sectors in different countries. The VE process is a creative process that brings about value and quality improvement. This process identifies opportunities to unnecessary cost reduction while maintaining or improving the desired level of performance. A value study job plan consist of six sequential phases including information phase, function analysis phase, creative phase, evaluation phase, development phase and presentation phase. One of the important phases is evaluation phase. Most of the time, multiple attributes with different unites and priorities are selected to assess the optimal alternatives. How the VE team can determine the optimal alternative, when there are different and even nondeterministic linguistic opinions. In this article, a new approach to alternative selection is proposed. A grey based approach with grey numbers is applied to improve the evaluation process.*

Keywords: *Optimal Selection, Grey Theory, Value Engineering.*

I. Introduction

VE is first introduced during World War II. Where there were lacks of recourses, so it was vital to change the traditional methods and designs and raw materials. In most cased these changes led to efficiency of products while the unnecessary costs were eliminated. Nowadays, application of value engineering is pervasive in construction projects and is categorized as a technique to achieve world class management (Farsijani, 2010). Value engineering is defined as "an organized effort directed at analyzing the functions of goods and services to achieve those necessary function and essential characteristics in the most profitable manner" (KAUFMAN, 1990). Different objectives are achieved via VE job plan: saving money, reduce time, improve quality, reliability, maintainability, performance, team work and creativity (Dell'Isola, 1997). Figure 1 depicts the different stages of value study:

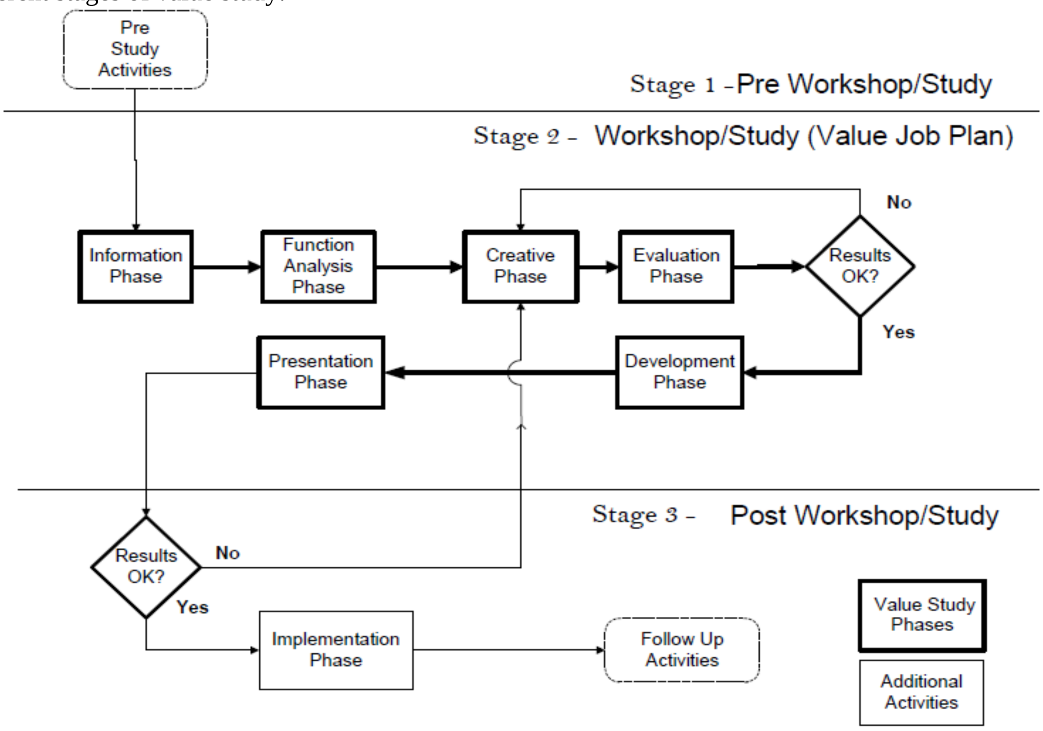

Figure1: Value Study Process Flow Diagram (SAVE International, 2007)

As shown in figure 1, value engineering or value study has three stages. The second stage, which is the main stage, is VE job plan comprises of six sequential phases. VE Job Plan is a road map towards function definition and evaluation and is the critical factor for success in VE study (Assaf, 2000). Evaluation phase is one of the important phases. The purpose of this phase is to reduce the mass of creative ideas that are generated in creative phase (SAVEInternational, 2007). In this phase attributes are weighted and the alternatives are prioritized regarding attributes using tools such as Pugh Analysis. But such a subjective linguistic data is exposed to fuzziness and the question that arises is that how the VE team can optimally propose the best alternative. Most of the methods lean upon the VE team scores that are not inherently crisp. The contribution of this paper is to introduce grey theory to evaluation phase of value engineering. First the extant literature on grey theory is reviewed. Then by an example the application of grey theory in VE is clarified.

II. Grey theory:

Grey theory is introduced by Deng (Deng, 1982). Its mathematical foundation is born out of the grey set. It is an effective technique that is applied to solve nondeterministic problems with discrete data. Grey theory contains five major parts: grey prediction, grey relational analysis (GRA) (Zhang, 2005; Chen, 2004), grey decision, grey programming and grey control. Here, some basic definitions of the grey system, grey set and grey number is presented.

A grey system is a system consists of uncertain information presented by a grey number and grey variables. The concept of a grey system is shown in Figure 2.

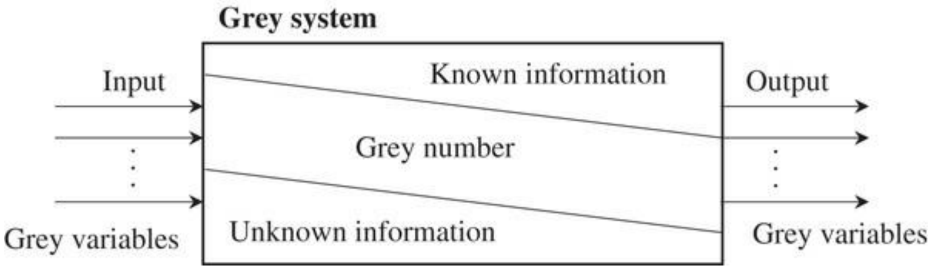

Figure 2: Grey System Concept

Considering X to be the universal set, a grey set G of X is defined by its two membership functions: $\underline{\mu_G}(x)$ and $\overline{\mu_G}(x)$ where $\underline{\mu_G}(x) \leq \overline{\mu_G}(x)\ x \in X, X = R$:

$$\begin{cases} \overline{\mu_G}(x) \colon x \ \to [0,1] \\ \underline{\mu_G}(x) \colon x \ \to [0,1] \end{cases}$$

When $\underline{\mu_G}(x) = \overline{\mu_G}(x)$, the grey set G becomes a fuzzy set. It shows that grey theory considers the condition of the fuzziness and can deal flexibly with the fuzziness situation.

The grey number can be defined as a number with uncertain information. For example, the ratings of attributes are described by the linguistic variables; there will be a numerical interval expressing it. This numerical interval will contain uncertain information. Generally, grey number is written as $\otimes G = G|_{\underline{\mu}}^{\overline{\mu}}$. When the lower and upper limits of G can be estimated then G is defined as an interval grey number $\otimes G = [\underline{G}, \overline{G}]$.

Grey number operation is an operation defined on sets of intervals, rather than crisp numbers. If $\otimes G_1 = [\underline{G_1}, \overline{G_1}]$ and $\otimes G_2 = [\underline{G_2}, \overline{G_2}]$ then four basic operations on grey numbers are the exact range of the corresponding real operation (Wu, 2005). In this paper, only two operations below are related:

$$\otimes G_1 + \otimes G_2 = [\underline{G_1} + \underline{G_2}, \overline{G_1} + \overline{G_2}] \tag{1}$$

$$\otimes G_1 - \otimes G_2 = [\underline{G_1} - \underline{G_2}, \overline{G_1} - \overline{G_2}] \tag{2}$$

The length of the $\otimes G_1$ is defined as $L(\otimes G) = [\overline{G} - \underline{G}]$. (3)

When comparing two grey numbers $\otimes G_1 = [\underline{G_1}, \overline{G_1}]$ and $\otimes G_2 = [\underline{G_2}, \overline{G_2}]$, the possibility degree of $\otimes G_1 \leq \otimes G_2$ is defined as follows (Shi, 2005):

$$P\{\otimes G_1 \leq \otimes G_2\} = \frac{\max\left(0, L^* - \max\left(0, \overline{G_1} - \underline{G_2}\right)\right)}{L^*} \tag{4}$$

Where $L^* = L(\otimes G_1) + L(\otimes G_2)$.

Regarding above equation there are four possible cases:

(1) If $\underline{G_1} = \underline{G_2}$ and $\overline{G_1} = \overline{G_2}$, then $\otimes G_1 = \otimes G_2$ and $P\{\otimes G_1 \leq \otimes G_2\} = 0.5$.

(2) If $\overline{G_1} < \underline{G_2}$ then $\otimes G_1 < \otimes G_2$ and $P\{\otimes G_1 \leq \otimes G_2\} = 1$.

(3) If $\underline{G_1} > \overline{G_2}$ then $\otimes G_1 > \otimes G_2$ and $P\{\otimes G_1 \leq \otimes G_2\} = 0$.

(4) If there is an overlap between two grey numbers, when

$P\{\otimes G_1 \leq \otimes G_2\} > 0.5$ then $\otimes G_1 < \otimes G_2$.

III. The suggested procedure:

A VE team composed of K members $\{T_t\}$ is considered. Each member proposes his/her preferences in linguistic manner according to table 1 (Li, 2007).

Scale	$\otimes G$
Very low (VL)	[0.0,0.1]
low (L)	[0.1,0.3]
Medium low (ML)	[0.3,0.4]
Medium (M)	[0.4,0.5]
Medium high (MH)	[0.5,0.6]
High (H)	[0.6,0.9]
Very high (VH)	[0.9,1.0]

Table 1: The scale of linguistic scores

First, each team member assigns desirable score from table 1 to define the weight of n independent attributes $\{A_j\}$. There are m alternatives $\{C_i\}$ that should be prioritized according to the set of attributes, considering table 1. The summery of procedure which is similar on fuzzy TOPSIS approch is as follows:

Weight calculation:

The weight of attribute A_j is calculated as below:

$$\otimes w_j = \frac{1}{K}\left[\sum_{t=1}^{K} \otimes w_j^t\right] \tag{5}$$

Where $\otimes w_j^t = \left[\underline{w}_j^t, \overline{w}_j^t\right]$ is the weight of attribute j according to linguistic opinion of VE team member T^t obtained from table 1.

Alternatives' Ratings:

Each team member T^i assigns its grey scores to each alternative C_i regarding attribute A_j. The total gray score for each alternative according to each alternative is calculated as below:

$$\otimes G_{ij} = \frac{1}{K}\left[\sum_{t=1}^{K} \otimes G_{ij}^t\right] \tag{6}$$

Where $\otimes G_{ij}^t = \left[\underline{G}_{ij}^t, \overline{G}_{ij}^t\right]$.

So decision matrix (D) will be produced:

$$D = \begin{bmatrix} \otimes G_{11} & \cdots & G_{1n} \\ \vdots & \ddots & \vdots \\ \otimes G_{m1} & \cdots & \otimes G_{mn} \end{bmatrix}$$

Normalizing Decision Matrix:

Each element of D is normalized as follows:

For positive (or profit) attribute:

$$\otimes G_{ij}^* = \left[\frac{\underline{G}_{ij}}{G_j^{max}}, \frac{\overline{G}_{ij}}{G_j^{max}}\right] \tag{7}$$

Where $G_j^{max} = \max_{1 \leq i \leq m}\{\overline{G}_{ij}\}$.

For negative (or cost) attribute:

$$\otimes G_{ij}^* = \left[\frac{G_j^{min}}{\overline{G}_{ij}}, \frac{G_j^{min}}{\underline{G}_{ij}}\right] \tag{8}$$

Where $G_j^{min} = min_{1 \leq i \leq m}\{\overline{G}_{ij}\}$.

The normalizing matrix is calculated for two reasons. First when there are different attributes with different units or properties (such as positive or negative properties), normalizing make them comparable. Also normalizing is done to put all variables between 0,1.

Weighted Matrix calculation:

Weighted decision matrix is calculated as below:

$$\otimes V_{ij} = \otimes G_{ij}^* \times \otimes w_j \tag{9}$$

Ideal alternative calculation:

In this step, there must find ideal alternative (C^+) to compare all alternatives with C^+. The ideal alternative is calculated as below:

$$C^{+} = \left\{[\underline{G}_j^{max}, \overline{G}_j^{max}]\right\} = \left\{[\max_{1\le i\le m} \underline{V}_{i1}, \max_{1\le i\le m} \overline{V}_{i1}], \dots, [\max_{1\le i\le m} \underline{V}_{in}, \max_{1\le i\le m} \overline{V}_{in}],\right\} \quad (10)$$

Comparing Alternatives with ideal:

For comparing alternatives with C^{+} the grey possibility degree is calculated as follows:

$$P\{C_i \le C^{+}\} = \frac{1}{n}\sum_{j=1}^{n} P\{\otimes V_{ij} \le \otimes G_j^{max}\} \quad (11)$$

The best alternative selection:

The alternatives are ranked according to their grey possibility degrees. Lower degree means that the distance from the ideal alternative is smaller, so the alternative is better.

IV. Application of the procedure:

In this paper, there are supposed to be eight VE team members, five attributes and five alternatives and the VE team is in evaluation phase of VE job plan. They want to decide about alternatives that are suitable for development phase of VE job plan. First, the team should calculate the weights of each attributes. Each team member assigns the linguistic score to each attributes according to table 1. Then the $\otimes w_j$s are calculated regarding formula 5. The results are shown in table 2.

Attribute	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	$\otimes w_j$
A	M	MH	M	ML	H	M	ML	MH	[0.425 0.550]
B	H	H	VH	MH	ML	H	M	M	[0.538 0.713]
C	ML	L	ML	ML	MH	M	L	ML	[0.288 0.413]
D	VH	MH	H	H	VH	MH	H	M	[0.625 0.800]
E	L	L	M	ML	VL	M	ML	M	[0.250 0.375]

Table 2: Weight Calculation

Second, each team member defines its linguistic variables about each alternative according to each attributes form table 1. Then $\otimes G_{ij}$s, or alternative ratings, are calculated according formula 7. The results are depicted in table 3.

Attribute	Alternative	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	$\otimes G_{ij}$
A	C_1	H	M	MH	M	H	M	ML	VH	[0.513 0.663]
	C_2	H	H	VH	M	M	H	H	MH	[0.575 0.775]
	C_3	M	MH	MH	MH	M	M	ML	M	[0.425 0.525]
	C_4	MH	MH	MH	H	ML	H	ML	L	[0.425 0.588]
	C_5	L	L	ML	MH	M	ML	ML	VL	[0.250 0.375]
B	C_1	VH	H	VH	MH	MH	H	M	M	[0.600 0.750]
	C_2	ML	M	M	M	MH	ML	ML	L	[0.338 0.450]
	C_3	M	MH	ML	MH	M	MH	M	ML	[0.413 0.513]
	C_4	M	M	ML	M	MH	MH	ML	ML	[0.388 0.488]
	C_5	ML	ML	L	L	M	ML	L	VL	[0.200 0.338]
C	C_1	VL	L	VL	L	L	ML	L	VL	[0.088 0.238]
	C_2	M	M	ML	ML	L	MH	ML	L	[0.300 0.425]
	C_3	MH	H	H	M	MH	M	ML	ML	[0.450 0.600]
	C_4	H	VH	VH	H	MH	MH	MH	M	[0.613 0.763]
	C_5	H	H	H	VH	H	MH	MH	M	[0.588 0.788]
D	C_1	H	M	M	H	MH	M	M	M	[0.463 0.613]
	C_2	L	L	L	ML	M	ML	L	L	[0.188 0.350]
	C_3	H	MH	MH	MH	MH	VH	MH	M	[0.550 0.675]
	C_4	M	M	M	MH	MH	H	H	ML	[0.463 0.613]
	C_5	M	ML	ML	MH	ML	L	L	VL	[0.250 0.375]
E	C_1	M	ML	ML	M	MH	MH	MH	M	[0.413 0.513]
	C_2	M	M	L	L	L	L	M	ML	[0.238 0.388]
	C_3	H	H	VH	H	VH	H	H	MH	[0.663 0.888]
	C_4	VH	VH	VH	VH	H	H	H	MH	[0.738 0.913]
	C_5	MH	M	ML	ML	M	M	M	ML	[0.375 0.475]

Table 3: Alternatives' Ratings

So the decision matrix is as Table 4:

Alternative	A (-)	B (+)	C (+)	D (-)	E (+)
C_1	[0.513 0.663]	[0.600 0.750]	[0.088 0.238]	[0.463 0.613]	[0.413 0.513]
C_2	[0.575 0.775]	[0.338 0.450]	[0.300 0.425]	[0.188 0.350]	[0.238 0.388]
C_3	[0.425 0.525]	[0.413 0.513]	[0.450 0.600]	[0.550 0.675]	[0.663 0.888]
C_4	[0.425 0.588]	[0.388 0.488]	[0.613 0.763]	[0.463 0.613]	[0.738 0.913]
C_5	[0.250 0.375]	[0.200 0.338]	[0.588 0.788]	[0.250 0.375]	[0.375 0.475]

Table 4: The Decision Matrix (D)

Then, the decision matrix is normalized considering formula 7 and 8. In this application attribute A and D are supposed to be negative attributes which the lower is better according to team members' opinions. The results are demonstrated in table 5:

Alternative	A (-)	B (+)	C (+)	D (-)	E (+)
C_1	[0.377 0.488]	[0.800 1.000]	[0.111 0.302]	[0.306 0.405]	[0.452 0.562]
C_2	[0.323 0.435]	[0.450 0.600]	[0.381 0.540]	[0.536 1.000]	[0.260 0.425]
C_3	[0.476 0.588]	[0.550 0.683]	[0.571 0.762]	[0.278 0.341]	[0.726 0.973]
C_4	[0.426 0.588]	[0.517 0.650]	[0.778 0.968]	[0.306 0.405]	[0.808 1.000]
C_5	[0.667 1.000]	[0.267 0.450]	[0.746 1.000]	[0.500 0.750]	[0.411 0.521]

Table 5: The Normalized Decision Matrix

At the fourth step, the weighted normalized decision matrix is calculated, regarding formula 9:

Alternative	A (-)	B (+)	C (+)	D (-)	E (+)
C_1	[0.160 0.268]	[0.430 0.713]	[0.032 0.124]	[0.191 0.324]	[0.113 0.211]
C_2	[0.137 0.239]	[0.242 0.428]	[0.110 0.223]	[0.335 0.800]	[0.065 0.159]
C_3	[0.202 0.324]	[0.296 0.487]	[0.164 0.314]	[0.174 0.273]	[0.182 0.365]
C_4	[0.181 0.324]	[0.278 0.463]	[0.224 0.399]	[0.191 0.324]	[0.202 0.375]
C_5	[0.283 0.550]	[0.143 0.321]	[0.214 0.413]	[0.313 0.600]	[0.103 0.195]

Table 6: The weighted Normalized Decision Matrix

The ideal alternative will be available, according to formula 10:

$$C^{+}=\{[0.283\ \ 0.550],[0.430\ \ 0.713],[0.224\ \ 0.413],[0.335\ \ 0.800],[0.202\ \ 0.375]\}$$

At this step, the grey possibility degree from ideal alternative is calculated according to formula 11:

$$P\{C_1 \le C^{+}\}=0.894 \qquad P\{C_2 \le C^{+}\}=0.900$$

$$P\{C_3 \le C^{+}\}=0.810 \qquad P\{C_4 \le C^{+}\}=0.770 \qquad P\{C_5 \le C^{+}\}=0.732$$

Finally, the priority of the alternatives is as follows:

$$C_5 > C_4 > C_3 > C_1 > C_2$$

V. Summery and Discussion:

This paper provides a grey alternative selection for value engineering workshop. Commonly, the selection process in VE is done by means of crisp numbers. But in reality the situation is not deterministic and is exposed to uncertainty. The VE team may not achieve to consensus over the scores given to each alternative regarding attributes. Also they may desire to give a range or continuum of numbers. So, a grey based alternative selection will be helpful.

Grey theory is a nascent area. It is one of the useful methods for covering uncertainty. In this article, an application of grey theory in VE is presented. The alternatives are ranked according to linguistic scores that are assigned by each team member. The experimental results show that this application is reasonable.

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