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Types of Task related to "Tangent to a Curve" Concept for 11th Grade Students in Vietnam: A Study based on the Anthropological Theory of the Didactic

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***Abstract**: Topic "the tangent to a curve" is one of main contents of Calculus. The problem "find the tangent to the graph of a function" was mentioned in the curriculum of Algebra and Calculus for 11th Grade in Vietnam when students begin to study "derivative" concept, geometrical meanings of derivative. We approached to the Anthropological Theory of the Didactic which considered a model of school mathematical activity as a "praxeology" consisting of a practical clock (know-how) and theoretical block (knowledge) to study the types of task related to "tangent to a curve" concept in Algebra and Calculus for 11th grade students in Vietnam. The results were that all the types of task in textbooks only had only mathematical contents. It showed that Vietnamese mathematics educators paid little attention to realistic mathematics in teaching mathematics to secondary school students. In addition, another investigation indicated that 11th grade students in Vietnam found it impossible to determine the tangent to the graph of a function defined implicitly.*

***Keywords**: Praxeology, mathematics organization, the tangent to a curve, teaching calculus, mathematics education, anthropological theory of the didactic.*

I. Background

A mathematics task

In [2], Gahamanyi defined a mathematical task as follows:

> *A mathematical task* can be viewed, in general terms, as any piece of mathematical work to be done by an individual or a group. In mathematics education, especially in teaching-learning context, a mathematical task normally refers to mathematical work or problems that are assigned to students, teachers or other concerned people (such as parents and mathematics curriculum makers) to be performed for the purpose of societal knowledge development in the subject of mathematics.

The role of mathematics task

Mathematical tasks play a core role to mathematics learners in the sense that they convey messages about "what doing mathematics entails" [6]. Mathematical tasks provide the contexts in which they learn to think about mathematics, and different tasks may place differing cognitive demands on students [5].

Mathematical praxeology

Chevallard considered a mathematical activity as a human activity situated in an institutional setting and any mathematics activity can be subsumed as a system of tasks [1]. According to the anthropological theory of the didactic, a model of mathematical activity (especially including school mathematical activity) in terms of praxeology [1], denoted by $[T, \tau, \theta, \Theta]$, has four main components: T - type of task (or problem), τ - technique used to solve the problem, θ - technology referring to the technique used and Θ – theory establishing profound justifications of the technology [1]. In other words, a mathematical praxeology consists of a practical block or "know-how" (the praxis) integrating types of problems and techniques used to solve them, along with a theoretical block or "knowledge" (the logos) integrating both the technological and the theoretical discourse used to describe, explain and justify the practical block. To clarify the model $[T, \tau, \theta, \Theta]$, we described it by the following diagram (see Figure 1).

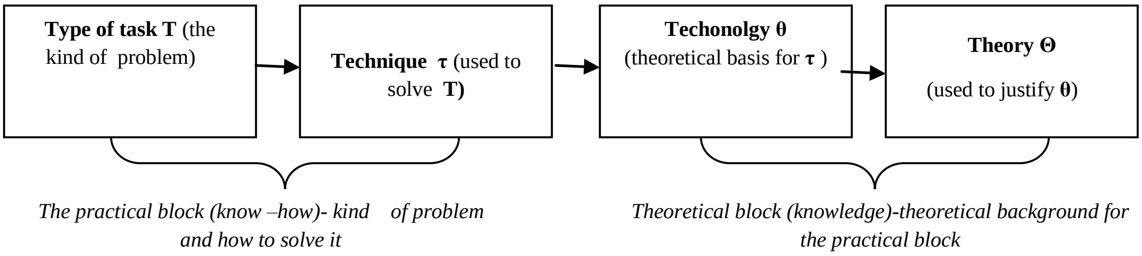

Figure 1. The structure of a praxeology

II. Statement of research problem

As we knew, topic "the tangent to a curve" is one of main contents of Calculus. In Vietnam, the problem "find the tangent to the graph of a function" was mentioned in the curriculum of Algebra and Calculus for 11^{th} Grade when students begin to study "derivative" concept, geometrical meanings of derivative. These contents are presented in the textbook "Đại số và Giải tích 11" (Algebra and Calculus 11) [3] as follows:

Definition of derivative:

Given function $y = f(x)$ that is defined on interval $(a; b)$ and $x_0 \in (a; b)$.

If the limit $\lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$ exists (finite), then the limit is called the derivative of the function $y = f(x)$ at x_0 and denoted by $f'(x_0)$ (or $y'(x_0)$); therefore

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}. \text{ [3]}$$

Geometrical meanings of derivative:

Theorem 2: The derivative of a function $y = f(x)$ at x_0 is the slope of the tangent M_0T of (C) at point $M(x_0; f(x_0))$, where (C) is the graph of the function. [3]

The tangent equation:

Theorem 3: The equation of tangent of the graph (C) of a function $y = f(x)$ at point $M(x_0; f(x_0))$ is

$$y - y_0 = f'(x_0)(x - x_0), \text{ where } y_0 = f(x_0). \text{ [3]}$$

The question is that "What kinds of tangent problem are introduced to students of 11^{th} grade in Vietnam?".To answer the above question, we conducted the study with the below two research questions:

The research question 1: Which types of task related to the tangent to the graph of a function were introduced to 11^{th} grade students in Vietnam ?

The research question 2: In the case of y as a function of x defined implicitly, can students find the tangent to the graph of the function at a given point?

III. Methodology

Content analysis: we analyzed two official materials for students and mathematics teachers: Textbook "Algebra and Calculus 11" (M_{11}) [3], the book "Exercises of Algebra and Calculus 11" (E_{11}) [4].

Survey: we surveyed theopinions of 11^{th} grade students towards how to find the tangent of the graph of a function in the case of the function defined implicitly. Particularly, we required students to answer the following two questions:

Question 2.1: Given $f(x) = 2x^2 - 1$ and $g(x) = f(x) + 3$. In your opinion, can we write the equation of tangent of the graph of function $y= g(x)$ at $M(-1; 4)$? Why?

Question 2.2: Given the function $y = f(x)$, where x and y satisfy the relation:

$$(y-1)x^2 - (x^2+1)x^3 + y - 1 = 0.$$

Can we write the tangent to the graph of the function? Why?

Students surveyed: 57 students who are studying in the Secondary School "Vĩnh Trạch", Thoại Sơn District, An Giang Province, Vietnam.

IV. Results and discussion

A. Types of task related to the tangent to the graph of a function in 11^{th} mathematics curriculum in secondary schools of Vietnam *(the answer to the research question 1)*

Type of task T_1: Write the equation of a tangent to the graph (C) of a function: $y=f(x)$ at a point M_0 $(x_0; y_0)$

T_1 consists of types of subtask: T_{1a}, T_{1b}, T_{1c} and T_{1d}:

T_{1a}: Write the equation of a tangent to the graph (C) of a function: y=f(x) at point with x= x_0.
τ_{1a}:

Step 1: Compute f(x_0);
Step 2: Find f'(x) and compute f'(x_0);
Step 3: The equation of the tangent is $y = f'(x_0)(x - x_0)$ +y_0.

θ_{1a}: Apply theorem 3 (M_{11}, p. 152).
Θ_{1a}: Derivative and geometrical meanings of derivative.

***Illustration*(T_{1a} ; τ_{1a}):** Write the equation of the tangent to (H): y= $y = \frac{1}{x}$ at point with x = -1. [Exercise (Ex) 6b, M_{11}, p. 156].

T_{1b}: Write the equation of a tangent to the graph (C) of a function: y=f(x) at a point M_0 whose ordinate is y_0
τ_{1b}:

Step 1: Solve the equation f(x) = y_0; (1)
Step 2: Find f'(x);
Step 3: Compute $f'(x_0)$, where x_0 is the root of (1);
Step 4: The equation of the tangent is $y = f'(x_0)(x - x_0)$ +y_0.

θ_{1b}: Apply theorem 3 (M_{11}, p. 152).
Θ_{1b}: Derivative and geometrical meanings of derivative.

***Illustration* (T_{1b} ; τ_{1b}):** Write the equation of the tangent to (P): $y = x^2 - 4x + 4$ at point with y = 1. [Ex 7c, M_{11}, p. 176].

T_{1c}: Write the equation of a tangent to the graph (C) of a function: y=f(x) at a point M_0 (x_0; f(x_0)).
τ_{1c}:

Step 1: Find f'(x); then compute $f'(x_0)$.
Step 2: The equation of the tangent is $y = f'(x_0)(x - x_0)$ +y_0.

θ_{1c}: Apply theorem 3 (M_{11}, p. 152).
Θ_{1c}: Derivative and geometrical meanings of derivative.

***Illustration* (T_{1c} ; τ_{1c}):** Write the equation of the tangent to (C): $y = x^3$ at point (-1; -1). [Ex 5a, M_{11}, p. 156].

T_{1d}: Write the equation of tangents to the graph (C): y=f(x) and (L): y=g(x) at their intersection.
τ_{1d}:

Step 1: Solve the equation f $(x) = g(x)$; (1)
Step 2: Compute f(x_0) (= g(x_0)), where x_0 is the root of (1);
Step 3: Find f'(x) and g(x); then compute $f'(x_0)$ and $g'(x_0)$;
Step 4: $y - y_0 = f'(x_0)(x - x_0)$.

θ_{1d}:: Apply theorem 3 (M_{11}, p. 152).
Θ_{1d}: Derivative and geometrical meanings of derivative.

Illustration* (T_{1d} ; τ_{1d}): *Given two functions: $y = \frac{1}{x\sqrt{2}}$ và $y = \frac{x^2}{\sqrt{2}}$. Write the equation of the tangent to the graph of each function at their intersection. Calculate the angle of the above two tangents. [Ex 9, M_{11}, tr. 177].

Type of task T_2: Write the equation of tangent to the graph (C): y=f(x) with the slope k
Types of subtask T_2:
T_{2a}: Write the equation of tangent to the graph (C): y=f(x) with the slope k = k_0.
τ_{2a}:

Step 1:: Find $f'(x)$;
Step 2: Solve the equation $f'(x) = k_0$ (1);
Step 3: Compute f(x_0), where x_0 is the root of (1);
Step 4: The equation of the tangent is $y = k_0(x - x_0)$ +y_0.

θ_{2a}: Apply theorem 3 (M_{11}, p. 152).
Θ_{2a}: Derivative and geometrical meanings of derivative.

***Illustration* (T_{2a} ; τ_{2a}):** Write the equation of the tangent to (C): $y = x^3$ where the slope of the tangent is 3. [Ex 5c, M_{11}, p. 156]

T_{2b}: Write the equation of a straight line both tangent to the graph (C) of a function: y=f(x) and perpendicular to the (d): y= ax + b.
τ_{2b}:

Step 1: Find $f'(x)$;
Step 2: Solve the equation $f'(x) = -\frac{1}{a}$ (1);
Step 3: Compute f(x_0), where x_0 is the root of (1);
Step 4: The equation of the tangent is $y = k_0(x - x_0)$ +y_0.

θ_{2b}: Apply theorem 3 (M_{11}, p. 152).
Θ_{2b}: Derivative and geometrical meanings of derivative.

> ***Illustration*** **$(T_{2b}\,;\ \tau_{2b})$*: Given function*** $y = -x^4 - x^2 + 6$. Write the equation of the tangent to the graph of the function perpendicular to the line $y = \frac{1}{6}x - 1$. [Ex 28c, E_{11}, p. 234]

T_3: Find the slope of the tangent to the graph of a function y = f(x) at x_0.
τ_3:

Step 1: Find $f'(x)$ and compute $f'(x_0)$ and $f(x_0)$;
Step 2: Conclude: $f'(x_0)$ is the slope of the tangent.

θ_3: Apply theorem 2 (M_{11}, p. 151).
Θ_3: Derivative and geometrical meanings of derivative.

> ***Illustration*** **$(T_{3a}\,;\ \tau_{3a})$**: Find the slope of the tangent to the graph of function $y = \tan x$ at point with x= $\frac{\pi}{4}$.
> [Ex 7, E_{11}, p. 215]

T_4: Find all points, if any, on the graph of y = f(x) where the slope equals k.
τ_4:

Step 1: Find $f'(x)$;
Step 2: Solve the equation $f'(x) = k$ (1);
Step 3: if x_0 is a root of (1), M $(x_0\ , f(x_0))$ is a solution.

θ_4: Apply theorem 2 (M_{11}, p. 151).
Θ_4: Derivative and geometrical meanings of derivative.

> ***Illustration*** **$(T_{4a}\,;\ \tau_{4a})$**: Find all points, if any, on the graph of $y = 4x^2 - 6x + 3$ where the tangent line is parallel to the line $y = 2x$. [Ex 8, E_{11}, p. 215]

T_5: Find the angle of the graph y= f(x) and x – axis.
τ_5:

Step 1: Find the intersection of the graph of the function y = f(x) and horizontal axis by solving the equation f(x) = 0 (1);
Step 2: If x_0 is a root of (1), the $tan\varphi$ = f'(x_0), where φ is the angle of the graph of the function y = f(x) and horizontal axis.

θ_5: Apply theorem 3 (M_{11}, p. 152).
Θ_5: Derivative and geometrical meanings of derivative.

> ***Illustration*** **$(T_5\,;\ \tau_5)$**: Compute the angle of the graph of the function y = $\frac{1}{\sqrt{3}}\sin 3x$ and x- axis at the origin O – the intersection of the graph of the function and x - axis? [Ex 9, E_{11}, p. 215]

Type of task T_6: Find the angle of two tangent lines
τ_6:

Step 1: Write the equations of two tangent lines;

Step 2: Apply the formula:

$$\cos\varphi = \frac{|\overrightarrow{n_1}.\overrightarrow{n_2}|}{|\overrightarrow{n_1}.||\overrightarrow{n_2}|}$$

where φ is a angle of two tangent lines and $\overrightarrow{n_1}, \overrightarrow{n_2}$ are normal vectors of the tangent lines.

θ_6: Apply theorem 3 (M_{11}, p. 152); formula for computing the angle of lines.
Θ_6: Derivative and geometrical meanings of derivative and definition of two lines.

Illustration $(\boldsymbol{T_6}\,;\,\boldsymbol{\tau_6})$***:*** Given two functions: $y = \frac{1}{x\sqrt{2}}$ và $y = \frac{x^2}{\sqrt{2}}$. Write the equation of tangent of the graph of each function at their intersection. Compute the angle of these tangents. [Ex 9, M_{11}, p. 177].

*Type of task T_7***:**Prove a property related the tangent to the graph (C) of a function y = f(x)

τ_7:

Step 1: Write the equation of tangent line at any point $M_0(x_0\,;f(x_0))$;

Step 2: Prove that the tangent line has the property required.

θ_7: Apply theorem 3 (M_{11}, p. 152).

Θ_7: Derivative and geometrical meanings of derivative.

Illustration $(\boldsymbol{T_7}\,;\,\boldsymbol{\tau_7})$***:*** Prove that the tangent to the hyperbola $y = \frac{a^2}{x}$ at any point together with x- axis and y-axis make a triangle having the constant area.[Ex11, E_{11}, p. 215].

Table 1. The number of types of task in M_{11} and E_{11}

Type of task		**Technique**	**The number of exercises in M_{11}**	**The number of exercises in E_{11}**	**Sum**
T_1	T_{1a}	$\boldsymbol{\tau_{1a}}$	6	4	10
	T_{1b}	$\boldsymbol{\tau_{1b}}$	1	0	1
	T_{1c}	$\boldsymbol{\tau_{1c}}$	2	2	4
	T_{1d}	$\boldsymbol{\tau_{1d}}$	1	0	1
T_2	T_{2a}	$\boldsymbol{\tau_{2a}}$	2	2	4
	T_{2b}	$\boldsymbol{\tau_{2b}}$	0	1	1
T_3		$\boldsymbol{\tau_3}$	0	1	1
T_4		$\boldsymbol{\tau_4}$	0	1	1
T_5		$\boldsymbol{\tau_5}$	0	2	2
T_6		$\boldsymbol{\tau_6}$	2	0	2
T_7		$\boldsymbol{\tau_7}$	0	2	2
Sum			**14**	**15**	**29**

From Table 1, we see that:

(1) The type of task T1 is mentioned much more in M11 and E11 than the other types of task. Besides, some types of task only are in M_{11} but not in E_{11} and vice versa. Thus, these two books complement each other to help students to know many different kinds of problems related to the tangent to a curve.

(2) All the above types of task only have only mathematical contents. It shows that Vietnamese mathematics educators pay little attention to realistic mathematics in teaching mathematics to secondary school students.

B. A limitation of students in finding the equation of the tangent of an implicit function *(the answer to the research question 2)*

Table 2 shows that only 10.53% students both gave the right answers to the question 2.1 and could explain correctly their answers to this question. For the question 2.2, all students could not both give the right answer and explain correctly their answers to this question. Therefore, we could conclude that the students found it confused in finding the tangent to the graph of an implicit function.

Table 2: Students' opinions on the solution of the equation of the tangent of an implicit function

Question	**The number of student (N=57)**				
	Choosing right answer and explaining correctly	Choosing right answer and not explaining	Choosing right answer but explaining wrongly	Choosing wrong answer	Not answering
The question 2.1	6 10.53%	23 40.35%	17 29.82%	4 7.02%	7 12.28%
The question 2.2	0 0 %	13 22.81%	7 12.28%	16 28.07%	21 36.84%

V. Conclusion

The results of this study showed that:

- Vietnamese mathematics educators focused much attention on pure mathematics problems; they paid attention little on realistic mathematics in teaching mathematics to secondary school students.
- In addition, in Vietnam, secondary school students found it confused in determining the tangent to the graph of a function defined implicitly.

Besides, an approach to the anthropological theory of the didactic to study learning – teaching materials which were published by curriculum designers and textbook authors could help teachers to figure out the core types of mathematical task in a school curriculum.

VI. References

[1] Y. Chevallard (1999). L 'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, 19(2), 221–266.
[2] M. Gahamanyi (2010). A Study of Mathematical Organisations in Rwandan Workplaces and Educational Settings. *Linköping Studies in Behavioural Science* No. 150 Linköping University, Department of Behavioural Sciences and Learning. Linköping 2010
[3] T.V. Hao et al (2011). *Algebra and Calculus 11*. Hanoi: Education publishing house. (in Vietnamese).
[4] T.V.Hao et al (2011). *Exercises for Algebra and Calculus 11*. Hanoi: Education publishing house. (in Vietnamese).
[5] M. Henningsen & M.K.Stein (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibithigh-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28(5), 524-549.
[6] National Council of Teachers of Mathematics (NCTM). (1991).*Professional standards for teaching mathematics*. Reston, VA: Author.