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Analytical Investigations and Properties of Hybrid Sumudu-Z Transform

Sadikali Latif Shaikh
Department of Mathematics
Maulana Azad College of Arts, Science and Commerce,
Dr. Rafiq Zakaria Campus, Aurangabad- 431001,
Maharashtra, INDIA

***Abstract**: A space of c-d functions which are continuous and discrete with respect to first and second variable respectively is presented. A new Hybrid Sumudu-Z transform defined over the space and investigated its properties.*

***Keywords**: Integral transform, Discrete transform , Hybrid systems.*

I. Introduction

In the last few decades, the theory of multidimensional hybrid control system is strongly developed because these systems appears as a model in many distinct branches such as image processing, geophysics, computer tomography, 2D hybrid linear systems [1][2][3], and many more. Such hybrid systems were studied in [4][5].
Classical multidimensional signals and systems are divided into two parts, one is continuous signals and systems and another is discrete signals and systems. In the classical 1D systems theory, the frequency domain method based on Laplace transform is available for continuous variable and Z-transform is popularly used for discrete case. In [5][6][7][8] generalized multidimensional hybrid Laplace-Z transform studied and developed complete theory of hybrid Laplace-Z transform.
In this paper we construct hybrid Sumudu-Z transform for suitable functions of continuous and discrete variables. The sumudu transform is widely popular now a days due to its scale and unit preserving properties , the sumudu transform in classical 1D theory may be used to solve the problems without resorting to a new frequency domain, Belgacem in [9] investigate fundamental properties and applications of the Sumudu transform. In this paper we construct the space A of HSZ transformable functions. Existence theorem and some operational properties are proved.

II. Preliminaries

Definition 2.1. Over the set of functions,

$$A = \{f(t):\ \exists\, M;\ \tau_{1,}\tau_2 > 0, |f(t)| < M\}e^{\frac{|t|}{\tau_j}}\ if\ t\ \ \in (-1)^j \times [0,\infty)$$

The sumudu transform is defined by

$$F(u) = S[f(t)] = \frac{1}{u}\int_0^\infty e^{\frac{-t}{u}} f(t)dt, \qquad Re\left(\frac{1}{u}\right) > \frac{1}{w}$$

Where u is any real number and w is some nonzero real number.
Definition 2.2. Z-transform of the function $g(n)$, where n is any positive integer is defined by

$$G(z) = Z[g(n)] = \sum_{n=0}^{\infty} z^{-n} g(n), \quad |g(n)| \le R$$

Where R is any positive real number
Definition 2.3. A function $f{:}\,\mathbb{R}\ \times\ \mathbb{Z}\ \to\ \mathbb{C}$ is said to be hybrid sumudu-z transformable continuous – discrete function or simply (c-d function) if f satisfying following conditions:

i) $f(t,n) = 0,\ \ if\ t < 0\ and\ n < 0.$

ii) $f(.,n) is\ smooth\ on\ \mathbb{R}_+\ \ for\ any\ n\ \in \mathbb{Z}_+$

iii) $\exists\, M > 0, \frac{1}{w} \in \mathbb{R}\ and\ R > 0, such\ that\ |f(t)| \le MRe^{\frac{1}{w}}, \forall\, t > 0, n \ge 0$

The set of all c-d functions is denoted by A.
Definition 2.4. For any c-d function $(t,n) \in A$, Hybrid Sumudu –Z transform (HS-Z) is defined by

$$F(u,z) = (HSZ)[f(t,n)] = \frac{1}{u}\int_0^\infty \sum_{n=0}^{\infty} e^{\frac{-t}{u}} z^{-n} f(t,n)dt$$

III. Existence Theorem for HSZ Transform

Theorem: If $f(t,n) \in A$ then (HSZ)-transform of $f(t,n)$ defined by

$$F(u,z) = \frac{1}{u}\int_0^\infty \sum_{n=0}^{\infty} e^{\frac{-t}{u}} z^{-n} f(t,n)dt$$

Is convergent in the domain

$$D(f) = \left\{(u,z) \in \mathbb{C}^2 \mid Re\left(\frac{1}{u}\right) > \frac{1}{w}, |z| > R\right\}$$

Also $F(u,z)$ is absolutely convergent in any domain

$$D^1(f) = \left\{(u,z) \in \mathbb{C}^2 \mid Re\left(\frac{1}{u}\right) > \frac{1}{w^1} > \frac{1}{w} > 0, |z| > R^1 > R > 0\right\}$$

Where $\frac{1}{w}, R$ are positive real numbers.

IV. Properties of HSZ-Transform

In this section we prove some operational properties of HSZ-transform.

4.1 Linearity

If $f, g \in$ A and α, β are any complex numbers then,

$$(HSZ)[\alpha.f + \beta.g] = \alpha(HSZ)[f] + \beta(HSZ)[g]$$

4.2 Homothety

If $f(t,n) \in$ A and a is any positive real number, b is any positive integer such that $n \neq b$ then

$$(HSZ)[f(at,bn)] = F\left(au, z^{\frac{1}{b}}\right) \quad for\ Re\left(\frac{1}{u}\right) > \frac{a}{w}\ , |z| > R^b$$

Proof: By using definition of HSZ-transform and substituting $x = at$, $l = bn$, we can prove the above property.

4.3 First time delay:

If α is any positive real number and b is any positive integer, then

$$(HSZ)[f(t-a, n-b)] = e^{\frac{-a}{u}}.z^{-b}F(u,z)$$

Where, $F(u,z) = (HSZ)[f(t,n)]$.

Proof: By using definition of HSZ-transform and substituting, $x = t - a$, $l = n - b$, we can prove the above property.

4.4 Second time delay:

If α is any positive real number and b is any positive integer, then

$$(HSZ)[f(t+a, n+b)H(t-a)] = e^{\frac{a}{u}}.[z^b F(u,z) - \sum_{t=0}^{b-1} z^{b-l}F(u,l)]$$

Where, $F(u,z) = (HSZ)[f(t,n)]$ and $H(t-a)$ is Heaviside unit step function.

Proof: By using definition of HSZ-transform and substituting, $x = t + a$, $l = n + b$, we can prove the above property.

4.5 Translation :

If $F(u,z) = (HSZ)[f(t,n)]$ then

$$(HSZ)[e^{at}.b^n f(t,n)] = \frac{1}{1-au}F\left(\frac{u}{1-au}, \frac{z}{b}\right)$$, where $a \in R_+ , b \in Z_+$

Proof: From the definition of HSZ-transform, we have

$$(HSZ)[e^{at}.b^n f(t,n)] = \frac{1}{u}\int_0^\infty \sum_{n=0}^{\infty} e^{\frac{-t}{u}} z^{-n}\, e^{at} b^n f(t,n)dt$$

$$= \frac{1}{u}\int_0^\infty \sum_{n=0}^{\infty} e^{-t(\frac{1}{u}-a)} (\frac{z}{b})^{-n}\, e^{at} f(t,n)dt$$

$$= \frac{1}{1-au}\left[\frac{1-au}{u}\int_0^\infty e^{-t\left(\frac{1-au}{u}\right)} \left(\frac{z}{b}\right)^{-n} f(t,n)dt\right]$$

$$= \frac{1}{1-au}F\left(\frac{u}{1-au}, \frac{z}{b}\right).$$

4.6 Average preserving property:

If $F(u,z) = (HSZ)[f(t,n)]$ then

$$(HSZ)\left[\frac{1}{t}\int_0^t f(\tau,n)d\tau\right] = \frac{1}{u}\int_0^u F(v,z)dv$$

Proof: Since Sumudu transform preserve average property, hence it is obvious to prove the theorem for HSZ-transform.

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