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Construction of Rational Quartic Trigonometric Bézier Curves

Mridula Dube[1]& Urvashi Mishra[2]

Professor, Department of Mathematics and Computer Science,
R.D. University, Jabalpur, Madhya Pradesh, India[1]

Assistant Professor, Department of Mathematics, Mata Gujri Mahila Mahavidyalaya,
Jabalpur,Madhya Pradesh, India[2]

Abstract: *In this paper a new kind of rational quartic trigonometric Bézier curve is defined. These curves not only have most properties of the quartic Bézier curves with the Bernstein basis in the polynomial space, but also have other properties for shape modelling. The shape parameter provides freedom for design and shape control of the curve. Here we give a shape preserving interpolation condition based on shape parameter. Thus we can easily construct smooth curves of any shape. The shape of the curve can be adjusted by altering the values of shape parameter while the control polygon is kept unchanged. These curves can be used as an efficient model for geometric design in the fields of CAGD.*

Keywords: *Quartic Trigonometric Bézier basis function, Quartic rational Trigonometric Bézier curve, Quartic rational Trigonometric Bézier surface, Shape parameter.*

I. INTRODUCTION

Interpolation of an order sequence of points is one of the most widely use of methods in practical curve modeling. Hence, there were a vast number of papers, books and chapters dealing with this topic. Designers generally prefer spline curves, where most of the methods work globally. Spline Interpolation is a significant tool in computer graphics, computer aided geometric design (CAGD) and engineering as well. The fitting of a spline curve to a set of data points has application in computer-aided-design (CAD), CAM, computer graphic system, robot path and trajectory planning. B-Spline curves and their rational generalization play a central role and are widely used in computer aided design today. These methods are excellent tool in design system to create new objects, but the modification and shape control of the existing objects are also essential. The Bezier-Curves and surfaces form a basic tool for constructing free form curves and surfaces. The study of curves and surfaces is key element in CAGD & CAD/CAM that has been around for quite some time. The method of CAGD has arisen from the need of efficient computer representation of practical curves and surfaces used in engineering design. Therefore, on this field it is desirable to generate a convexity preserving interpolating curves and surfaces based on given data. Convexity is a substantial shape characteristic of the data. The significance of the convexity preserving interpolation problem in industry can not be denied. A number of examples can be quoted in this regard, like the modeling of cars in automobile industry, aero plane and ship design. Designing well shaped smooth curve and surfaces also arise in manufacturing the TV-Screens. In the surface designing sense we say that the screens must preserve the convexity. Shape control, shape design, shape representation and shapes preservation are important areas for graphical representation of data .See [1-25]. Monotonicity is a prevailing shape property of curve. There are many physical situations that arise from different sciences and art where entities only have a meaning when their values are monotone .Examples include approximations of couple and quasi couples in statistics, approximation of potential functions in physical and chemical systems and does response curves in biochemistry and pharmacology. The specification of certain devices like digital-to-analog (DAC- used in audio/video devices) and analog-to-digital (ADC- used in music recordings, digital signal processing) requires monotonicity, which seems to be a sort of confusion. In these devices the output direction is supposed to be the same as that of input direction. In terms of monotonicity, as the input to the device increases (decreases) the output must also increase (decrease) accordingly. Monotonicity of monotone data is also involved in some other areas like, the level of blood uric acid in gout patients, data generated from stress and strain of a material, graphical display of Newton's law of cooling, medical diagnosis and economic forecasting. Positivity is a prevailing shape property of curves and surfaces. Positivity preserving problems occur in visualizing a physical quantity that cannot be negative which may arise if the data is taken from some scientific ,social or business environments .There are some physical quantities which are always positive: In the modeling of earth surface, measurement of altitude above sea level at different positions on the surface of earth, carbon dating used to measure the age of mummies and fossils, terrain modeling, formation of geological crust movement to forecast earth quake and volcanic eruptions, the rate of dissemination of drugs in the blood, depreciation of the price of computers, probability distributions and resistance offered by an electric circuit. Bézier curves and surfaces are the basic tools for modelling in Computer Aided Geometric Designing (CAGD) and Computer Graphics (CG).

The Problem of shape preservation has been discussed by a number of authors. In recent years a good amount of work has been published [1-25] that focuses on shape preserving curves and surfaces .In many interpolation problems it is important that the solution preserves some shape properties such as convexity or monotonicity. Classical methods usually ignore these kinds of condition and thus yield solution exhibiting undesirable inflections. This is the reason why many investigations during the last years have been directed towards interpolation by means of shape- preserving polynomial spline functions. During the last few years, a major research focus has been the use of trigonometric functions or the blending of polynomial and trigonometric functions. Trigonometric B-splines were first presented in [1] and there recurrence relation for the trigonometric B-splines of arbitrary order was established in [2]. An extension of the Bézier model is studied in [18]. In [19], [20], and [21] quartic and cubic trigonometric Bézier curve respectively with shape parameter is presented and the effect of shape parameter is studied. In this sequence a new Quartic rational trigonometric Bézier Curve with a shape parameter is presented.

The purpose of this paper is to present new rational quartic trigonometric polynomial blending functions, which are useful for constructing rational quartic trigonometric B´ezier curves and can be applied to construct F^3 continuous shape preserving interpolation spline curves with shape parameters. The changes of a local shape parameter will only affect two curve segments.

In this paper a new kind of rational quartic trigonometric Bézier Curve with a shape parameter is presented. The paper is organized as follows. In section II, quartic trigonometric Bézier basis functions are established and the properties of the basis functions are shown. In section III, rational quartic trigonometric Bézier curves are given and some properties are discussed. By using shape parameter, shape control of the quartic Trigonometric Bézier curves is studied. In section IV, Rational Quartic Trigonometric Parametric Curve Segments are shown. the representation of quartic trigonometric Bézier surface has been shown. In section V, shape Preserving Interpolation Spline Curves with local shape parameter are constructed. Numerical examples and applications are discussed in section VI. Conclusion is presented in Section VII.

II. QUARTIC TRIGONOMETRIC BÉZIER BASIS FUNCTIONS

Firstly, the definition of quartic trigonometric Bézier basis functions is given as follows.

The construction of the basis functions

Definition 1. For $t\in [0,\pi/2]$, the following six functions are defined as quartic Trigonometric Bézier basis functions:

$$
\begin{aligned}
\mathbf{b_0} &= (1-\sin t)^4 \\
\mathbf{b_1} &= 4\sin t\,(1-\sin t)^3, \\
\mathbf{b_2} &= (1-\sin t)^2(1-\cos t)(9+8\sin t+3\cos t), \\
\mathbf{b_3} &= (1-\sin t)(1-\cos t)^2(9+3\sin t+8\cos t), \\
\mathbf{b_4} &= 4\cos t\,(1-\cos t)^3, \\
\mathbf{b_5} &= (1-\cos t)^4, \qquad \textbf{(1)}
\end{aligned}
$$

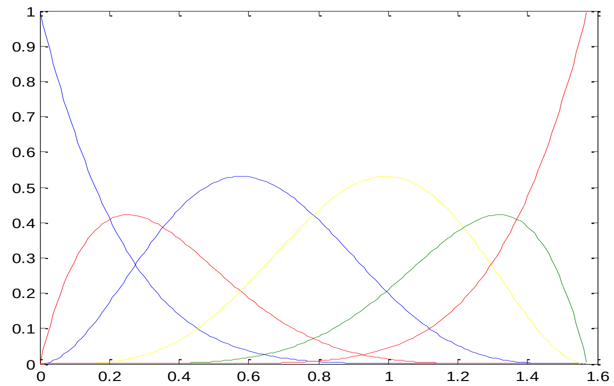


Fig. 1 shows the curves of the blending functions.

From the definition of the blending functions, We can know that the blending functions have the following properties analogous to that of the quintic Bernstein basis functions:

(a) **Non-negativity***:* $b_i(t) \geq 0,$ for i = 0, 1, 2, 3, 4, 5.

(b) **Partition of unity**: $\sum_{i=0}^{5} b_i(t) = 0,$

(c) **Symmetry:** $b_i(t) = b_{5-i}(\pi/2 - t)$, for i = 0, 1, 2.

(d) **Maximum**: Each $b_i(t)$ has one maximum value in [0, π/2].

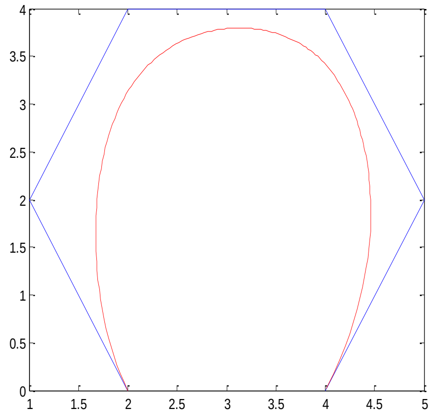
III. RATIONAL QUARTIC TRIGONOMETRIC BÉZIER CURVE

2.1. The construction of the Rational Quartic Trigonometric Bézier curve:
We define a rational quartic trigonometric Bezier curve as follows:

Definition: Given points P_i (i = 0,1,2,3,4,5) in R^2 or R^3 . For t∈ [0, π/2], then

$$R(t) = \frac{\sum_{i=0}^{5} w_i P_i b_i(t)}{\sum_{i=0}^{5} w_i b_i(t)} \qquad \textbf{(2)}$$

is called a Rational Quartic Trigonometric Bézier curve (RQTB , for short). Where the weight $w_i \succ 0$ for i= 0,1,2,3,4,5 and the basis function $b_i(t)$ for i= 0,1,2,3,4,5 are defined in Eq. (1).Figure (2) shows that a Rational Quartic Trigonometric Bezier Curve and Figure (3) shows that comparison between Quartic Trigonometric Bezier curve and Rational Trigonometric Bezier Curve.



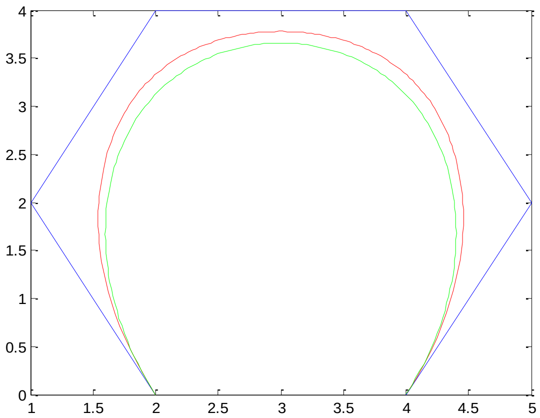


Fig. 2 Rational Quartic Trigonometric Bezier Curve **Fig. 3 Compare Quartic and RQTB**

2.2.The Properties of the Rational Quartic TrigonometricBezier Curve

From the definition of the basis function, some properties of the Rational Quartic Trigonometric Bézier curve can be obtained as follows:
Theorem 2 The Rational Quartic Trigonometric Bézier curves (2) have the following properties:

(1) **Terminal Properties**:

$$R(0) = P_0,$$

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$$R(\frac{\pi}{2}) = P_{5},$$

$$R^{'}(0) = 4\frac{w_1}{w_0}(P_1 - P_0),$$

$$R^{'}(\frac{\pi}{2}) = 4\frac{w_4}{w_5}(P_5 - P_4),$$

$$R^{''}(0) = \frac{1}{w_0^2}\Big[(32w_1^2 - 8w_0w_1)(P_0 - P_1) - 12w_0w_2(P_0 - P_2)\Big],$$

$$R^{''}(\frac{\pi}{2}) = \frac{1}{w_5^2}\Big[(32w_4^2 - 8w_4w_5)(P_5 - P_4) - 12w_3w_5(P_5 - P_3)\Big],$$

(2) Symmetry: Assume we keep the location of control points ***Pi*** (*i* = 0,1,2,3,4) fixed, invert their orders, and then obtained curve coincides with the former one with opposite directions. In fact, from the symmetry of Rational Quartic Trigonometric Bézier curve, we have

$$\mathbf{R}(\frac{\pi}{2} - \mathbf{t}\ ,\mathbf{P}_5,\mathbf{P}_4,\mathbf{P_3},\mathbf{P_2},\mathbf{P_1},\mathbf{P_0}) = \mathbf{R}(\ \mathbf{t}\ ,\mathbf{P_0},\mathbf{P_1},\mathbf{P}_2,\mathbf{P_3},\mathbf{P}_4,\mathbf{P}_5,)\ ;\ \mathbf{t} \in [\mathbf{0},\frac{\pi}{2}]\ ,$$

(3)Geometric Invariance: The shape of a Rational Quartic Trigonometric Bézier curve is independent of the choice of coordinates, i.e. (2) satisfies the following two equations:

$$\mathbf{R}(\frac{\pi}{2} - \mathbf{t}\ ,\mathbf{P}_5,\mathbf{P}_4,\mathbf{P_3},\mathbf{P_2},\mathbf{P_1},\mathbf{P_0}) + q = \mathbf{R}(\ \mathbf{t}\ ,\mathbf{P_0} + q,\mathbf{P_1} + q,\mathbf{P_2} + q,\mathbf{P_3} + q,\mathbf{P}_4 + q,\mathbf{P}_5 + q\ )\ ;\ \mathbf{t} \in [\mathbf{0},\frac{\pi}{2}]\ ,$$

$$\mathbf{R}(\frac{\pi}{2} - \mathbf{t}\ ,\mathbf{P}_5,\mathbf{P}_4,\mathbf{P_3},\mathbf{P_2},\mathbf{P_1},\mathbf{P_0}) \bullet T = \mathbf{R}(\ \mathbf{t}\ ,\mathbf{P_0} \bullet T,\mathbf{P_1} \bullet T,\mathbf{P_2} \bullet T,\mathbf{P_3} \bullet T,\mathbf{P}_4 \bullet T,\mathbf{P}_5 \bullet T)\ ;\ \mathbf{t} \in [\mathbf{0},\frac{\pi}{2}]\ ,$$

Where q is arbitrary vector in R2 or R3 and T is an arbitrary d * d matrix, d = 2 or 3.
(4)Convex Hull Property: The entire Quartic Rational Trigonometric Bézier curve segment lies inside its control polygon spanned by P0, P1, P2, P3,P_4, P_5.
(5)Variation diminishing property: The curves generated by the normalized totally positive B-basis have variation diminishing property. In this paper, each of the Rational quartic trigonometric polynomial blending functions constructed in the space *{*1*;* sin *t;* cos *t;* cos 2*t;* sin 3*t;* cos 3*t;* sin 4*t;* cos 4*t}* has one maximum value in [0,*π*/2]. The Rational quartic trigonometric B´ezier curves lie between the quintic B´ezier curves and the controlling polygon. And the Rational quartic trigonometric B´ezier curves are closer to the control polygon than the quintic B´ezier curves, which indicates that the Rational quartic trigonometric B´ezier curves can preserve the feature of the control polygon better than the quintic B´ezier curves.
(6)Convexity-preserving property: The variation diminishing property means the convexitypreservingproperty holds.

IV. Rational Quartic Trigonometric Parametric Curve Segments

Given the interpolation points Vi (i = 0; 1; 2; 3) and the rational quartic trigonometric B´ezier control points Pi(i = 0; 1; 2; 3; 4; 5), allowing for the continuity and the shape preserving property, the terminal points requirements are given in the following:

$$\mathbf{R(0) = P_0 = V_1},$$

$$\mathbf{R^{'}(0) = 4\frac{w_1}{w_0}(P_1 - P_0) = \alpha_1(V_2 - V_0)},$$

$$\mathbf{R^{''}(0) = \frac{1}{w_0^2}\Big[(32w_1^2 - 8w_0w_1 - 12w_0w_2)P_0 + (8w_0w_1 - 32w_1^2)P_1 + (12w_0w_2)P_2\Big] = \alpha_1(V_0 - 2V_1 + V_2)}, \qquad (3)$$

$$\mathbf{R^{''}(\frac{\pi}{2}) = \frac{1}{w_5^2}\Big[(12w_3w_5)P_3 + (8w_4w_5 - 32w_4^2)P_4 + (32w_4^2 - 8w_4w_5 - 12w_3w_5)P_5\Big] = \alpha_2(V_1 - 2V_2 + V_3)},$$

$$\mathbf{R^{'}(\frac{\pi}{2}) = 4\frac{w_4}{w_5}(P_5 - P_4) = \alpha_2(V_3 - V_1)},$$

$$\mathbf{R(\frac{\pi}{2}) = P_5 = V_2},$$

where $\alpha_1, \alpha_2 \in [\mathbf{0}, \infty]$, we call α_1, α_2 as shape parameters.

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The curve segment can be generate using Eq. (3) and the blending functions, as follows:

PROPOSITION 4.1 : Let $\alpha_1, \alpha_2 \in [0, \infty]$, be the shape parameters , Pi(i = 0; 1; 2; 3; 4; 5), the rational quartic trigonometric B´ezier control points and Vi(i = 0; 1; 2; 3) the corresponding interpolation points, then for
t∈ [0, π/2] we have

$$Q(t,\alpha_1,\alpha_2) = \frac{\sum_{i=0}^{5} w_i\, b_i(t) P_i}{\sum_{i=0}^{5} w_i\, b_i(t)} = \sum_{i=0}^{3} Tb_i(t,\alpha_1,\alpha_2) V_i, \tag{4}$$

Where

$$Tb_0(t,\alpha_1,\alpha_2) = \frac{1}{\sum_{i=0}^{5} w_i\, b_i(t)}\left[-\frac{\alpha_1 w_0}{4} b_1(t) + \left(\frac{\alpha_1 w_0}{4} - \frac{2}{3}\alpha_1 w_1\right) b_2(t)\right],$$

$$Tb_1(t,\alpha_1,\alpha_2) = \frac{1}{\sum_{i=0}^{5} w_i\, b_i(t)}\left[w_0 b_0(t) + w_1 b_1(t) + \left(-\frac{\alpha_1 w_0}{6} + w_2\right) b_2(t) + \left(-\frac{\alpha_2 w_5}{12} + \frac{2}{3}\alpha_2 w_4\right) b_3(t) + \frac{\alpha_2 w_5}{4} b_4(t)\right], \tag{5}$$

$$Tb_2(t,\alpha_1,\alpha_2) = \frac{1}{\sum_{i=0}^{5} w_i\, b_i(t)}\left[\frac{\alpha_1 w_0}{4} b_1(t) + \left(-\frac{\alpha_1 w_0}{12} + \frac{2}{3}\alpha_1 w_1\right) b_2(t) + \left(-\frac{\alpha_2 w_5}{6} + w_3\right) b_3(t) + w_4 b_4(t) + w_5 b_5(t)\right],$$

$$Tb_3(t,\alpha_1,\alpha_2) = \frac{1}{\sum_{i=0}^{5} w_i\, b_i(t)}\left[\left(\frac{\alpha_2 w_5}{4} - \frac{2}{3}\alpha_2 w_4\right) b_3(t) - \frac{\alpha_2 w_5}{4} b_1(t)\right],$$

It is easy to proof that $\sum_{i=0}^{3} Tb_i(t,\alpha_1,\alpha_2) = 1.$

V. Shape Preserving Interpolation Spline Curves

Given interpolation points V$i \in R^d$(d = 2; 3; i = 0; 1; · · ·; n), knot vector U = (u_1; u_2; · · · ; u_{n-1}),and shape parameters α_i(i = 1; 2; · · · ; n − 1), where, $u_1 < u_2 < \cdots < u_{n-1}$, and $\alpha_i \in [0,+\infty)$.For i = 1; 2; · · ·; n − 2, the *i*-th rational quartic trigonometric parametric curve segment is

$$Q_i(t,\alpha_i,\alpha_{i+1}) = \sum_{j=0}^{3} Tb_i(t,\alpha_i,\alpha_{i+1}) V_{i+j-1}, \quad 0 \le t \le \pi/2, \tag{6}$$

Where $Tb_i(t,\alpha_i,\alpha_{i+1})$ is given in Eq. (5). The corresponding rational quartic trigonometric parametric spline curve composed by all of the trigonometric parametric curve segments are defined as follows:

$$Q(u) = Q_i\left(\frac{\pi}{2}.\frac{u-u_i}{\Delta u_i}, \alpha_i, \alpha_{i+1}\right), u \in [u_i, u_{i+1}] \tag{7}$$

Where $\Delta u_i = u_{i+1} - u_i$ and i = 1,2,…n – 1.

Theorem 1: The spline curve Q(u) has F^3continuity at the inner knots u_i; i = 1; 2; · · · ; n−1.

Proof: Consider the continuity at the knot u_{i+1}. For $u \in [u_i, u_{i+1}]$, $t = \frac{\pi}{2}.\frac{u-u_i}{\Delta u_i}$, we have

$$Q^{(k)}(u)=\left(\frac{\pi}{2}\cdot\frac{1}{\Delta u_i}\right)^{(k)} Q_i^{(k)}\left(t,\alpha_i,\alpha_{i+1}\right), k=0,1,2,3,$$

By simple calculations we have

$$\begin{aligned}
&Q\left(u_{i+1}^{+}\right)=Q\left(u_{i+1}^{-}\right)=Q_{i+1}\\
&Q^{(1)}\left(u_{i+1}^{+}\right)=\frac{\Delta u_i}{\Delta u_{i+1}}Q^{(1)}\left(u_{i+1}^{-}\right)\\
&Q^{(2)}\left(u_{i+1}^{+}\right)=\left(\frac{\Delta u_i}{\Delta u_{i+1}}\right)^2 Q^{(2)}\left(u_{i+1}^{-}\right)\\
&Q^{(3)}\left(u_{i+1}^{+}\right)=\left(\frac{\Delta u_i}{\Delta u_{i+1}}\right)^3 Q^{(3)}\left(u_{i+1}^{-}\right)-\frac{4\pi(\Delta u_i)^2}{(\Delta u_{i+1})^3}Q^{(2)}\left(u_{i+1}^{-}\right)
\end{aligned} \tag{8}$$

From here, the theorem follows at the knot u_{i+1}. We can deal with other knots in the same way.

From Eq. (7) and Eq. (8), it is clear that Q(u) interpolates the interpolation points $Q_i(i=1; 2; \cdot\cdot\cdot; n-1)$. To generate an open curve Q(u) interpolating all of the points $Q_i(i=0; 1; 2; \cdot\cdot\cdot; n)$, we can add two control interpolation points Q_{-1}; Q_{n+1}, two knots u_0; u_n, and two shape parameters α_0; α_n. For generating a closed curve Q(u) interpolating all of the points $P_i(i=0; 1; 2; \cdot\cdot\cdot; n)$, we can add three interpolation points $Q_{-1}=Q_n$; $Q_{n+1}=Q_0$; $Q_{n+2}=Q_1$, three knots u_0; u_n; u_{n+1}, and three shape parameters $\alpha_0;\alpha_n;\alpha_{n+1}$.

VI. Numerical Examples and Applications

Taking into account the various applications of B-spline functions, it can see that the properties of B-spline functions mentioned above can be useful in solving some problem related to approximation theory, numerical analysis or computer graphics, for example representation of splines. To compare our computed results and justify the accuracy and efficiency of our presented trigonometric functions we consider the following examples. Figure 4and Figure 5show open and closed trigonometric polynomial planar curves generated by using the shape preserving trigonometric Interpolation spline curves. Figure 6 and Figure 7 show that glass model and Flower model.

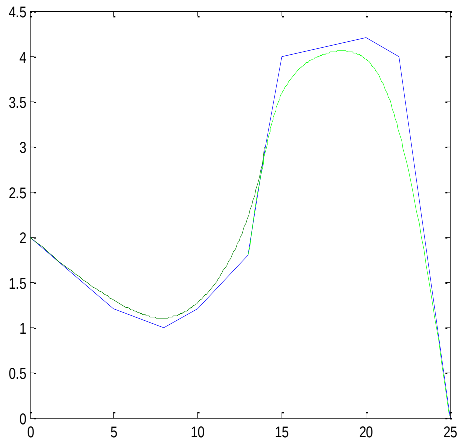

Fig. (4) Open curve

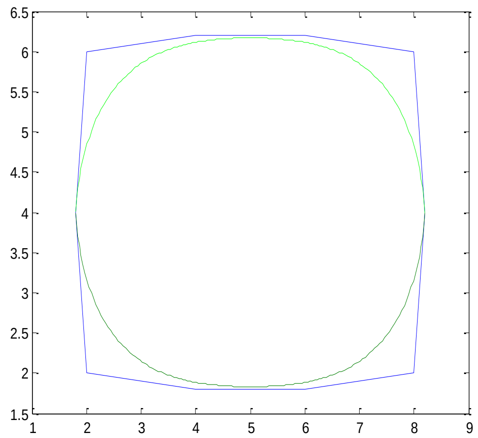

Fig. (5) Closed curve

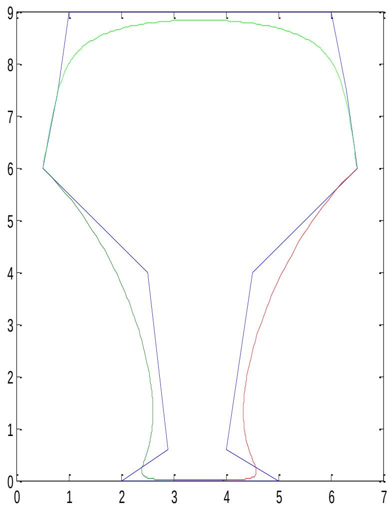

Fig. (6) Wine glass model

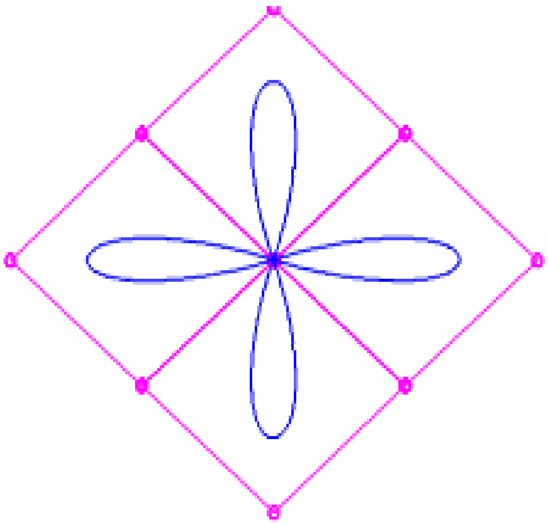

Fig. (7) Flower model

VII. Conclusion

As mentioned above Rational Quartic Trigonometric Bézier curve has all the geometric properties that classical quartic Bézier curves have. The shape of the curve can be flexibly controlled by the shape parameter without altering the control points. Since there is nearly no difference in structure between a Rational Quartic Trigonometric Bézier curve and a classical quartic Bézier curve, it is not difficult to adapt a Rational Quartic Trigonometric Bézier curve to a CAD/CAM system that already uses the classical quartic Bézier curves.The Rational quartic trigonometric polynomial blending functions constructed in this paper have the properties analogous to that of the quarticBernstein basis functions. And the Rational quartic trigonometric B´ezier curves are also analogous to the quartic B´ezier curves. Specially, Rational quartic trigonometric B´ezier curves are closer to the control polygon than the quarticand quintic B´ezier curves. Therefore, therational quartic trigonometric B´ezier curves can better preserve the shape of the control polygon. For any shape parameters satisfying the shape preserving conditions, the obtained shape preservingRational quartic trigonometric interpolation spline curves are all F^3 continuous. Although the shape preserving property is discussed on planar curves, the numerical example indicates that our method can be also applied to generate nice feature preserving space curves.

References

1. J. Schoenberg, 1964, On Trigonometric Spline Interpolation, J. Math. Mech., Vol. 13, pp. 795-825.
2. T. Lyche, R. Winther, 1979, A Stable recurrence relation for trigonometric B-splines, J. Approx. Theory, Vol. 25, pp. 266-279.
3. X. Han, 2012, Quartic Trigonometric B_ezier Curves and ShapePreserving Interpolation, Journal of Computational Information Systems 8: 2 (2012) 905–914
4. X. Han, 2004, Cubic Trigonometric Polynomial Curves with a Shape Parameter, Computer Aided Geometric Design, Vol. 21, pp. 535-548.
5. X. Han, 2006, Quadratic trigonometric polynomial curves concerning local control, Applied Numerical Mathe-matics, Vol. 56, pp. 105-115.
6. X. A. Han, Y. C. Ma, X. L. Huang, 2009, The Cubic Trigonometric Bézier Curve with Two Shape Parameters, Applied MathematicalLetters, Vol. 22, pp. 226-231.
7. Xi-An Han, YiChen Ma, XiLi Huang, 2008, A novel generalization of Bézier curve and surface, Journal of Computational and AppliedMathematics, 217, pp. 180-193.
8. Cheng Wei Wang, Jul.2007, Cubic Trigonometric Polynomial Spline Curves with Shape Parameter, Journal of Beijing Institute of Clothing Technology, vol.28, pp. 50-55.
9. Benyue Su and Youdu Huang, 2005, Properties and Applications of T-B Splines, College Mathematics, 21(1), pp.87-90 (in Chinese).
10. Xuli Han, 2011, A class of general quartic spline curves with shape parameters, Computer Aided Geometric Design, 28, pp. 151-163.
11 Lanlan Yan, Jiongfeng Liang, 2011, A Class of Algebraic-Trigonometric Blended Splines, Journal of Computa-tional and AppliedMathematics, 235, pp. 1713-1729.
12. Huayong Liu, Lu Li, Daming Zhang, 2011, Study on a Class of T-C Bézier Curve with Shape Parameters, Jour-nal of Information andComputational Science, 8: 7, pp. 1217-1223.
13. Wei Xiang Xu, Liu Qiang Wang, Xu Min Liu, 2011, Quadratic TC-Bézier Curves with Shape Parameter, Advanced Materials Research,vols. 179-180, pp. 1187-1192.

14. L.L. Yan, X.B. Yang, L.Z. Song, 2008, Bézier Curves with Two Shape Parameters, Journal of Engineering Graphics, vol.29, pp. 88-92.
15. Xiaoqin Wu, Xuli Han, 2007, Cubic Trigonometric Polynomial Spline Curves with a Shape Parameter, Com-puter Applications andSoftware, vol.24, pp.62-64.
16. XiaoqinWu, Xuli Han, Shanming Luo, 2008, Quadratic Trigonometric Polynomial Bézier Curves with a Shape Parameter, Journal ofEngineering Graphics, vol.29, pp. 82-87.
17. Xi-An Han, XiLi Huang, YiChen Ma, 2010, Shape Analysis of Cubic Trigonometric Bézier Curves with a Shape Parameter, AppliedMathematics and Computation, 217, pp.2527-2533.
18. L.Yan and J. Liang, An extension of the Bézier model, Applied Mathematics and Computation, vol. 218, no. 6, pp. 2863-2879.
19. Mridula Dube, Reenu Sharma, 2013, Quartic Trigonometric Bézier Curve with a shape parameter, International Journal of Mathematicsand Computer Applications Research (IJMCAR), Vol. 3, Issue 3, Aug 2013, pp. 89-96.
20. Qin Xinqiang, Shen Xiaoli, Hu Gang, Shape modification for quartic C-Bézier curves [J]. Computer Engineering and Applications, 2014,50(13): 178-181.
21. Y. Zhu, X. Han and J. Han, Quartic Trigonometric Bézier Curves and Shape Preserving Interpolation Curves, Journal of ComputationalInformation Systems, 8 (2),(2012) 905-914.