Teaching Topic “Ellipse” with the Help of Mathematics Software

“GeoGebra”: A Try Out

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Abstract: Topic “Ellipse” is in Geometry 10 curriculum for Vietnamese high schools; mathematics contents of the topic are difficult for learners to remember. In order to increase the quality of teaching this topic, we used the GeoGebra as a teaching tool. In this paper, we report the how to apply the GeoGebra dynamic software to teaching contents related to Ellipse to 10th grade students and the results of this application. The remarkable findings obtained were:
(1) Students’ learning outcomes were relatively high;
(2) Students’ attitude towards teaching “Ellipse” with GeoGebra was positive.

Keywords: GeoGebra, dynamic mathematics software, mathematics education, Ellipse, educational technology, ICT.

I. An introduction to GeoGebra

GeoGebra - dynamic software
GeoGebra is dynamic mathematics software. This software was developed by Hohenwarter in 2001. About characteristics of GeoGebra, Hohenwarter, M. and Lavicza, Z. (2007) showed that “The basic idea of software development was created a dynamic software that incorporates geometry, algebra, and calculus, which other packages treat separately (spreadsheet and computer algebra extensions are soon to be added to the software), into a single easy-to-use package”. [2]

Applications of GeoGebra in schools
According to M. Hohenwarter and K. Fuchs (2004), GeoGebra offered the following applications in teaching mathematics in schools [1]:
1. GeoGebra can be used for demonstration and visualization;
2. GeoGebra is a construction tool;
3. GeoGebra can be used for discovering mathematics;
4. GeoGebra can be used for preparing teaching materials.

II. Statement of research problem
Towards applications of GeoGebra to teaching mathematics, we had some works published such as: Loc and Triet (2014), Loc (2014), Loc and Phuong (2015), Loc and Tuan (2015) (see [3]-[7]). Also studying the use of this software to teaching school mathematics, we continued to do an experiment on teaching topic “Ellipse” with the following research questions:
1. The 1st question: What are students’ opinions on learning “Ellipse” with the help of GeoGebra?
2. The 2nd question: Is teaching “Ellipse” with assistance of GeoGebra to students in schools effective?

III. Methodology
Mathematics contents taught experimentally:
Topic “Ellipse” is in Geometry 10 – Vietnam, which consists of:
1. Definition of an ellipse (time for teaching: 50 minutes);
2. The shape of an ellipse (time for teaching: 50 minutes).
3. Teaching strategies with GeoGebra
In experimental teaching, we made use of the following three applications of GeoGebra:
1. GeoGebra was used for demonstration and visualization (see Illustration 1);
2. GeoGebra was used as a construction tool (see Illustration 2);
3. GeoGebra was used for discovering mathematics (see Illustration 3).

Illustration 1: A real image of an ellipse

Illustration 2: How to draw an ellipse

<table>
<thead>
<tr>
<th>$\mu_M$</th>
<th>$MF_1$</th>
<th>$MF_2$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.858</td>
<td>1.738</td>
<td>8.262</td>
<td>1.738</td>
<td>8.262</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.737</td>
<td>8.263</td>
<td>1.737</td>
<td>8.263</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.736</td>
<td>8.264</td>
<td>1.736</td>
<td>8.264</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.735</td>
<td>8.265</td>
<td>1.735</td>
<td>8.265</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.734</td>
<td>8.266</td>
<td>1.734</td>
<td>8.266</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.733</td>
<td>8.267</td>
<td>1.733</td>
<td>8.267</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.732</td>
<td>8.268</td>
<td>1.732</td>
<td>8.268</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.731</td>
<td>8.269</td>
<td>1.731</td>
<td>8.269</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.730</td>
<td>8.270</td>
<td>1.730</td>
<td>8.270</td>
</tr>
<tr>
<td>-4.858</td>
<td>1.729</td>
<td>8.271</td>
<td>1.729</td>
<td>8.271</td>
</tr>
</tbody>
</table>

$T_1 = a + \frac{e \cdot \mu_M}{a}$

$T_2 = a - \frac{e \cdot \mu_M}{a}$

Illustration 3: How to discover the formula for computing $MF_1$ and $MF_2$

Experimental model: See Table 1.

Table 1: Experimental model of teaching topic “Ellipse” with GeoGebra

<table>
<thead>
<tr>
<th>Experimental class</th>
<th>Treatment (teaching with GeoGebra, denoted X)</th>
<th>Posttest (Mathematics test scores, and attitudes of students, denoted O)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 10A, Class 10B1</td>
<td>X</td>
<td>O</td>
</tr>
</tbody>
</table>

Experimental classes:
- Class 10A of 40 students - “Trung học cơ sở và trung học phổ thông Long Phú” School (Tam Bình, Vĩnh Long).
- Class 10B1 of 40 students - “Trung học thực hành sự phạm” School (Can Tho University).

After finishing 1st semester of academic year 2014-2015, the learning level of the above students was relatively high (see Table 2).

Table 2. The learning ranks of students in two experimental classes

<table>
<thead>
<tr>
<th>Class 10A</th>
<th>Class 10B1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very good</td>
<td>36 (students)</td>
</tr>
<tr>
<td>Good</td>
<td>4</td>
</tr>
<tr>
<td>medium</td>
<td>0</td>
</tr>
<tr>
<td>Weak</td>
<td>0</td>
</tr>
<tr>
<td>Sum</td>
<td>40</td>
</tr>
</tbody>
</table>

Posttest:
- **Questionnaire**: We used questionnaire to find out the students’ opinions towards lessons with the help of GeoGebra.
- **Test of students’ learning outcomes**: In order to evaluate how effective our teaching was, we asked the students to do a test of ten questions which we constructed after doing experiment (see Appendix)

The pedagogical experiment was implemented in the second semester of academic year 2014 -2015.
IV. Results and discussion

A. The results of investigating students’ opinions

Students’ opinions on the lessons with GeoGebra

The opinions of 80 students in two experimental classes towards lessons with the help of GeoGebra were showed in Table 3. Accordingly, a majority of students (80%) of the two experimental classes said that they felt interesting in learning with using GeoGebra. In particular, about 70% of students felt understandable after two experimental lessons although nearly 50% of students said that the content of the topic “Ellipse” was difficult.

Table 3. Students’ opinions on the lessons with GeoGebra

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Do you feel interesting to study mathematics with the help of GeoGebra?</td>
<td>35 (43.75%)</td>
<td>29 (36.25%)</td>
<td>15 (18.75%)</td>
<td>1 (1.25%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>b. How difficult is topic “Ellipse”?</td>
<td>1 (1.25%)</td>
<td>36 (45%)</td>
<td>28 (35%)</td>
<td>15 (18.75%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>c. After studying Ellipse with the help of GeoGebra, what is your level of understanding this topic?</td>
<td>12 (15%)</td>
<td>47 (58.75%)</td>
<td>18 (22.5%)</td>
<td>3 (3.75%)</td>
<td>0 (0%)</td>
</tr>
</tbody>
</table>

Note: Numbers (1) – (5) indicate the following contents:
- (1): Very interesting (a); very difficult (b); very good (c).
- (2): Interesting (a); difficult (b); good (c).
- (3): Normal (a, b); medium (c).
- (4): Not interesting (a); Easy (b); bad (c).
- (5): Not interesting completely (a); very easy (b); very bad (c).

Students’ opinions on the GeoGebra software

When asked to comment on GeoGebra, 90% of students said that this software was interesting, had many useful features but not unwieldy; and only a small number of children (about 15%) felt that the software was difficult to use and its interface was complicated. Results of student surveys about GeoGebra were detailed in Table 4:

Table 4. Statistical table of students’ opinions about software GeoGebra

<table>
<thead>
<tr>
<th></th>
<th>Strongly agreed</th>
<th>Agreed</th>
<th>undecided</th>
<th>disagreed</th>
<th>Strongly disagreed</th>
</tr>
</thead>
<tbody>
<tr>
<td>GeoGebra is an interesting software</td>
<td>28 (35%)</td>
<td>47 (58.75%)</td>
<td>5 (6.25%)</td>
<td>0 (0%)</td>
<td>28 (0%)</td>
</tr>
<tr>
<td>GeoGebra is helpful for self-study</td>
<td>29 (36.25%)</td>
<td>47 (58.75%)</td>
<td>4 (5%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>GeoGebra has many useful functions</td>
<td>30 (37.5%)</td>
<td>43 (53.75%)</td>
<td>7 (8.75%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>It is difficult to operate GeoGebra</td>
<td>1 (1.25%)</td>
<td>10 (12.5%)</td>
<td>38 (47.5%)</td>
<td>27 (33.75%)</td>
<td>4 (5%)</td>
</tr>
<tr>
<td>Interface of GeoGera is complicated</td>
<td>1 (1.25%)</td>
<td>9 (11.25%)</td>
<td>28 (35%)</td>
<td>38 (47.5%)</td>
<td>4 (5%)</td>
</tr>
<tr>
<td>Other opinions</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
</tbody>
</table>

B. The results of mathematics test

The students’ score results after they finished the test are shown in Table 5.

Table 5. Statistical table of students’ score totals

<table>
<thead>
<tr>
<th>Scale of 15</th>
<th>Scale of 10</th>
<th>The number of student obtaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>14</td>
<td>9.33</td>
<td>19</td>
</tr>
<tr>
<td>13</td>
<td>8.67</td>
<td>13</td>
</tr>
<tr>
<td>12</td>
<td>8.0</td>
<td>9</td>
</tr>
<tr>
<td>11</td>
<td>7.33</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>6.67</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>6.0</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>5.33</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>4.67</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3.33</td>
<td>1</td>
</tr>
<tr>
<td>Average mark: 8.4</td>
<td>Sum: 80</td>
<td></td>
</tr>
</tbody>
</table>
The reliability of the test

To calculate the reliability of the test, we divided test into two groups: group A of the even numbered questions and group B of the odd numbered questions. Meanwhile, the correlation coefficient between the test scores of two groups is \( R(A, B) = 0.56 \). According to Spearman-Brown Prophecy formula, the reliability of the test is

\[
\frac{2R(A, B)}{1 + R(A, B)} = 0.72, \text{ which is good for a small number of test items (10 items)}.
\]

Comment

Test results show that the majority of students obtained high scores: 60% of students obtained the number of mark from 8 to 10 (according to scale of 10); only 0.05% of students obtained the number of mark under 5. This suggested that most students could master the relatively less lesson contents. In addition, the reliability of the test reached 0.72, so these facts could be trusted.

C. Experiences of the process of experimental teaching

Through experimental lessons, we had the following experiences:

- Each lesson was completed exactly according to the specified time. This indicated that the use of software GeoGebra in teaching process did not affect the time for lesson.
- In the experimental lessons, although some students were not active, but when asked, the majority of them could answer the teacher’s questions correctly. Moreover, students could discover the typical features of an ellipse while observing the dynamic figures which offered by GeoGebra

V. Conclusion

From the results received from the pedagogic experiment, we believe that the application of GeoGebra to teaching topic “Ellipse” in particular and teaching mathematics in general is feasible and effective. So, in Vietnam, GeoGebra should be introduced to mathematics teachers of schools as a measure for innovating teaching methods of mathematics.

VI. References


Appendix: QUESTIONS FOR TESTING (in Vietnamese)

Cho trước hai điểm \( F_1, F_2 \) với \( F_1F_2 = 2c(c > 0) \) và một số \( a > c \). Hãy sử dụng giá trị này cho câu 1 và câu 2.

1. Elip (E) được định nghĩa là:
   a. Tấp hợp các điểm \( M \) sao cho \( MF_1 + MF_2 = a \)
   b. Tấp hợp các điểm \( M \) sao cho \( MF_1 + MF_2 = 2a \)
   c. Tấp hợp các điểm \( M \) sao cho \( MF_1 + MF_2 = a^2 \)
   d. Tấp hợp các điểm \( M \) sao cho \( MF_1 + MF_2 = 4a \)

2. Tiêu cự của elip (E) có độ dài là:
   \( a \)        \( b \)        \( c \)        \( 2a \)        \( 2c \)

Trong hệ trực toạ Oxy, cho elip (E) có hai điểm \( F_1(-c; 0), F_2(c; 0) \) với \( c \) là một số dương cho trước và một điểm \( M(x; y) \) thuộc (E). Hãy sử dụng giá trị này cho câu 3 và câu 4.

3. Hai bán kính qua tiêu \( MF_1, MF_2 \) của (E) có công thức tính độ dài lần lượt là:
   a. \( MF_1 = a + \frac{cx}{a} \) ; \( MF_2 = a - \frac{cx}{a} \)
   b. \( MF_1 = a + cx \) ; \( MF_2 = a - cx \)

References


\[ c. \quad MF_1 = a - \frac{cx}{a}; \quad MF_2 = a + \frac{cx}{a} \quad d. \quad MF_1 = c + \frac{ax}{c}; \quad MF_2 = c - \frac{ax}{c} \]

4. Hai bán kính qua tiêu \( MF_1 \) và \( MF_2 \) của (E) có giá trị bằng nhau khi:
   a. \( M \) là giao điểm của trực hoành với (E)
   b. \( M \) là giao điểm của trực tung với (E)
   c. \( MF_1 \) vuông góc với trực hoành
   d. \( MF_2 \) vuông góc với trực hoành

Trong hệ tọa độ Oxy, cho elip (E) có phương trình chính tác là \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) và có hai tiêu điểm \( F_1(-c; 0), F_2(c; 0). \) Sử dụng giả thiết này cho các câu: 5-6-7.

5. Mô liên hệ giữa các số đường \( a, b, c \) của elip (E) là:
   a. \( a > b > c \)
   b. \( a > b \); \( a > c \); \( b^2 = a^2 + c^2 \)
   c. \( a > c > b \)
   d. \( a > b \); \( a > c \); \( b^2 = a^2 - c^2 \)

6. Hãy chọn phát biểu đúng trong các phát biểu sau:
   a. Elip (E) có vô số trực đi xứng.
   b. Elip (E) có 2 trực đi xứng nhưng không có tâm đi xứng.
   c. Elip (E) không có tâm đi xứng.
   d. Elip (E) có 1 tâm đi xứng và 2 trực đi xứng.

7. Công thức xác định tâm sai của (E) là:
   a. \( e = \frac{a}{c} \)
   b. \( e = \frac{b}{c} \)
   c. \( e = \frac{b}{a} \)
   d. \( e = \frac{c}{a} \)

8. Trong các phương trình sau, phương trình nào là phương trình chính tác của một elip:
   a. \( \frac{x^2}{9} + \frac{y^2}{16} = 1 \)
   b. \( \frac{x^2}{4} + y^2 = 1 \)
   c. \( 16x^2 + 25y^2 = 400 \)
   d. \( x^2 + y^2 = 1 \)

9. Phương trình chính tác của elip có tâm sai \( e = \frac{2}{5} \) là:
   a. \( \frac{x^2}{5} + \frac{y^2}{\sqrt{21}} = 1 \)
   b. \( \frac{x^2}{25} + \frac{y^2}{21} = 1 \)
   c. \( \frac{x^2}{25} + \frac{y^2}{\sqrt{21}} = 1 \)
   d. \( \frac{x^2}{25} + \frac{y^2}{4} = 1 \)

10. Cho elip (E) có phương trình chính tác \( \frac{x^2}{100} + \frac{y^2}{64} = 1 \). Khí đó:
   i. Tọa độ hai tiêu điểm là: \( F_1(____;____) \) và \( F_2(____;____) \).
   ii. Tiêu cự của (E) là: ____.
   iii. Đô đại trực lớn là: ____.
   iv. Đô đại trực bé là: ____.
   v. Các đỉnh của (E) là: (____;____); (____;____); (____;____); (____;____).
   vi. Tầm sai của (E) là: \( e = ____ \).