International Association of Scientific Innovation and Research (IASIR)
(An Association Unifying the Sciences, Engineering, and Applied Research)

ISSN (Print): 2279-0020
ISSN (Online): 2279-0039

International Journal of Engineering, Business and Enterprise Applications (IJEBEA)

www.iasir.net

Research on fluid in an open container with a complex force field applied

Penev Mariel, Stoyanov Ivaylo
Department "Machine Components"
Technical University of Gabrovo, Bulgaria

Abstract: *This article is a study of the behavior of fluid under the influence of a force field. Examined is the behavior of the fluid during the reciprocal linear movement, by simulating different laws of acceleration of various sizes. Conclusions are made for the application of this study.*

Keywords: *CFD, Simulation, VOF, Slosh, Field of Force.*

I. Introduction

In modern devices for packaging fluids require the use of positioning systems. These in turn are influenced by the behavior of the fluid in the container undergoing complicated force field. In [3] Armenio and La Rocca, is considered modeling of fluid in two-dimensional space by Reynolds Averaged Navier-Stokes Equations (RANSE) for incompressible flows and shallow water equations for surfaces - Shallow Water Equations (SWE). These problems are solved numerically FEM. Digital solutions compared to the experimental show that the model RANSE is closer to the real model than SWE [4], is considered a free surface with a moving range. Digital method requires working with two streams that are incompressible or one is incompressible, and the other - a collapsible. Move grid is used to clarify the boundaries of the field. The technique (volume-of-fluid - VOF) is used to track the boundaries of the two fluids. In [5] describes the potential flow equations using Laplace. The dynamic boundary conditions at the free surface is used so-called damped modified equation of Bernoulli, which is included the viscosity of the fluid. This method is solved by BEM (boundary element method). The boundary of the free surface is adjusted by the movement of the free surface.

II. Summary

The movement of the fluid is described by the equations of the Navier-Stokes [1, 2]. They are a set of nonlinear partial differential equations, in his first equation represents the law of conservation of mass (equation of the continuous environment):

$$\frac{d\rho}{dt} + \nabla(\rho.v) = 0 \tag{1}$$

where ρ is the density of the fluid, $v=(v_x,v_y,v_z)$ - velocity vector of the fluid in the spatial coordinate system, (x, y, x).

Here the law of conservation of energy is given with an equation having four unknowns. More conditions are required, assuming that the density is known. These conditions are given by Newton's second law, giving three equations. For viscous fluids, while retaining the currently recorded:

$$\rho\left(\frac{\partial v}{\partial t} + (v.\nabla)v\right) = -\nabla \mathrm{p} + (\beta + \mu)\nabla(\nabla.v) + \beta\nabla^2 v + \rho F \tag{2}$$

where: p is pressure; β coefficient of volume contraction; μ - viscosity, $F=(F_x, F_y, F_z)$ – intensity of mass force.

If the fluid is ideal and incompressible, ie: (β = 0 and μ = 0) the equations of Navier-Stokes transform into Euler's equations:

$$\nabla.v = 0 \tag{3}$$

$$\frac{\partial v}{\partial t} + (v.\nabla)v = -\frac{1}{\rho}\nabla p + F \tag{4}$$

In the expression (4), vector $(v.\nabla)v = \frac{1}{2}\nabla|v|^2 + (\nabla \times v) \times v$ and taking account of the potential external forces, such as is converted in the type:

$$\frac{\partial v}{\partial t} + \frac{1}{2}\nabla|v|^2 + (\nabla \times v) \times v = -\frac{1}{\rho}\nabla p - F \tag{5}$$

The value of the velocity potential ϕ is determined by the expression:

$$v = \nabla\phi \tag{6}$$

After substitution of (6) to (3) to be recorded:

$$\nabla.\nabla\phi = \nabla^2\phi = 0, \tag{7}$$

which is the familiar equation of Laplace. After substitution of (6) and ∇ × v = 0 in the (5) and subsequent integration of the equation is obtained Bernoulli's equation:

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \frac{p}{\rho} - F = C(t) \tag{8}$$

where: $C(t)$ is a random function of time.

Figure 1 Diagram of the container

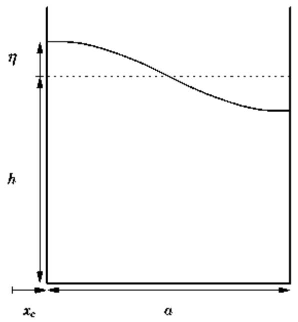
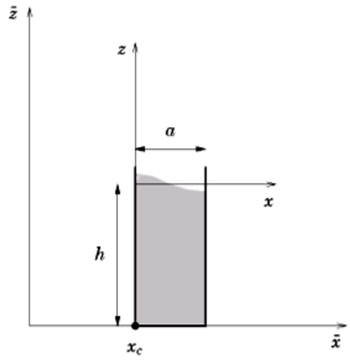


Figure 2 Correlation between the two coordinate systems



Further approximations can be made for the width and depth of the container. Horizontal velocity of the fluid is assumed to be independent of the vertical position. This is described by equations of the thin film fluid. The limit of the fluid are considered walls of the container, the bottom and the free surface. Condition of boundary walls and bottom of the container is zero velocity in the direction normal to them ie:

$$n.v = 0 \Rightarrow n.\nabla\phi = 0 \qquad (9)$$

where n normal vector to the wall.

Fig.1 shows a diagram of the container. Fluid surface z = η(*t, x, y*) has two components - one is dynamic, and the other - kinematics. Dynamic component arises from external forces acting on the surface and can be expressed by the equation of Bernoulli:

$$\frac{\partial\phi(t,x,y,\eta)}{\partial t} + \frac{1}{2}|\nabla\phi(t,x,y,\eta)|^2 + \frac{p(t,x,y,\eta)}{\rho} + V(t,x,y,\eta) = C(t). \qquad (10)$$

Kinematic components defines a fluid particle on the free surface, i.e.:

$$\frac{\partial\eta}{\partial t} + \frac{\partial\phi}{\partial z}\frac{\partial\eta}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial y} = \frac{\partial\phi}{\partial z} \qquad (11)$$

Consider the motion of a fluid under the influence of horizontal acceleration in open container. The container has a rectangular cross-section with a length a, width - b and height of the fluid - h and accelerates horizontally, parallel to two of its walls (see Figure 1.).

The movement of the container is parallel to two of the walls, it is assumed that the flow velocity is zero in a direction perpendicular to the movement. Then solve the problem can be seen in two-dimensional space. Used a fixed (absolute) coordinate system ($\bar{x}$, $\bar{z}$) and relative coordinate system (moving together with the container) - (x, z), the relationship between coordinate systems is

$$x = \bar{x} - x_c \quad e_x = e_{\bar{x}}$$
$$z = \bar{z} - h \quad e_z = e_z,$$

where: x_c position of the container, e_* corresponding basic vectors.

FIG. 2 shows the relationship between the two coordinate systems.

The fluid is accepted as incompressible and ideal. Surface level should be $(x, 0)$, and defines from $\eta(t, x)$. The forces acting on the fluid are gravity and horizontal acceleration. Gravity force $F_g(\bar{x},\bar{z}) = -\ddot{x}_c e_{\bar{x}}$, and horizontal acceleration is defined by the expression: $F_a(\bar{x},\bar{z}) = -\ddot{x}_c . e_{\bar{x}}$. Transformation to relative coordinate system is:

$$F(x,z) = -ge_z - \ddot{x}_c e_c \qquad (12)$$

Equations (7), (8), (9), (10), (11) и (12) describe the flow of fluid in the container

$$\nabla^2.\phi(t,x,z) = 0 \qquad (13)$$

$$\frac{\partial\phi(t,x,z)}{\partial t} + \frac{p(t,x,z)}{\rho} + \frac{1}{2}|\nabla\phi(t,x,z)|^2 + gz + u(t)x = C(t) \qquad (14)$$

$$\frac{\partial\eta(t,x)}{\partial t} + \frac{\partial\phi(t,x,z)}{\partial x}\frac{\partial\eta(t,x)}{\partial x} = \frac{\partial\phi(t,x,z)}{\partial z} \qquad (15)$$

$$\frac{\partial\phi(t,0,z)}{\partial z} = 0, \frac{\partial\phi(t,a,z)}{\partial z} = 0, \frac{\partial\phi(t,x,-h)}{\partial z} = 0 \qquad (16)$$

where the expressions (14) and (15) describe the free surface of the fluid i.e. η(*t, x*)) и $u(t) = \ddot{x}_c$ - horizontal acceleration of the container.

It's solved by separating the variables, substituting $\phi(t,x,z) = T(t)X(x)Z(z)$ in (13) and is seeking of type $T(t)X''(x)Z(z) + T(t)X(x)Z''(z) = 0$ i.e.:

$$\frac{X''(x)}{X(x)} = -\frac{Z''(z)}{Z(z)} = -\beta \qquad (17)$$

The following equations are derived from (16) and (17) i.e.

$$\begin{cases} X'' + \beta X = 0 \\ X'(0) = 0 \\ X'(a) = 0 \end{cases} \qquad (18)$$

$$\begin{cases} Z'' - \beta Z = 0 \\ Z'(-h) = 0 \end{cases} \tag{19}$$

Solutions of the expressions (18) and (19) are:

$$\beta = 0 \Rightarrow \begin{cases} X(x) = a_1 x + a_2 \\ Z(z) = b_1 z + b_2 \end{cases}$$

After entering the boundary conditions:

$$\begin{cases} X'(0) = a_1 = 0 \\ X'(a) = a_1 = 0 \\ Z'(-z) = -b = 0 \end{cases} \Rightarrow \begin{cases} a_1 = 0 \\ b_1 = 0 \end{cases}$$

determine the harmonic flow:

$$X_0(x) = a_0(t),\, Z_0(z) = b_0(t) \tag{20}$$

$$\beta > 0 \Rightarrow \begin{cases} X(x) = a_1 \cos\sqrt{\beta}x + a_2 \sin\sqrt{\beta}x \\ Z(z) = b_1 \cosh\sqrt{\beta}z + b_2 \sinh\sqrt{\beta}z \end{cases}$$

Boundary conditions are:

$$\begin{cases} X'(0) = a_2\sqrt{\beta} = 0 \\ X'(a) = a_1\sqrt{\beta}\sin\sqrt{\beta}a + a_2\sqrt{\beta}\cos\sqrt{\beta}a = 0 \\ Z'(-h) = -b_1\sqrt{\beta}\sinh\sqrt{\beta}h + b_2\sqrt{\beta}\cosh\sqrt{\beta}h = 0 \end{cases}$$

Individual harmonics are determined by the expressions:

$$X_n(x) = a_n(t)\cos\frac{n\pi}{a}x; Z_n(y) = b_n(t)\frac{\cosh\frac{n\pi}{a}h}{\sinh\frac{n\pi}{a}h}\cosh\frac{n\pi}{a}(z+h) \tag{21}$$

The potential given in (20) and (21) is defined:

$$\phi(t,x,z) = T_0(t) + \sum_{n=1}^{\infty} T_n(t)\cos\frac{n\pi}{a}\cosh\frac{n\pi}{a}(z+h) \tag{22}$$

wherein all constants are collected together in $T_n(t)$.

Function $T_n(t)$ can be found from (14). It is assumed that the speed of the power flow is low, and in the square (14) is even smaller. Furthermore, the pressure is evenly across surface, thus to $C(t)=p[t, x, \eta(t, x)]/\rho$:

$$\frac{\partial\phi(t,x,\eta(t,x))}{\partial t} + g\,\eta(t,x) + u(t)x = 0 \tag{23}$$

After differentiation of (23) over time

$$\frac{\partial^2\phi(t,x,\eta(t,x))}{\partial t^2} + \frac{\partial^2\phi(t,x,\eta(t,x))}{\partial t^2}\dot{\eta}(t,x) + g\,\dot{\eta}(t,x) + \dot{u}(t)x = 0$$

After substituting (15) and neglecting all nonlinear members and the assumption that$(\eta(t,x) \approx 0)$ is very small i.e.:

$$\frac{\partial^2\phi(t,x,0)}{\partial t^2} + g\frac{\partial\phi(t,x,0)}{\partial z} = -\dot{u}(t)x$$

Replace (22) to give:

$$T_n''(t) + \sum_{n=1}^{\infty}\left[T_n''(t)\cosh\frac{n\pi h}{a} + T_n(t)\frac{n\pi h}{a}\sinh\frac{n\pi h}{a}\right]\cos\frac{n\pi}{a}x = -\dot{u}(t)x \tag{24}$$

Decomposes right side baseline function $\cos\frac{n\pi}{a}x$ т.е.:

$$-\dot{u}(t)x = -\dot{u}(t)a\left[\frac{1}{2} + 2\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2\pi^2}\cos\frac{n\,\pi}{a}x\right]$$

For the differential equation (24) after decomposition by the cosine function is obtained:

$$T_n''(t) + \omega_n^2.T_n(t) = \begin{cases} -\frac{a}{2}.\dot{u}(t), \text{при } n = 0 \\ b_n.\dot{u}(t), \text{при } n\ odd \\ 0 \text{ при } n\ even \end{cases} \text{ при}$$

$$\omega_n = \sqrt{\frac{n\pi g}{a}\tanh\frac{n\pi h}{a}},\; b_n = \frac{4a}{n^2\pi^2\cosh\frac{n\pi h}{a}}.$$

It should be noted that the applied horizontal acceleration excites only odd harmonics.

Modification of the surface of the fluid relative to the equilibrium position can be determined by (23), i.e.:

$$\eta(t,x_m) = -\frac{1}{g}\left[\frac{\partial\phi(t,x_m,0)}{\partial t} + u(t)x_m\right] \tag{25}$$

Assuming that the increase in the surface of the fluid from the equilibrium position is not significant $\eta(t,x) \approx 0$ and after substitution with potential $\phi(t,x_m,0)$ to give:

$$\eta(t,x_m) = -\frac{1}{g}\left[T_0'(t) + \sum_{n=1}^{\infty} c_n(x_m)T_n'(t) + u(t)x_m\right] \tag{26}$$

where: $c_n(x_m) = \cos\frac{n\pi x_m}{a}\cosh\frac{n\pi h}{a}$

Combining the input and output equations defines the model:

$$\eta(t,x_m) = -\frac{1}{g}\left[\frac{a}{2} - x_m - \sum_{n=1,\,odd}^{\infty}\frac{b_n.c_n.x_m.p^2}{p^2+\omega_n^2}\right]u(t) \tag{27}$$

where p is the differential operator d/dt,

Substitution coefficients b_n и $c_n(x_m)$ therefore:

$$\eta(t,x_m) = -\frac{1}{g}\left[\frac{a}{2} - x_m - \sum_{n=1,\,odd}^{\infty}\frac{4a}{n^2\pi^2}\cos\frac{n\pi x_m}{a}\left(1 - \frac{\omega_n^2}{p^2+\omega_n^2}\right)\right]u(t) \tag{28}$$

The first member of the sum is the order of the cosine in the form$\frac{a}{2} - x_m$, ctherefore, the surface of the fluid is raised above the point x_m for a rectangular section of the container, ie.:

$$\eta(t,x_m) = \frac{4a}{g.\pi^2}\left[\sum_{n=1,\,odd}^{\infty}\frac{1}{n^2}\cos\frac{n\pi x_m}{a}\frac{\omega_n^2}{p^2+\omega_n^2}\right]u(t) \tag{29}$$

III. Numerical experiments

In the numerical experiments made a glass container is used in which it is poured fluid (water) – the length of the fluid is $a = 0{,}070\ m$, and the height - $h = 0{,}200\ m$ (see fig. 3a).

Figure 3. General formulation of the research: a) Geometry of the fluid б) Height of Fluid

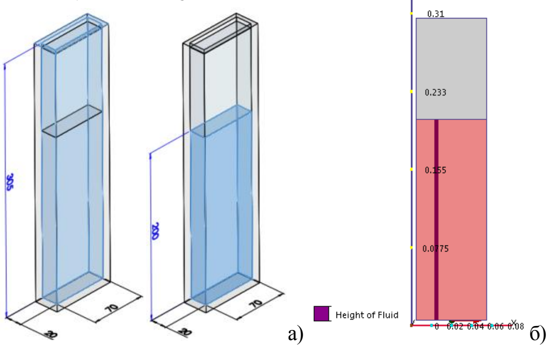
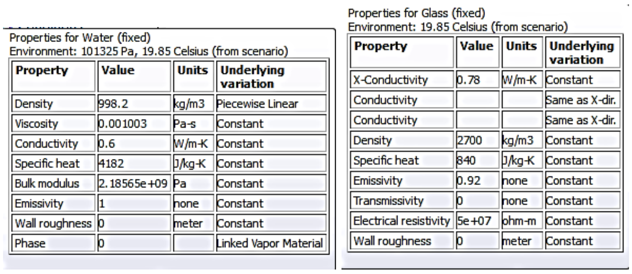


Figure 4. Material characteristics of the fluid and the open container



Properties for Water (fixed)
Environment: 101325 Pa, 19.85 Celsius (from scenario)

Property	Value	Units	Underlying variation
Density	998.2	kg/m3	Piecewise Linear
Viscosity	0.001003	Pa-s	Constant
Conductivity	0.6	W/m-K	Constant
Specific heat	4182	J/kg-K	Constant
Bulk modulus	2.18565e+09	Pa	Constant
Emissivity	1	none	Constant
Wall roughness	0	meter	Constant
Phase	0		Linked Vapor Material

Properties for Glass (fixed)
Environment: 19.85 Celsius (from scenario)

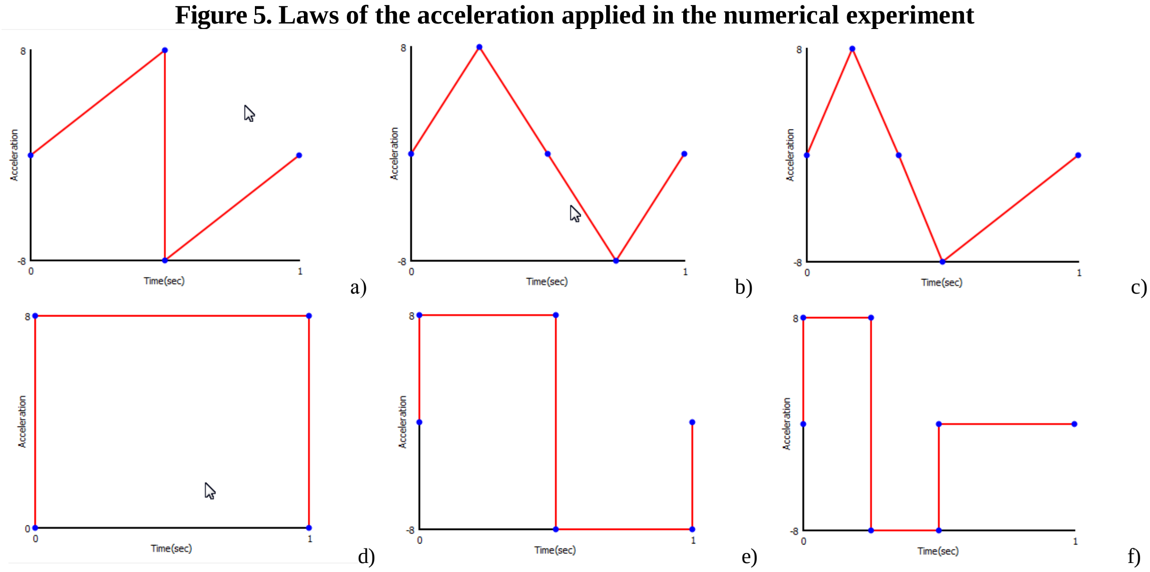
Property	Value	Units	Underlying variation
X-Conductivity	0.78	W/m-K	Constant
Conductivity			Same as X-dir.
Conductivity			Same as X-dir.
Density	2700	kg/m3	Constant
Specific heat	840	J/kg-K	Constant
Emissivity	0.92	none	Constant
Transmissivity	0	none	Constant
Electrical resistivity	5e+07	ohm-m	Constant
Wall roughness	0	meter	Constant

The material properties of the fluid (water) and the open container (glass) are shown in Fig. 4.

To study the sloshing into an open container is necessary that it is partially filled. This must be specified by setting the initial state of the fluid in Height of Fluid (HOF). Define the initial conditions for the height of the fluid (Fig. 3b), requires the use of separate volumes for both it and the volume located above it, where the fluid enters. The open container is moving linearly with velocity $v = 1\frac{m}{s}$, at a distance 0.2 m.

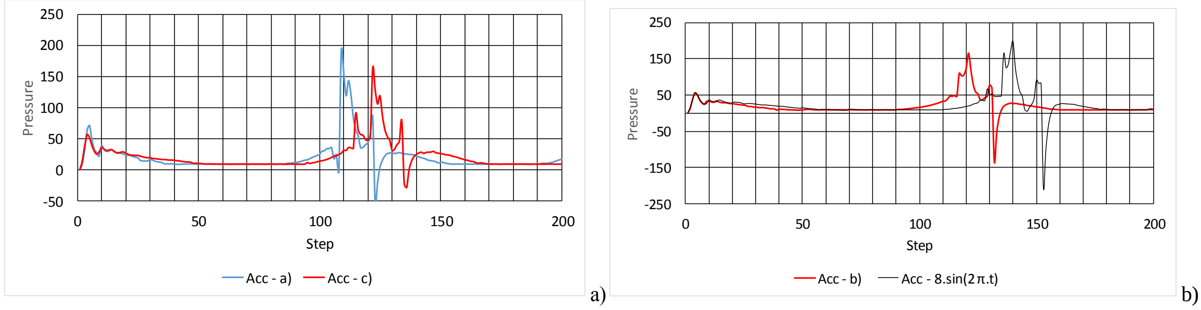
For this numerical study of the behavior of interest is fluid due to the different laws of acceleration shown in Fig. 5, The maximum value of the applied acceleration is $8\ m.s^{-2}$.

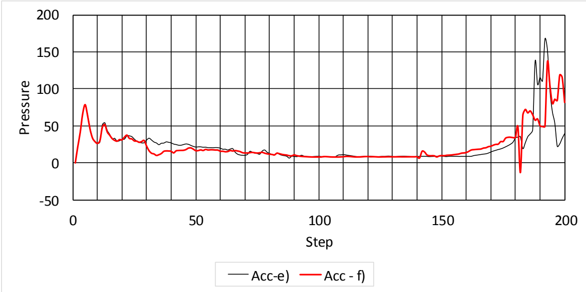
Figure 5. Laws of the acceleration applied in the numerical experiment

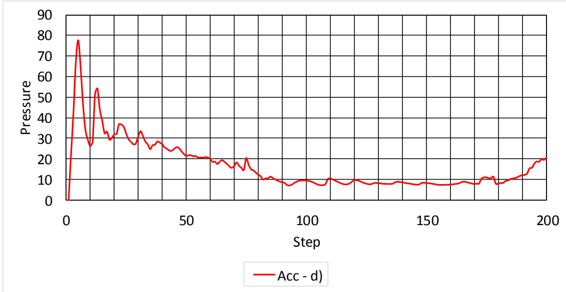


The results of the numerical experiments are shown graphically in FIG. 6. FIG. 6a has shown in the pressure of application of the acceleration of the type on Fig. 5c The change in pressure depending on the acceleration applied to the type of Fig. 5b compared with the acceleration and the sinusoidal law (8.sin (2π.t) shown in Fig.. 6b. After applying the acceleration to a rectangular law see Fig. 5e,f the pressure change shown in fig.6c, if law of acceleration is linear according to Fig. 5d then the pressure changes according to the graph in Fig. 6d.

Figure. 6 The change in pressure during acceleration shown in Figure 5







c)

d)

Using the technique of Volume of Fluid (VOF) numerical modeling results can be displayed and shown in Fig. 7. Study was done in “reciprocating with half period time equal 2 seconds” move to distance 0,2 m. with acceleration applied to the type of Fig. 5c. FIG. 8 shows the pressure variation depending on the magnitude of the acceleration during the acceleration of the type on Fig. 5c.

Figure 7 Volume of Fluid (VOF) on one of the numerical studies.

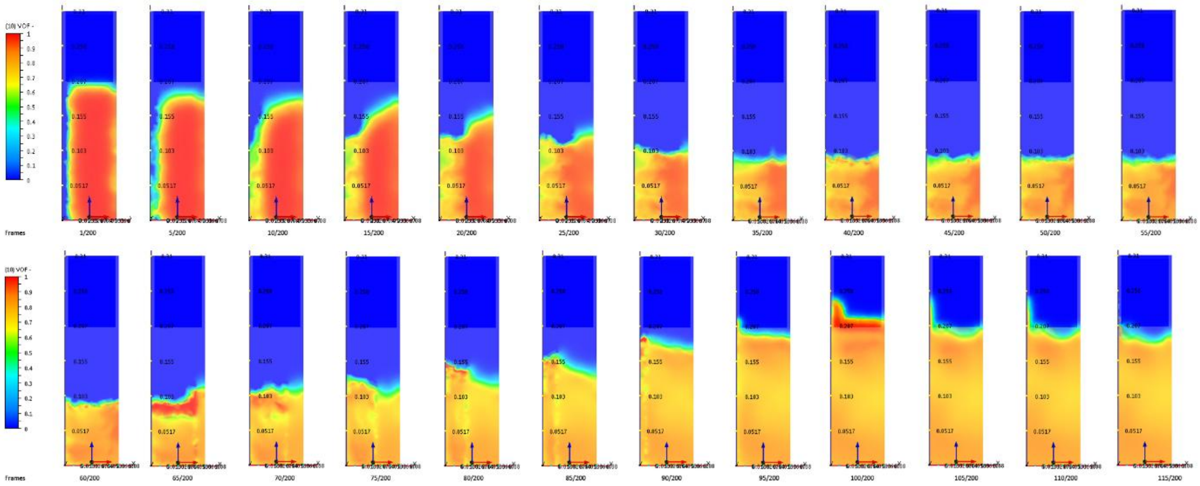
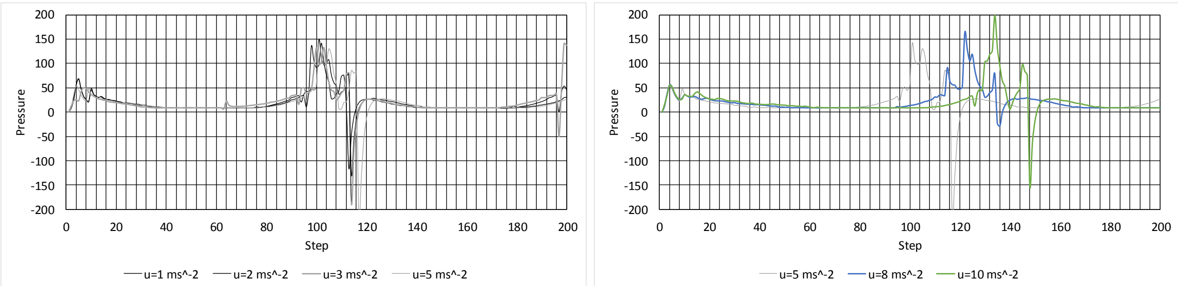


Figure 8. Pressure change depending on the magnitude of the applied acceleration



IV. Conclusions

The analytical conclusions made may be used to study sloshing on fluid located in an open container under the influence of a complex force field, which can be relevant for the packaging machines and systems for finding the optimal amplitudes of splashing depending on the type and magnitude of the applied acceleration, and also the viscosity of the fluid.

From the completed studies is seen that the least pressure <78 Pa is observed in the linear acceleration Fig. 5.

When applying different sized accelerations (Fig. 8) shows that the peaks of pressure in the time delay, and this most clearly appears when accelerations are greater than $5\,\frac{m}{s^2}$.

V. References

[1] Madzhirski C. H., Hydrodynamics, Sofia, Technics, 1979.

[2] Povh I.L. Tehnicheskaya Fluid Mechanics, Mechanical Engineering, L. 1976

[3] Armenio, V. and M. La Rocca (1996): “On the analysis of sloshing of water in rectangular containers: Numerical study and experimental validation.” Ocean Engineering, **23:8**, pp. 705–739.

[4] M.Penev, H. Slavchev, N. Marchev, “Conduct Container with Fluid in Complexity a Fluid of Force”, International Science Conference Unitech’06 Gabrovo, vol. II., ISBN 13: 978-954-683-352-5, 24-25 november 2006, pp. II-422 … II-426

[5] Kelkar, K. M. and S. V. Patankar (1997): “Numerical method for the prediction of free surface flows in domains with moving boundaries.” Numerical Heat Transfer, Part B: Fundamentals, **31:1**, pp. 387–399.

[6] Romero, V. J. and M. S. Ingber (1995): “A numerical model for 2D sloshing of pseudoviscous liquids in horizontally accelerated rectangular containers.” In Brebbia et al., Eds., Boundary Elements XVII, pp. 567–583. Computational Mechanics Publications, Southampton, England.