Special Pythagorean Triangles in Connection with the Narcissistic Numbers of Order 3 and 4

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Abstract: We aim to find special pythagorean triangles in connections with the Narcissistic number of order 3 and 4. Also we present the number of primitive and non-primitive pythagorean triangles.

Keywords: Narcissistic number, primitive and non-primitive, pythagorean triangles

I. Introduction

Number theory is about properties of natural numbers, integers or rational numbers. The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true.

For various problems and ideas one may refer [1-3]. Always there is someone to one correspondence between pythagorean triangles and special numbers. From [4] to [11] one can understand the relations between pythagorean triangles and special polygonal numbers.

In [12], [13] special relations between pythagorean triangles with Jarasandha numbers and Harshad numbers are studied.

Recently in [14] special pairs of pythagorean triangles with Narcissistic numbers are obtained.

In this paper we aim to find special pythagorean triangles in connection with the Narcissistic numbers of order 3 and 4. We have presented pythagorean triangles each with a leg, hypotenuse, generators represented by 3rd and 4th order narcissistic numbers 153, 370, 371 and 8208 respectively and some expressions representing the considered narcissistic numbers.

Also we present the number of primitive and non-primitive pythagorean triangles.

II. Basic Definitions

Definition 2.1: Narcissistic Numbers:

An n-digit number which is the sum of nth power of its digits is called an n-narcissistic number. It is also known as Armstrong number.

Definition 2.2:

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle.

Definition 2.3:

Most cited solution of the Pythagorean equation is \( x = m^2 - n^2, y = 2mn, z = m^2 + n^2 \) where m, n are generators (m > n > 0). This solution is called primitive, if m, n are of opposite parity and gcd (m, n) = 1.

III. Method of Analysis

Case 1:

1. \( x = 153 \) (3rd order narcissistic number), i.e., \( m^2 - n^2 \) = narcissistic number

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>( x^2 )</th>
<th>( y^2 )</th>
<th>( x^2 + y^2 = z^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>4</td>
<td>153</td>
<td>104</td>
<td>185</td>
<td>23409</td>
<td>10816</td>
<td>34225</td>
</tr>
<tr>
<td>27</td>
<td>24</td>
<td>153</td>
<td>1296</td>
<td>1305</td>
<td>23409</td>
<td>1679616</td>
<td>1703025</td>
</tr>
<tr>
<td>77</td>
<td>76</td>
<td>153</td>
<td>11704</td>
<td>11705</td>
<td>23409</td>
<td>136983616</td>
<td>137007025</td>
</tr>
</tbody>
</table>

Out of these 3 pythagorean triangles 2 are primitive and one is non-primitive.

2. \( x = 370 \) is not possible, since 370 cannot be written as difference of two square numbers.

3. \( x = 371 \) (3rd order narcissistic number), i.e., \( m^2 - n^2 \) = narcissistic number
Note that since 371 and 8208 cannot be written as sum of two squares in this case all the pythagorean triangles are non-primitive.

Case 2:

In this case both the triangles are primitive.

4. \( x = 8208 \) (4\(^{th}\) order narcissistic number) ie, \( m^2-n^2 = \) narcissistic number

\[
\begin{array}{cccccccc}
\text{m} & \text{n} & \text{x} & \text{y} & \text{z} & \text{x}^2 & \text{y}^2 & \text{x}^2+\text{y}^2 = \text{z}^2 \\
30 & 23 & 371 & 1380 & 1429 & 137641 & 190440 & 2042041 \\
186 & 185 & 371 & 68820 & 68821 & 137641 & 4736192400 & 4736330041 \\
\end{array}
\]

In this case all the pythagorean triangles are non-primitive.

Case 3:

In this case both the numbers 370 and 8208 are only considered, since 153,371 cannot be written in the form of \( 2mn \)

1. \( y = 370 \) (3\(^{rd}\) order narcissistic number) ie, \( 2mn = \) narcissistic number

\[
\begin{array}{cccccccc}
\text{m} & \text{n} & \text{x} & \text{y} & \text{z} & \text{x}^2 & \text{y}^2 & \text{x}^2+\text{y}^2 = \text{z}^2 \\
92 & 16 & 8208 & 2944 & 8720 & 67371264 & 8667136 & 76038400 \\
93 & 21 & 8208 & 3906 & 9090 & 67371264 & 15256836 & 82628100 \\
103 & 49 & 8208 & 10094 & 13010 & 67371264 & 101888836 & 169260100 \\
127 & 89 & 8208 & 22606 & 24050 & 67371264 & 51103126 & 578402500 \\
132 & 96 & 8208 & 25344 & 26640 & 67371264 & 642318336 & 709689600 \\
183 & 159 & 8208 & 58194 & 58770 & 67371264 & 3386541636 & 3453912900 \\
237 & 219 & 8208 & 103806 & 104130 & 67371264 & 10775685636 & 10843056900 \\
348 & 336 & 8208 & 233856 & 234000 & 67371264 & 5468628736 & 54756000000 \\
517 & 509 & 8208 & 526306 & 526370 & 67371264 & 276998005636 & 277065376900 \\
687 & 681 & 8208 & 935694 & 935730 & 67371264 & 875522613636 & 875956329000 \\
1028 & 1024 & 8208 & 2105344 & 2105360 & 67371264 & 4432473358336 & 4432540729600 \\
2053 & 2051 & 8208 & 8421406 & 8421410 & 67371264 & 70920079016836 & 70920146388100 \\
\end{array}
\]

Out of these 11 pythagorean triangles 3 are primitive and the remaining are non-primitive.

Case 3:

In this case all the pythagorean triangles are non-primitive

Note that since 371 and 8208 cannot be written as sum of two squares, \( z \neq 371 \cdot 8208 \)
Case 4: The generators m, n are narcissistic numbers.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>(x^2 + y^2 = z^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>370</td>
<td>153</td>
<td>113491</td>
<td>113220</td>
<td>160309</td>
<td>25698975481</td>
</tr>
<tr>
<td>371</td>
<td>153</td>
<td>114232</td>
<td>113526</td>
<td>161050</td>
<td>25937102500</td>
</tr>
<tr>
<td>8208</td>
<td>153</td>
<td>67347855</td>
<td>2511648</td>
<td>67394673</td>
<td>454204194876930</td>
</tr>
<tr>
<td>8208</td>
<td>370</td>
<td>67234364</td>
<td>6073920</td>
<td>67508164</td>
<td>4557352206650900</td>
</tr>
<tr>
<td>8208</td>
<td>371</td>
<td>6723623</td>
<td>6900336</td>
<td>67508905</td>
<td>4557452254299020</td>
</tr>
</tbody>
</table>

Out of these 5 pythagorean triangles 2 triangles are primitive and the remaining are non-primitive.

Case 5: The expressions a). \(\frac{4A}{P} - y + z, \ x\) b). \(x - \frac{2A}{P} \cdot \frac{1}{2} [z + x - y]\) c). \(\frac{2A}{P} \cdot \frac{1}{2} [y + x - z]\) represents the narcissistic number 153 which is presented below in the Table (a), Table (b), Table (c) respectively.

Table (a)

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>4</td>
<td>153</td>
<td>104</td>
<td>185</td>
</tr>
<tr>
<td>27</td>
<td>24</td>
<td>153</td>
<td>1296</td>
<td>1305</td>
</tr>
<tr>
<td>77</td>
<td>76</td>
<td>153</td>
<td>11704</td>
<td>11705</td>
</tr>
</tbody>
</table>

Out of these 3 pythagorean triangles 2 triangles are primitive and one is non-primitive.

Table (b)

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>3</td>
<td>2907</td>
<td>324</td>
<td>2925</td>
</tr>
<tr>
<td>26</td>
<td>9</td>
<td>595</td>
<td>468</td>
<td>757</td>
</tr>
<tr>
<td>26</td>
<td>17</td>
<td>387</td>
<td>884</td>
<td>965</td>
</tr>
<tr>
<td>54</td>
<td>51</td>
<td>315</td>
<td>5508</td>
<td>5517</td>
</tr>
<tr>
<td>154</td>
<td>153</td>
<td>307</td>
<td>47124</td>
<td>47125</td>
</tr>
<tr>
<td>154</td>
<td>1</td>
<td>23715</td>
<td>308</td>
<td>23717</td>
</tr>
</tbody>
</table>

Out of these 6 pythagorean triangles, 4 triangles are primitive and 2 are non-primitive.

Table (c)

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>8</td>
<td>225</td>
<td>272</td>
<td>353</td>
</tr>
<tr>
<td>51</td>
<td>48</td>
<td>297</td>
<td>4896</td>
<td>4905</td>
</tr>
</tbody>
</table>

Out of these 2 pythagorean triangles one triangle is primitive and the other is non-primitive.

For simplicity, in Table (d), we exhibit the connections between special pythagorean triangles and the other narcissistic numbers 370, 371 and 8208 respectively.

Table (d)

<table>
<thead>
<tr>
<th>Narcissistic Number</th>
<th>Expressions</th>
<th>No. of Triangles</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>370</td>
<td>(x - \frac{2A}{P} \cdot \frac{1}{2} [z + x - y])</td>
<td>4</td>
<td>2-primitive 2-non-primitive</td>
</tr>
<tr>
<td>371</td>
<td>(\frac{4A}{P} - y + z, \ x)</td>
<td>2</td>
<td>2-primitive</td>
</tr>
<tr>
<td>8208</td>
<td>(x - \frac{2A}{P} \cdot \frac{1}{2} [z + x - y])</td>
<td>12</td>
<td>12-non-primitive</td>
</tr>
</tbody>
</table>

IV. Conclusion

One can also search for connections between the Pythagorean triangles and other Narcissistic number of higher orders.
References