Seepage Analysis of an Embankment

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Abstract: The stability of an embankment to a large extend depends on quantity of seepage. Flow nets are drawn to estimate the amount of seepage through an embankment and hence its stability. The conventional methods of drawing a flow net and to establish the top flow line are essentially trial and sketchy. In the present study, an attempt has been made to establish the top flow line by using the equation of cubic parabola. The top flow line thus established is very accurate as compared to the conventional graphical methods. Thus, estimation of seepage and consequently uplift pressure becomes very accurate.

Key Words: seepage pressure, phreatic line.

I. INTRODUCTION
A new method is established for the determination of top flow line in a homogeneous embankment. Although a method is available in literature for locating the top flow line for a homogeneous embankment but is trial and sketchy. In the present study equation of cubic parabola is used to establish the top flow line. The method discussed in the present study involves determining the value of constants in the equation of cubic parabola. The value of constants is determined by applying boundary conditions, for example to determine the value of four constants in the equation of cubic parabola, four boundary conditions are established. The results obtained are compared with those given by Casagrande(1937).

II. METHODOLOGY: – Using the Equation of Cubic Parabola
In Fig. 1.0 is shown a schematic diagram of an embankment; the other details for which are as given below. 1) TW – top width, 2) BW – bottom width, 3) α – angle made by the u/s surface with the horizontal, 4) β - angle made by the d/s surface with the horizontal, 4) H = water level and 5) H₁ = H + freeboard.
Figure 1.0 Determination of top flow line for seepage through an embankment

As discussed earlier, shape of the top flow line is approximated by parabola. Therefore, referring to Fig. 1.0, the equation of the top flow line is obtained as

\[ y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \]  
(cubic parabola) \hspace{1cm} (1.0)

The constants, \( a_{(i=0, 3)} \) in Eq. (1.0) are determined using the boundary conditions as given below.

1. At \( x = 0 \), \( y = H \)
   \[ H = a_0 + a_1 0 + a_2 0 + a_3 0 = a_0 \]
   \[ \therefore a_0 = H \]
   \[ \therefore y = H + a_1 x + a_2 x^2 + a_3 x^3 \]

2. At \( x = 0 \), \( dy / dx = - \tan \theta \)
   \[ dy / dx = a_1 + 2a_2 x + 3a_3 x^2 \]
   \[ \therefore at \ x = 0, \ dy / dx = a_1 = - \tan \theta \]
   \[ \therefore y = H - \tan \theta \cdot x + a_2 x^2 + a_3 x^3 \]

3. At \( x = BW - a \cdot \cos \beta - H \cdot \cot x \), \( y = a \cdot \sin \beta \)
   \[ \therefore a \cdot \sin \beta = H - \tan \theta \cdot x_1 + a_2 x_1^2 + a_3 x_1^3 \] \hspace{1cm} (2.0)

4. At \( x = x_1 \), \( dy / dx = - \tan \beta \)
   \[ y = H - \tan \theta \cdot x + a_2 x^2 + a_3 x^3 \]
   \[ dy / dx = - \tan \theta + 2a_2 x + 3a_3 x^2 \]
   \[ - \tan \beta = - \tan \theta \cdot 2a_2 x_1 + 3a_3 x_1^2 \] \hspace{1cm} (3.0)

Multiplying Eq. (3.0) by \( 0.5x_1 \)
\[ - 0.5x_1 \cdot \tan \beta = - 0.5x_1 \cdot \tan \theta + a_2 x_1^2 + 1.5a_3 x_1^3 \] \hspace{1cm} (4.0)

Subtracting Eq. (2.0) from Eq. (4.0), the constant, \( a_3 \) is obtained as
\[ 0.5a_3 x_1^3 = H - 0.5x_1 \cdot \tan \theta - 0.5x_1 \cdot \tan \beta - a \cdot \sin \beta \]
\[ \therefore a_3 = \frac{\left\{ 2H - x_1 (\tan \theta + \tan \beta) - 2a \cdot \sin \beta \right\}}{x_1^3} \]

Now, rewriting Eqs. (2.0) and (3.0),
\[ - \tan \beta = - \tan \theta \cdot 2a_2 x_1 + 3a_3 x_1^2 \]
\hspace{1cm} (3.0)
\[ a \cdot \sin \beta = H - \tan \theta \cdot x_1 + a_2 x_1^2 + a_3 x_1^3 \] \hspace{1cm} (4.0)

Multiplying Eq. (2.0) by Eq. (3.0),
\[ 3a \cdot \sin \beta = 3H - 3 \cdot \tan \theta \cdot x_1 + 3a_2 x_1^2 + 3a_3 x_1^3 \] \hspace{1cm} (5.0)

Multiplying Eq. (3.0) by \( x_1 \)
\[ - \tan \beta \cdot x_1 = - \tan \theta \cdot x_1 + 2a_2 x_1^2 + 3a_3 x_1^3 \] \hspace{1cm} (6.0)

Subtracting Eq. (6.0) from Eq. (5.0), the constant, \( a_2 \) is obtained as
\[ 3a \cdot \sin \beta + \tan \beta \cdot x_1 = 3H + \tan \theta \cdot x_1 - 3 \cdot \tan \theta \cdot x_1 + a_2 x_1^2 \]
\[ a_2 x_1^2 = 3a \cdot \sin \beta + \tan \beta \cdot x_1 - 3H - \tan \theta \cdot x_1 + 3 \cdot \tan \theta \cdot x_1 \]
\[ \therefore a_2 = \frac{\left\{ 3(a \cdot \sin \beta - H) + x_1 (\tan \beta + 2 \cdot \tan \theta) \right\}}{x_1^2} \]

Thus, the equation for the top flow line is obtained as
III. RESULTS AND DISCUSSIONS

The following is an example that demonstrates the procedure for establishing the top flow line for an embankment using the methods developed in the present study.

**Given Data:**
- Water head (u/s) $H = 10$ m
- Freeboard = 2 m
- $H_1 = 10 + 2 = 12$ m
- $\alpha = 45^\circ$
- $B = 45^\circ$
- Top width (TW) = 3 m
- Bottom width (BW) = $H_1 \cot \alpha + H_1 \cot \beta + TW = 27$ m

**Solution:**

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>$y$ (m) [Proposed Method, Eq.(7.0)]</th>
<th>$y$ (m) Casagrande’s (1937) Method</th>
</tr>
</thead>
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<td>0</td>
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<td>9.0064</td>
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<tr>
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<td>4</td>
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<td>5</td>
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<td>6</td>
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</tr>
<tr>
<td>13</td>
<td>3.9898</td>
<td>4.9455</td>
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</tbody>
</table>

**Figure 2.0** Generation of top flow lines using the proposed methods and construction of flow nets

In Fig. 2.0 is shown a plot of the top flow line as $\Phi = 9$ using proposed method $\Phi = 8$ and Casagrande’s (1937) method. As seen from this figure, the top flow line $\Phi = 9$ compare $\Phi = 8$ in the one plot $\Phi = 7$ the Casagrande’s (1937) method. Also, the proposed closed-form solution facilitates the construction of a very accurate flow net, which otherwise would not be possible using the available graphical methods.
IV. SUMMARY

In the present study and attempt has been made to obtain closed-form solutions for determining the top flow line in an embankment. For this purpose, new method is developed using the equation of cubic parabola and Laplace’s equation. The proposed results is observed to be in good agreement reported by casagrande(1987)

V. CONCLUSIONS

The conclusions that are drawn from the present study are as follows.

1. New method is proposed to establish the top flow line in the seepage zone of an embankment. For this purpose, the equation of cubic parabola and Laplace’s equation are used.

2. The closed-form solution derived using the Laplace’s equation facilitates the construction of a very accurate flow net, which otherwise would not be possible using the available graphical methods.

REFERENCES

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