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Quasistatic stationary growth of elastoplastical single crack

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Abstract: *The boundary value problem with relations of the theory of flow with nonlinear hardening in derivatives stress and strain tensors in the parameter loading is formulated to estimate local mechanical properties in the vicinity of the crack tip of mode I loading for the plane stress state of the elastic-plastic material at the stage of quasi-static growth. Complete solutions are obtained by the method of asymptotic decompositions. The redistribution of stress and strain fields in the plastic region at a quasi-static growing crack for the intermediate structure is investigated. The form of the plastic zones was found. Direct estimates of the errors were obtained.*

Keywords: *subcritical crack propagation, the stress-strain state, the method of asymptotic decompositions*

Cracks significantly affect the structural strength of the material. At the same time, the distribution of strain and stress fields near the propagating crack considerably changes compared with a fixed crack in the theory of plasticity. Therefore, to obtain adequate estimates of the stress- strain state in the vicinity of singular points of the boundary, including the tip of the crack at various modes of crack growth is crucial for solving boundary value problems of nonlinear fracture mechanics. In the vicinity of the crack tip in such materials there are plastic zones, the boundaries of which have a complex shape and form [1]. Moreover, experimental data about the presence of the area of stable (slow) crack growth under monotonic loading, as well as repeated (cyclic) loads [2,3]. As are known for the features of the first type there have been two ways to account for the plastic properties of the material. One of them is the solution of elastoplastic problems for a hardening body with a crack, where, during the solution the boundary, separating the elastic and plastic zones is determined besides the definition of emerging stress and strain fields [4]. This way is associated with considerable mathematical difficulties, because of the need to formulate the boundary value problem in the incremental theory of plasticity, taking into account the fact, that the mapping of a plastic zone on a plane stress is not degenerated and it is necessary to search for corresponding representations of the solutions, defined both in the elastic and plastic zones, with their subsequent gluing along the line separating areas of active loading and unloading. To account for the characteristics of the second type requires the consideration of various modes of crack propagation, such as quasi-static or dynamic nonstationary and stationary growth.

At the same time, finding exact or approximate solutions in the problems of stress concentrators such as cracks is complicated because of the singularity of stress and strain fields at a singular boundary point and non-linearity and nonholonomicity of resolving equations. The development of asymptotic methods, occupying an intermediate position between the exact analytical approaches, used in the linear fracture mechanics [5], and direct numerical methods for nonlinear problems [6], is a topical problem.

One of the variants of the method of asymptotic decompositions, used in the problems for elastic-plastic hardening material is the method, connected with finding the hyperfine structure – the main term of the asymptotic of stress and strain [7]. The latter allows to get the values of the local characteristics of the stress state for an infinitely small vicinity of the crack tip, but does not give the values of those at the final (although, perhaps, small) distances and errors, occurring in these situations. The study of the asymptotics of intermediate structure levels is of considerable interest for their combined use with the fracture criteria and the construction of efficient algorithms for the analysis of complete solutions while taking into account the unloading in the case of elastoplastic material with hardening. For the construction of approximate solutions at stresses we applied the method of the asymptotic decompositions, when in the vicinity of the singular point complete series of stresses and strains, including along with the main part the correct one, are considered [8]. The accuracy of solutions is characterized by the direct estimates of the errors of stress and strain for the finite increments of the crack length, taking into account the next terms of the series. The above method allows, in particular, to determine the geometry of small plastic zones.

The definition of stress and strain fields near the fixed crack-like defects in elastic-plastic bodies is an important part of the problem of estimating the strength of structural elements. Experiment, show that the crack remains fixed for sufficiently small values of a monotonically varying loading parameter (for example, for the values of the stress intensity factor $K \le K_c$ – fracture toughness for quasibrittle fracture) [1]. This stage can contain the subcritical growth mode, when the stress-strain state, typical of a small vicinity of the tip of a fixed cut, is

realized in the smooth growth of cracks at a sufficiently large distance from the singular point and has the meaning of to some intermediate asymptotics of the full elastic-plastic solution.
Further, under the conditions, providing a sustainable growth, the stages of a rapid crack propagation in elastic-plastic body can take turns with the areas of braking and stop or slow propagation, which makes necessary the formulation of the problem.

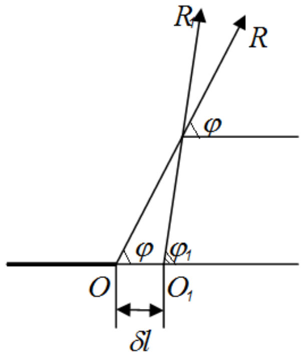
I. Formulation of the problem

Let's consider a stationary mode of crack propagation, where the stress and strain fields do not depend explicitly on the loading parameter. The case of elastoplasticity is distinguished by the presence of the initial stage of crack propagation in subcritical growth. We apply the variant of asymptotic decomposition method for loading parameters for evaluating the stress-strain state near the crack tip. The elastoplastic, incompressible material with a hardening power law is considered. Cartesian coordinate system is applied to the end of the crack. System axes $\xi_i\,(i=1,2)$ are located in such a way that ξ_1 is oriented along the crack in the direction of its end. We assume that the state near the tip is controlled by the load parameter $K = p\sqrt{\pi l}$, here p - tension at infinity, 2ℓ - length of crack in a plate, which will be interpreted as the stress intensity factor in the elastic region, surrounding the zone of plastic deformation. The only independent parameter of the problem with the dimensions of length is the value of K^2/G^2, where G – shear modulus, that's why the required functions of the problem depend on the load only through the dimensionless variables

(2.1) $$x_i = \xi_i G^2/K^2 \quad (i=1,2),\ r=\sqrt{x_1^2+x_2^2};\ \varphi = arctg\frac{x_2}{x_1}.$$

(O, r,φ) is the mobile polar coordinate system with the pole at the end of the crack, $r=\rho/2\ell$ being a dimensionless radius – the ratio of the radius vector of the point to the length of the crack 2ℓ; The crack tip is moved along with the associated coordinate system (O_1, R_1, φ_1) at the subcritical growth.
Unlike the Cartesian coordinate system, where the orientation of the area, on which the changes of stress are considered, remains fixed and the coordinates of the normal vector to this area do not change, when the origin together with the crack tip move, coordinates of the normal fixed-area change, when you move the pole O to the point O_1 in the case of the polar coordinate system, that is they are a function of the crack length (fig. 1).

Figure 1 The movable polar coordinate system



This fact must be taken into account when choosing stress and displacement representations, when the function $f(l,r,\varphi)$ implicitly depends on the crack length, and its derivative is represented by

(2.2) $$\dot{f}(r(l),\varphi(l)) = -\frac{\partial f}{\partial r}\cos(\varphi) + \frac{\partial f}{\partial \varphi}\frac{\sin(\varphi)}{r}.$$

The length derivative of strains (speed) in the polar coordinate system is understood as follows:

(2.3) $$\dot{\sigma}_{ij}(l,R,\varphi) = \underset{\delta l\to 0}{Lim}\frac{\sigma_{ij}(l+\delta l, R_1,\varphi_1) - \sigma_{ij}(l,R,\varphi)}{\delta l},\ i,j=r,\varphi.$$

Elastoplastic, incompressible material with a quadratic hardening law under a uniform load at the infinity is considered. The strain and stress tensors and deviators components under the plane stress conditions in the polar coordinate system can be expressed by:

(2.4) $$\varepsilon_{ij} = e_{ij}, e_{ii} = 0,\ e_{zz} = -(\varepsilon_{rr}+\varepsilon_{\varphi\varphi}) = -(e_{rr}+e_{\varphi\varphi}),$$

$$\sigma = \frac{1}{3}(\sigma_{rr}+\sigma_{\varphi\varphi}),\ s_{r\varphi} = \sigma_{r\varphi}, s_{rr} = \frac{2}{3}\left(\sigma_{rr} - \frac{1}{2}\sigma_{\varphi\varphi}\right), s_{\varphi\varphi} = \frac{2}{3}\left(\sigma_{\varphi\varphi} - \frac{1}{2}\sigma_{rr}\right), \sigma = -s_{zz}.$$

In addition, it is necessary to include the equation in the length derivatives of the strain

(2.5) $$2\frac{\partial}{\partial r}\left(r\frac{\partial \dot{e}_{r\varphi}}{\partial \varphi}\right)=\frac{\partial^2 \dot{e}_{rr}}{\partial \varphi^2}-r\frac{\partial \dot{e}_{rr}}{\partial r}+r\frac{\partial^2 (r\dot{e}_{\varphi\varphi})}{\partial r^2}$$

and, writing the constitutive equations of the flow theory with isotropic hardening, taking into account availability of the zone of active loading and the unloading zone, in derivatives with respect to the crack length:

(2.6) $$\dot{e}_{rr}=\dot{s}_{rr}+\mu\dot{F}(T)s_{rr},\ \dot{e}_{r\varphi}=\dot{s}_{r\varphi}+\mu\dot{F}(T)s_{r\varphi},\ \dot{e}_{\varphi\varphi}=\dot{s}_{\varphi\varphi}+\mu\dot{F}(T)s_{\varphi\varphi},$$

where $\mu=\vartheta(\frac{G}{M}-1)$, $\vartheta=\begin{cases}1, \delta T\geq 0\\ 0, \delta T<0\end{cases}$, where M - hardening modulus,

$F(T)=\frac{A_3}{2}(\sigma_{rr}^2-\sigma_{rr}\sigma_{\varphi\varphi}+\sigma_{\varphi\varphi}^2+3\sigma_{r\varphi}^2)$ – the function of the shear stress intensity, $A_3 = 2A_2$, here A_2 — material constant, characterizing the non-linearity of the deformation diagram, determined, for example, from the experience of a simple stretching.

The boundary value problem is formulated under the condition, that the crack surfaces are free from effort:

(2.7) $$\dot{\sigma}_{\varphi\varphi}\,|_{\varphi=\pi}=0,\ \dot{\sigma}_{r\varphi}\,|_{\varphi=\pi}=0,\ \dot{\sigma}_{rr}\,|_{\varphi=0}=0,\ \dot{\sigma}_{\varphi\varphi}\,|_{\varphi=0}=0,$$

where σ_{ij} − dimensionless stress components, related to the shear modulus G .

II. Numerical-analytical solution

Let's introduce the stress function $\Phi(r,\varphi)$ as a full decomposition on power parameter in the vicinity of the singular point, $\Phi(r,\varphi)=\sum_{k\geq 0}\psi_k(\varphi)r^{\lambda_k}$, r and φ being the functions of the crack length. The values λ_k and $\psi_k(\varphi)$ are expected to be determined while solving the problem.

Then the expressions for the stress, the function of hardening and the stress and strain derivatives of length for the field of active loading are as follows:

(3.1) $$\sigma_{rr}=\sum_{k\geq 0}(\psi_k\lambda_k+\psi_k'')r^{\lambda_k-2},\ \sigma_{\varphi\varphi}=\sum_{k\geq 0}\lambda_k(\lambda_k-1)\psi_k r^{\lambda_k-2},\ \sigma_{r\varphi}=\sum_{k\geq 0}(1-\lambda_k)\psi_k' r^{\lambda_k-2},$$

$$F(T)=\sum_{k\geq 0}\sum_{l\geq 0}a_{kl}r^{\lambda_k+\lambda_l-4},$$

where $a_{kl}=\frac{A_3}{2}(\psi_k''+\lambda_k\psi_k)(\psi_l''+\lambda_l\psi_l-\lambda_k(\lambda_k-1)\psi_k)+\lambda_k(\lambda_k-1)\psi_k\lambda_l(\lambda_l-1)\psi_l+3(1-\lambda_k)\psi_l'(1-\lambda_l)\psi_l'$.

(3.2) $$\dot{e}_{rr}=\frac{1}{3}\sum_{k\geq 0}((2\psi_k'''+\lambda_k(3-\lambda_k)\psi_k')\sin(\varphi)-(\lambda_k-2)(2\psi_k''+\lambda_k(3-\lambda_k)\psi_k)\cos(\varphi))r^{\lambda_k-3}+$$
$$\frac{\mu}{3}\sum_{k\geq 0}\sum_{l\geq 0}\sum_{m\geq 0}(-(\lambda_k+\lambda_l-4)a_{kl}\cos(\varphi)+a_{kl}'\sin(\varphi))(2\psi_k''+\lambda_k(3-\lambda_k)\psi_k))r^{\lambda_k+\lambda_l+\lambda_m-7}$$

(3.3) $$\dot{e}_{\varphi\varphi}=\frac{1}{3}\sum_{k\geq 0}((\lambda_k-2)(-\psi_k''+\lambda_k(-3+2\lambda_k)\psi_k)\cos(\varphi)-(-\psi_k'''+\lambda_k(-3+2\lambda_k)\psi_k')\sin(\varphi))r^{\lambda_k-3}+$$
$$\frac{\mu}{3}\sum_{k\geq 0}\sum_{l\geq 0}\sum_{m\geq 0}((\lambda_k+\lambda_l-4)a_{kl}\cos(\varphi)-a_{kl}'\sin(\varphi))(-\psi_k''+\lambda_k(-3+2\lambda_k)\psi_k)r^{\lambda_k+\lambda_l+\lambda_m-7}$$

(3.4) $$\dot{e}_{r\varphi}=\sum_{k\geq 0}(-(\lambda_k-2)(1-\lambda_k)\psi_k'\cos(\varphi)+(1-\lambda_k)\psi_k''\sin(\varphi))r^{\lambda_k-3}+$$
$$\mu\sum_{k\geq 0}\sum_{l\geq 0}\sum_{m\geq 0}(-(\lambda_k+\lambda_l-4)a_{kl}\cos(\varphi)+a_{kl}'\sin(\varphi))(1-\lambda_m)\psi_m' r^{\lambda_k+\lambda_l+\lambda_m-7}$$

Thus we have a recurrent sequence of boundary value problems on the eigenvalues for any approximation of the equation

(3.5)

$$\sum_{n\geq 0}(2(\lambda_n-2)(-(\lambda_n-2)(1-\lambda_n)(\psi''_n\cos(\varphi)-\psi'_n\sin(\varphi))+(1-\lambda_n)(\psi'''_n\sin(\varphi)+\psi''_n\cos(\varphi)))-\frac{1}{3}((2\psi_n^{(5)}+\lambda_n(3-$$
$$-\lambda_n)\psi'''_n)\sin(\varphi)+(2\psi_n^{(4)}+\lambda_n(3-\lambda_n)\psi''_n)\cos(\varphi)+(2\psi_n^{(4)}+\lambda_n(3-\lambda_n)\psi''_n)\cos(\varphi)-(2\psi'''_n+\lambda_n(3-\lambda_n)\psi'_n)\times$$
$$\times\sin(\varphi)-(\lambda_n-2)(2\psi_n^{(4)}+\lambda_n(3-\lambda_n)\psi''_n)\cos(\varphi)+(\lambda_n-2)(2\psi'''_n+\lambda_n(3-\lambda_n)\psi'_n)\sin(\varphi)+(\lambda_n-2)(2\psi'''_n+$$
$$+\lambda_n(3-\lambda_n)\psi'_n)\sin(\varphi)+(\lambda_n-2)(2\psi''_n+\lambda_n(3-\lambda_n)\psi_n)\cos(\varphi))+\frac{1}{3}(\lambda_n-3)((2\psi'''_n+\lambda_n(3-\lambda_n)\psi'_n)\sin(\varphi)-$$
$$-(\lambda_n-2)(2\psi''_n+\lambda_n(3-\lambda_n)\psi_n)\cos(\varphi))-\frac{1}{3}(\lambda_n-2)(\lambda_n-3)((\lambda_n-2)(-\psi''_n+\lambda_n(-3+2\lambda_n)\psi_n)\cos(\varphi)-(-\psi'''_n+$$
$$+\lambda_n(-3+2\lambda_n)\psi'_n)\sin(\varphi)))r^{\lambda_n-3}+\sum_{k+l+m=n+1}(2\mu(\lambda_k+\lambda_l+\lambda_m-6)(-(\lambda_k+\lambda_l-4)(1-\lambda_m)((a'_{kl}\cos(\varphi)-a_{kl}\sin(\varphi)+$$
$$+a''_{kl}\sin(\varphi)+a'_{kl}\cos(\varphi))\psi'_m+(a_{kl}\cos(\varphi)+a'_{kl}\sin(\varphi))\psi''_m))-\frac{\mu}{3}((-(\lambda_k+\lambda_l-4)(a'_{kl}\cos(\varphi)-a_{kl}\sin(\varphi))+(a''_{kl}\sin(\varphi)+$$
$$+a'_{kl}\cos(\varphi)))(2\psi'''_k+\lambda_k(3-\lambda_k)\psi'_k)+(-(\lambda_k+\lambda_l-4)a_{kl}\cos(\varphi)+a'_{kl}\sin(\varphi))(2\psi^{(4)}{}_k+\lambda_k(3-\lambda_k)\psi''_k)+(-(\lambda_k+\lambda_l-$$
$$-4)(a'_{kl}\cos(\varphi)-a_{kl}\sin(\varphi))+a''_{kl}\sin(\varphi)+a'_{kl}\cos(\varphi))(2\psi'''_k+\lambda_k(3-\lambda_k)\psi'_k))+(-(\lambda_k+\lambda_l-4)(a''_{kl}\cos(\varphi)-a'_{kl}\sin(\varphi)-$$
$$-a'_{kl}\sin(\varphi)-a_{kl}\cos(\varphi))+a'''_{kl}\sin(\varphi)+a''_{kl}\cos(\varphi)+a''_{kl}\cos(\varphi)-a'_{kl}\sin(\varphi))(2\psi''_k+\lambda_k(3-\lambda_k)\psi_k)))+\frac{\mu}{3}(\lambda_k+\lambda_l+\lambda_m-$$
$$-7)(-(\lambda_k+\lambda_l-4)a_{kl}\cos(\varphi)+a'_{kl}\sin(\varphi))(2\psi''_k+\lambda_k(3-\lambda_k)\psi_k)-\frac{\mu}{3}(\lambda_k+\lambda_l+\lambda_m-6)(\lambda_k+\lambda_l+\lambda_m-7)((\lambda_k+$$
$$+\lambda_l-4)a_{kl}\cos(\varphi)-a'_{kl}\sin(\varphi))(-\psi''_k+\lambda_k(-3+2\lambda_k)\psi_k))r^{\lambda_k+\lambda_l+\lambda_m-7})=0$$

The boundary conditions (2.7) will take the form:

(3.6) $$\psi_n\mid_{\varphi=\pi}=0\,,\;\psi'_n\mid_{\varphi=\pi}=0\,,\;\psi_n\mid_{\varphi=0}=0\,,\;\psi'_n\mid_{\varphi=0}=0\,,\;\psi''_n\mid_{\varphi=\pi}=0\,.$$

Asymptotic problem solution of hard concentrators such as cracks include some degree of arbitrariness. In the case of linear analysis of the stress-strain state the corresponding uncertainty arises due to the stress intensity factor with the geometry of the solid and the boundary conditions [1]. In the elastic-plastic materials a similar situation appears while considering the hyperfine structure, when the main term in the expansion near the point of singular - the crack tip contains along with an undetermined factor the previously unknown parameter (the degree of singularity). At the same time for sufficiently long cracks and small plastic zones there may also exist an asymptotic solution of the level of the fine structure. The degree of uncertainly in the case of propagation is even more profound – the number of characteristic asymptotic parameters may increase up to three. The energy invariants of the type J-integral Cherepanov-Rice are used to resolve this arbitrariness. In addition, in elastic-plastic problems there arise the inconsistency of solutions with energy criteria of the crack growth, since the energy flow into the tip of a growing crack is not sufficient to compensate the energy for the formation of new crack surfaces.

The value λ_0 is determined from the invariance of the J-integrals, for each the integration is along the contour γ surrounding the crack tip [9]

$$\oint_{\gamma}((E+\frac{1}{2}\rho\dot{l}^2u_{i,x_1}u_{i,x_1})n_1-\sigma_{ij}n_ju_{i,x_1})d\Gamma=-\frac{\partial\Pi}{\partial l}\,,$$

here E – strain energy, Π - potential energy, $\sigma_{ij}n_j$ - vector of efforts on γ by the normal vector in the positive direction $\bar{n}=(n_1,n_2)$, u_i - displacements, $\overset{\circ}{l}$ - the speed of propagation of the crack tip, ρ - material constant.

First, we find the value of the parameter λ_0, corresponding to the nontrivial solution, and the function ψ_0 in the interval $\varphi\in[0;\pi]$. Some additional conditions must be carried out for the solving these problems. These conditions occur because the functions $a_{klm}(\varphi)$ depend on $\psi_n(\varphi)$ with the indexes $\min\{k,l,m\}\leq n\leq\max\{k,l,m\}$ .Then $\lambda_1=4-\lambda_0$, $\lambda_2=8-3\lambda_0$. In general, this condition has the form $\lambda_n-3=\lambda_k+\lambda_l+\lambda_m-7$ $(k+\ell+m=n+1,\ n=0,1,2\ldots)$, whence $\lambda_n=4n+(2n-1)\lambda_0$, ($n=1,2\ldots$).

We have a recurrent sequence of a boundary value problems for systems of differential equations of fourth order. At the initial stage, this is problem on eigenfunctions of nonlinear homogeneous differential operator, and the next stages - linear problems for nonhomogeneous differential operator. The boundary problem is solved numerically using the modified shooting method.

The following notations while solving the boundary value problem has been introduced:

$f00=\psi_0,\ f01=\psi_0',\ f02=\psi_0'',\ f03=\psi_0''',\ f04=\psi^{(4)}{}_0$, and obtain an equivalent system of differential equations with five boundary conditions.

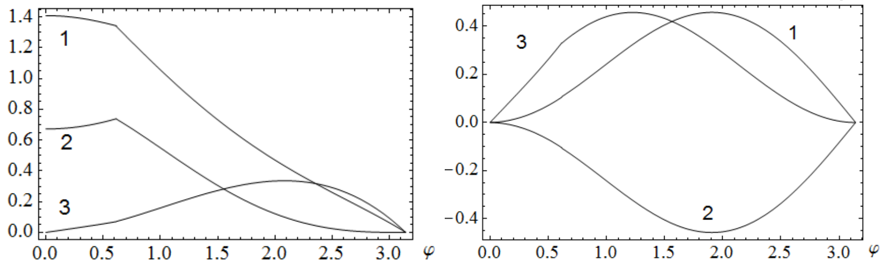
(3.7) $$f00'=f01,\ f01'=f02,\ f02'=f03,\ f03'=f04,\ f04'=f(f00,f01,f02,f03,f04)\,.$$

(3.8) $$f00|_{\varphi=\pi}=0\,,\ \ f01|_{\varphi=\pi}=0\,,\ \ f00|_{\varphi=0}=0\,,\ \ f01|_{\varphi=0}=0\,,\ \ f02|_{\varphi=\pi}=0\,.$$

III. The analysis of the results

The calculations are made for copper alloy with the following material constants:
G=0.424·1011N/m², hardening modulus M=1000MPa, the Poisson coefficient ν=0.365. These obtained graphs correspond to the case P=300MPa, elastoplastic material constant A_2 = 1.3270501·10^{-5}. The obtained value of the parameter $\lambda_0=5/3$. Using this λ_0, it is possible to get a continuously splicing zones of active loading and unloading.
The graphs of the related components of the stress and stain tensor of the intermediate structure (Fig. 2) are given for $dl=1\cdot 10^{-7}$ depending on the angle φ for mode I crack (Notations on the graph: 1 – $\sigma_{rr}(\varphi)$, 2 – $\sigma_{\varphi\varphi}(\varphi)$, 3 – $\sigma_{r\varphi}(\varphi)$, 1 – $\varepsilon_{rr}(\varphi)$, 2 – $\varepsilon_{\varphi\varphi}(\varphi)$, 3 – $\varepsilon_{r\varphi}(\varphi)$).

Figure 2 Graphs of the component dependence of stresses and strains tensors on the angle for $r=1\cdot 10^{-5}$



The contours of the stress distribution (Fig. 3) and strain (Fig. 4) in the region of the intermediate structure are represented. From the graph analysis, we conclude that for the case of the plane strain and stress component, $\sigma_{\varphi\varphi}$ has a maximum value in the direction of the supposed development of the crack. Besides, the stress concentration occurs near the crack, and a smoothed decrease of the distance from it in comparison with the deformation theory of plasticity [10].

Figure 3 Contour plots of the stresses σ_{rr}, $\sigma_{\varphi\varphi}$, $\sigma_{r\varphi}$ in the vicinity of the crack tip

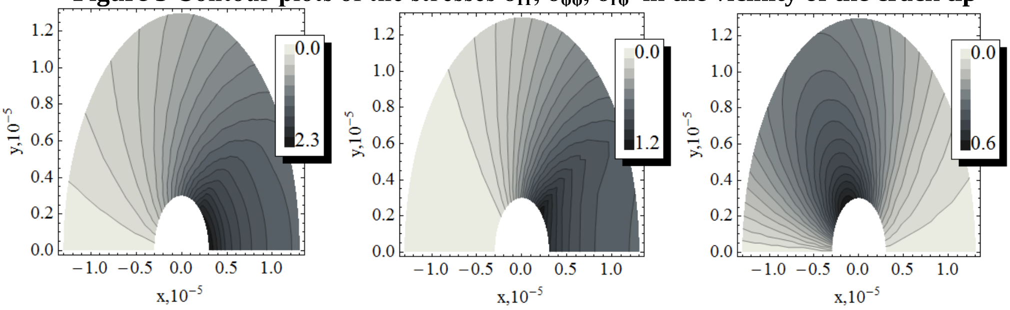
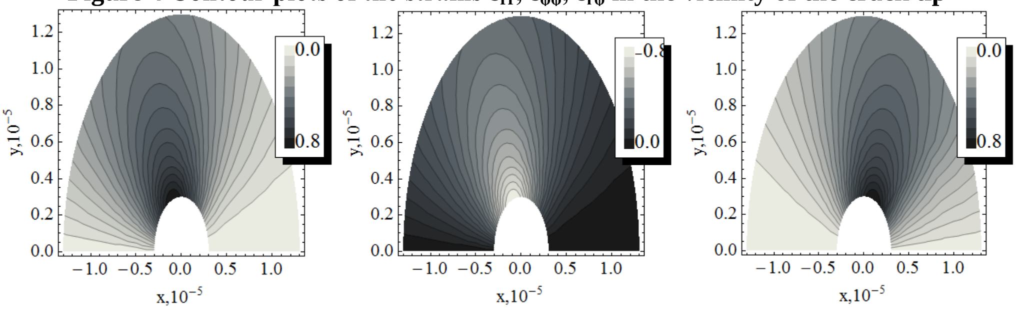


Figure 4 Contour plots of the strains ε_{rr}, $\varepsilon_{\varphi\varphi}$, $\varepsilon_{r\varphi}$ in the vicinity of the crack tip



It should be noted that, with increasing external load we have a sharp expansion of the plastic region. The area of plastic deformation, including the zone of active loading and unloading, is shown in Fig. 5. The area degenerates into a circular sector, limited by the diameter of the intermediate structure and the critical value of the angle at which unloading takes place.

Figure 5 The boundaries of the plastic zone in the region of the intermediate structure

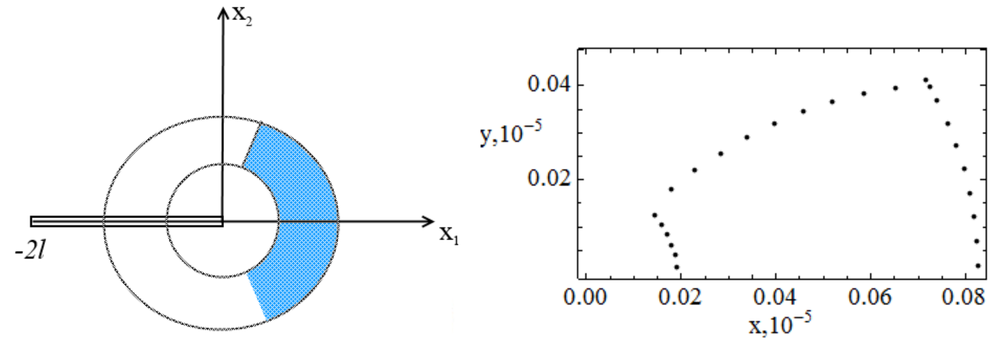


Table 1 shows the values of the angle of the boundary between the zones of active loading and unloading.

Table 1. The critical value of the angle $\varphi = \varphi_r$ for the unloading area, at an external effort *P=300MPa*

r (m)	$0.3\cdot10^{-5}$	$0.5\cdot10^{-5}$	$0.7\cdot10^{-5}$	$0.9\cdot10^{-5}$	$1.1\cdot10^{-5}$	$1.3\cdot10^{-5}$
φ_r	0.823573	0.767061	0.710548	0.647756	0.581825	0.503335

The tables below show the relative quantities of each of the approximations in general asymptotics. In general, the values are for $\sigma_{rr}(\varphi)$ from 2% to 7% for the first approximation, from 0,05% to 0,6% for the second approximation. For $\sigma_{r\varphi}$: from 1% to 8%– for the first approximation and from 0,1% to 1% – for the second approximation. For $\sigma_{\varphi\varphi}$: from 2% to 8% – for the first and from 0,01% to 1% – for the second approximation.

Table 2. The direct estimation of convergence for $\sigma_{rr}(\varphi)$

r(m)	$0.3\cdot10^{-5}$	$0.5\cdot10^{-5}$	$0.7\cdot10^{-5}$	$0.9\cdot10^{-5}$	$1.1\cdot10^{-5}$	$1.3\cdot10^{-5}$
the zero approximation	0.98031	0.96787	0.95559	0.94309	0.93070	0.91835
the first approximation	0.01923	0.03108	0.04253	0.05388	0.06488	0.07558
the second approximation	0.00046	0.00105	0.00188	0.00304	0.00444	0.00609
norm of the three approximations	2.30377	1.81736	1.59099	1.44935	1.35390	1.29699

Table 3. The direct estimation of convergence for $\sigma_{r\varphi}$

r(m)	$0.3\cdot10^{-5}$	$0.5\cdot10^{-5}$	$0.7\cdot10^{-5}$	$0.9\cdot10^{-5}$	$1.1\cdot10^{-5}$	$1.3\cdot10^{-5}$
the zero approximation	0.99440	0.98676	0.97596	0.96046	0.94002	0.91257
the first approximation	0.01033	0.01929	0.02991	0.04294	0.05900	0.07995
the second approximation	0.00190	0.00255	0.00287	0.00415	0.00727	0.01201
norm of the three approximations	0.20304	0.14355	0.11055	0.08188	0.06371	0.04503

Table 4. The direct estimation of convergence for $\sigma_{\varphi\varphi}$

r(m)	$0.3\cdot10^{-5}$	$0.5\cdot10^{-5}$	$0.7\cdot10^{-5}$	$0.9\cdot10^{-5}$	$1.1\cdot10^{-5}$	$1.3\cdot10^{-5}$
the zero approximation	0.98179	0.96676	0.95188	0.93733	0.92325	0.90930
the first approximation	0.01824	0.03238	0.04583	0.05849	0.07034	0.08157
the second approximation	0.00019	0.00091	0.00234	0.00418	0.00642	0.00913
norm of the three approximations	1.16081	0.95609	0.83888	0.76592	0.72988	0.69238

The form of the plastic zones, taking into account the evolution of unloading process of the fracture has been found. One can conclude that there is a reduction of the order of singularity stress and strain fields of the main term within the boundaries of the intermediate structure.

Their local geometry depends greatly on the choice of the defining relations and a steady-state modes of stationary in comparison with the deformation theory. In [11], suggesting that the stress-strain state, prior to the breakaway of crack is reached under loading conditions, close to the radial, the stresses in the plastic region at this moment along the linearity of the asymptotic of material chart were determined according to the formulas of the deformation theory, which does not allow to this technique to the materials with a non-linear diagram of straining, when the linear portion is absent. At the same time, due to disproportionality of the loading process with the breakaway crack the redistribution of the stress and strain requires of their finding within a nonholonomic flow theory. Comparative analysis showed significant relative differences in the zone structure of the active loading and unloading – the reducing of the areas of an active loading compared with the deformation theory and no discharge areas near the edges of the crack. The order singularity component of the main term is reduced, resulting in a more uniform distribution of stress fields within the boundaries of the intermediate structure in the vicinity of the crack tip.

IV. Concluding remarks

The obtained results allow us to formulate the following conclusions. We have found an asymptotic solution of the boundary elastic-plastic problem of the propagating crack. An iterative process, in which an initial approximation is considered as the solution of the eigenvalue problem of the nonlinear differential operator, has been constructed. From the condition of the finiteness J-integral the order of the singularity was numerically-analytically obtained. It is necessary to take into account the singularity of the displacement gradients and stresses distributed along the crack edges, as well as the concept of potential deformations for materials described by nonintegrable differential equations. At the subsequent stages of the algorithm, recurrent representations for the eigenvalues are determined. The estimation accuracy of the solution is given. On the basis of these solutions obtained, we conclude that the direction in which the maximum values of the stress intensity and strain intensity are reached may not coincide. The account of the stress state for the elastic-plastic materials leads to reducing the of the critical stress at which the crack growth occurs.

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