A Study of quasi C-reducible Finsler space

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Abstract: The main aim of this paper is to study the theory of quasi C-reducible Finsler space and T tensor in a quasi C-reducible Finsler space. In this paper we have obtained some important results.

Keywords: Quasi C-reducibility, Finsler space, T tensor, Landsberg space, hv-torsion tensor.

I. Introduction:
Let n-dimensional Finsler space be $F^n (M^n, L)$ with fundamental function is $L(x, y)$ and underlying manifold is $M^n$.

The metric tensor $g_{ij}$ of Finsler space $F^n$ is defined as

$$g_{ij} = \frac{1}{2} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} L^2$$

$$h_{ij} = g_{ij} - 1, 1_{ij}$$

II. Quasi C-Reducibility In Finsler Space:

Definition 2.1:
If the C–tensor $C_{ijk}$ of any two dimensional Finsler space can be written the form

$$(2.1) \quad C_{ijk} = \left(\frac{1}{C^2}\right) C_i^j C_k^k$$

Then the Finsler space $F^n$ $(n\geq2)$ is termed as C-2 like Finsler space.

Definition 2.2:
A Finsler space $F^n$ is said to be C-reducible, if the following three conditions holds good

(i) Finsler space $F^n$ is not Riemannian,
(ii) The dimension is more than two,
(iii) the (h)hv-torsion tensor $C_{ijk}$ of $F^n$ may be written as

$$(2.2) \quad C_{ijk} = \left\{1/(n+1)\right\} \left[ h_{ij} C_{k}^{k} + h_{jk} C_{i}^{i} + h_{ki} C_{j}^{j} \right]$$

Wherein

$$(2.3) \quad h_{ij} = g_{ij} - 1_{ij} 1_{ij}$$

and $C_{i}^{i}$ is the contracted torsion tensor.
Definition 2.3:
An n-dimensional Finsler Space $F^n$ is said to be S-3 like Finsler space, if the v-curvature tensor $S_{ijkl}$ is of the form

\[(2.4) \quad S_{ijkl} = \left\{1/(n-1)(n-2)\right\} S \left( h_{il} h_{jk} - h_{ik} h_{jl} \right)\]

Definition 2.4:
An n-dimensional Finsler Space $F^n$ is said to be S-4 like Finsler space, if the v-curvature tensor $S_{ijkl}$ is written as

\[(2.5) \quad S_{ijkl} = \left( h_{il} Q_{jk} + h_{jk} Q_{il} - h_{ik} Q_{jl} - h_{jl} Q_{ik} \right)\]

Definition 2.5:
A Finsler space $F^n$ is said to be quasi C-reducible, if a Finsler space of n-dimensional whose hv-torsion tensor $C_{ijk}$ is written in the form

\[(2.6) \quad C_{ijk} = \left( B_{ij} C_{ik} + B_{jk} C_{i} + B_{ki} C_{j} \right)\]

Wherein $C_{i}$ is a covariant vector and $B_{ij}$ is a symmetric tensor.

Contracting equation (2.6) with $g_{jk}$ yields

\[(2.7) \quad (1 - B)C_{i} = 2 B_{ij} C_{j}\]

Wherein

\[(2.8) \quad B = B_{ij} g^{ij}\]

If we take $(1 - B)/2 = \mu$ then equation (2.7) becomes

\[(2.9) \quad B_{ij} C_{j} = \mu C_{i}\]

If an n-dimensional Finsler space is C-reducible then $B_{ij}$ becomes [10]

If an n-dimensional Finsler space is C-2 like then $B_{ij}$ becomes in the form [10]

\[(2.11) \quad B_{ij} = \left\{1/3C^2\right\} C_{i} C_{j}\]

Contracting equation (2.9) by $g_{jk}$, we get

\[(2.12) \quad B_{ik} C_{j} = \mu g_{jk} C_{i}\]

Contracting equation (2.9) by $g^{ik}$, we get

\[(2.13) \quad B^{jk} C_{j} = \mu C^{k}\]
Contracting equation (2.10) by $g^i_j$ we obtain

\[ (2.14) \quad g^i_j \cdot h_{ij} = (n + 1) B \]

Contracting equation (2.11) by $g^i_j$, we get

\[ (2.15) \quad C_i^i = 3BC^2 \]

From equations (2.14) and (2.15), we obtain

\[ (2.16) \quad C_i^i = \left\{ \frac{3}{n + 1} \right\} C^2 g^i_j \cdot h_{ij} \]

Contracting equation (2.16) by $g^i_j$ yields

\[ (2.17) \quad C_i^j = \left\{ \frac{3n}{(n + 1)} \right\} C^2 h_{ij} \]

III. \quad T TENSOR IN A QUASI C-REDUCIBLE FINSLER SPACE:

The T tensor in quasi C-reducible Finsler space $F^n$ is defined as

\[ (3.1) \quad C_{ijk}, l = B_{jk,l} C_i^i + B_{ki,l} C_j^j + B_{ij,l} C_k^k \]

Contracting equation (3.1) by $g^{jk}$, we get

\[ (3.2) \quad C_i^i = B_{i,j} C_i^i + B_{i,k} C_j^j + B_{i,l} C_k^k \]

Using the relation $C_i^i = 0$, then the equation (3.2) reduces in the form

\[ (3.3) \quad B_{i,j} C_i^i + B_{i,k} C_j^j + B_{i,l} C_k^k = 0 \]

Contracting equation (3.3) with $C_i^i$ and using the equation (2.15), we get

\[ (3.4) \quad 3B C^2 B_{i,j} + 2B_{i,j} C^i C_j^j + = 0 \]

IV. \quad THE LANDSBERG SPACES IN A QUASI C-REDUCIBLE FINSLER SPACE:

Quasi C-reducible Finsler space $C_{ijk,l}$ is the h-covariant derivative of $C_{ijk}$ and it is expressed as
(4.1) \[ C_{ijkl} = \left( B_{ijk} + B_{jkl} + B_{klj} + B_{lji} \right) \]

We have by Berwald’s space

(4.2) \[ C_{ijkl} = 0 \]

And

(4.3) \[ C_{i,kl} = 0 \]

In view of equations (4.2) and (4.3), equation (4.1) assumes the form

(4.4) \[ B_{ijk} + B_{jkl} + B_{klj} + B_{lji} = 0 \]

Contracting equation (4.4) by \( g_{kn} \) yields

(4.5) \[ B_{ij} + B_{j}^{n} + B_{i}^{k} + B_{k}^{l} = 0 \]

Next, contracting equation (4.5) with \( C^{i}_{j} \) and using equation (2.15), we obtain

(4.6) \[ B_{ij} + C^{k}_{j} + 3B^{n}_{j} + B^{k}_{i} + B^{l}_{j} = 0 \]

In view of relation \( B_{ij} + C^{i}_{j} = -\left(1/2\right)B_{i}^{l} + C_{j}^{l} \), the equation (4.6) assumes the form

(4.7) \[ B_{ij} + C^{i}_{j} = \left(1/3BC^{2}\right)B_{i}^{l} + C_{j}^{l} = 0 \]

Contracting equation (4.7) by \( g^{ik} \) yields

(4.8) \[ B_{jk} = \left(1/3BC^{2}\right)B_{j}^{k} \]

**Theorem 4.1:**

If the n-dimensional Finsler space \( F^{n} \) is C-reducible as well as C-2 like then the following relation

\[ B_{jk} = \left\{n/(n+1)\right\}B^{-1}_{j} \quad h_{jk} \]

Holds good.

**Proof:**

From equations (2.17) and (4.8)

(4.9) \[ B_{jk} = \left\{n/(n+1)\right\}B^{-1}_{j} \quad h_{jk} \]

**REFERENCES:**