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Glauber Modeling in Heavy Ion Collisions

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Abstract: *With increasing search of critical point at RHIC fluctuations in flow, Glauber model provides a baseline over which effects of fluctuations start gaining importance. Glauber model is used to have a quantitative idea of the geometrical configuration of nuclei, when nuclei collide at ultra relativistic energies, and treats the collision as a sequence of nucleon-nucleon collision. This method is able to calculate the number of participating nucleons and the number of binary collisions analytically, for a given impact parameter and centre of mass energy. To compare the results to real experimental data, the nuclear density profile and inelastic nucleon-nucleon cross-section is given as an input to the model. Glauber model comes into two variants-The Optical Glauber model, which assumes smooth matter density described by Fermi distribution function, and The Monte Carlo Glauber model which analyses the quantities mentioned, by considering individual nucleons as distributed stochastically event by event.*

This analytical study of the flow mechanism of heavy ion collisions with the help of Glauber model is highly significant for the study of high energy collisions. This work will also help scientific society for the search of parameters yet not explored at high energy.

Keywords: *Heavy Ion collisions, Quark Gluon Plasma, Glauber Modeling, fluctuations, Global variables.*

I. Introduction

The Glauber model was developed by Roy Glauber (Nobel Prize winner in Physics, 2005) for the analytical study of high energy collusions. Glauber model is used to calculate the geometric parameters in the initial state of heavy ion collisions. This model provides a quantitative idea of geometrical configuration of nuclei, when they collide at high energies. It is based on the assumption that baryon-baryon interaction cross-section is constant throughout and nuclei move along the direction of collision in a straight line path. It helps to determine the number of participating nucleons, initial eccentricity, the number of binary collisions among the nucleons for the two nuclei, colliding with fixed energy and impact parameter, and obey the nuclear density distribution. This model falls into two main classes-

The Optical Glauber model is one in which a smooth matter density is assumed, described by Fermi distribution function in the radial direction and uniform over solid angle. The Monte Carlo based model considers that individual nucleons are stochastically distributed event by event and the collision properties are calculated by averaging over multiple events.

Both the model provide mostly similar results for number of participating nucleons and impact parameter, but give different results in the quantities, where event by event fluctuations are significant. This is a semi classical model and to compare its results with the experimental data, few model inputs are required like nuclear density profile of colliding nuclei and the energy dependence of inelastic nucleon-nucleon cross-section.

Few model inputs for calculations:

Nuclear charge density: The nucleon density inside the nucleus in Wood-Saxon form

$$\rho(r) = \frac{\rho_0}{1+e^{\frac{r-R}{a}}} \qquad (1)$$

Where R: Nuclear radius.

a : Diffuseness parameter, which measures how fast the nuclear density falls off at the nuclear surface and ρ_0 can be calculated by normalization condition.

For a spherical nucleus-

$$\int \rho . dv = \int_0^{\infty} 4\pi r^2 \rho(r). dr = A \quad . \qquad (2)$$

Where A is the mass number

Following formula [5] can express nuclear radius R in terms of A as-

$$R = 1.12\,A^{1/3} - \frac{0.86}{A^{1/3}} \quad (3)$$

From electron scattering experiment more realistic form of ρ (r) is given as-

$$\rho(r) = \frac{c_1+c_2r+c_3r^2}{1+e^{(r-R)/a}} \quad (4)$$

Where c_1, c_2 & c_3 are the parameters which has different values for different nuclei.
Sometimes foe deformed nuclei (nuclei which are not symmetric) a deformed Wood-Saxon profile is used [6]

$$\rho(\mathrm{r}) = \frac{\rho_0}{1+e^{(r-R')/a}} \quad (5)$$

Where R' =R $[1 + \beta_2 Y_2^0(\theta) + \beta_4 Y_4^0(\theta)]$ (6)

Here $Y_l^m(\theta)$ indicates the spherical harmonics, the polar angle with respect to the symmetry axis is θ.
Inelastic nucleon-nucleon cross-section: The model is based on the assumption of nucleons inelastically and on an average the number of charged particle produced in each collision remains same. Also the static cross-section is assumed to be independent of nuclear environment and same as that of single p+p collision and interchangeability of proton and neutron is inherent to the model.
The experimentally measured values of inelastic nucleon-nucleon cross-section (σ_{inel}^{NN}) is used as input. This provides for the only non-trivial dependence of the Glauber calculations on the beam energy [7][8].

i. Optical Glauber Model

Semi classical concepts are used to get the dependence of the number of participating nucleons and the number of binary nucleon collisions, on the impact parameter, for a given nuclear density profile and inelastic nucleon-nucleon cross-section. When the target and projectile heavy ions collide with small impact parameter it is called central collision and when takes place with large impact parameter, termed as peripheral collisions. Consider two heavy ions, target (A) and projectile (B) colliding at relativistic speed with an impact parameter (b) as shown in Fig. 1.

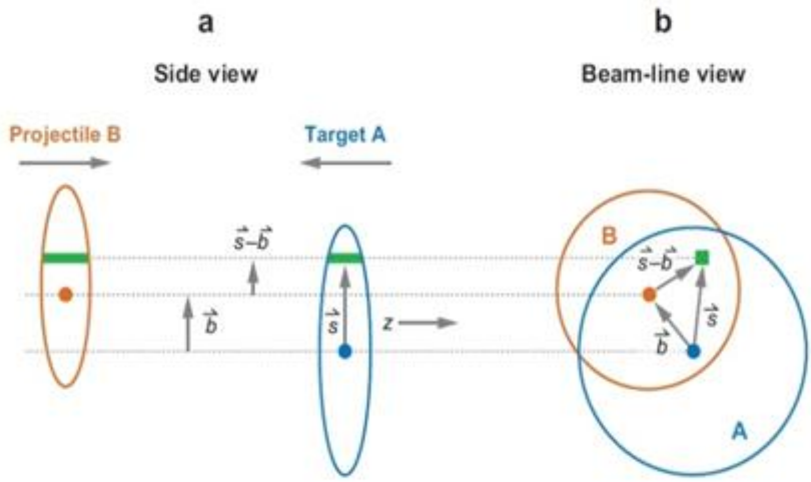

Fig. 1 Schematic representation of the optical Glauber model geometry, with transverse (a) and longitudinal (b) views.

Let two flux tubes are there at a displacement s and (s-b) from the centre of target and projectile nuclei respectively. Then the probability per unit transverse area of a nucleon being located in the flux tube can be given by the nucleus thickness function as-

$$T_{\frac{A}{B}}(b) = \int \rho_{\frac{A}{B}}(b, Z_{A/B})\, dZ_{A/B} \quad (7)$$

Where $\rho_{\frac{A}{B}}(b, Z_{A/B})$ is the probability of finding the nucleon at the point (s,$Z_{A/B}$) per unit volume in projectile (A) or target (B) nucleus, normalized to unity. Thus the joint probability per unit area of finding nucleons in the respective overlapping region is defined as thickness function & given as-

$$T_{\frac{A}{B}}(b) = \int T_A(s).T_B(s-b)\,d^2s \quad (8)$$

The number of inelastic nucleon-nucleon collision is given as[3]-

$$N_{coll}(b) = \sigma_{incl}^{NN}.T_{\frac{A}{B}}(b) \quad (9)$$

The probability of n elastic collisions at an impact parameter b and when summed over all probabilities, we get,

$$\sum_{n=1}^{A} P(n,b) = 1 - \exp\,[-\sigma_{incl}^{NN}.\,T_A\,(b)] \quad (10)$$

The number of participants in nucleus A is proportional to the nuclear profile function $T_A\,(s)$, at transverse position s, weighted by the sum over the probability for a nucleon-nucleon collision at transverse position (b-s) in nucleus B.

So the number of participants at a given impact parameter b

$$N_{part}(b) = \int T_A(s)(1 - \exp[-\sigma_{incl}^{NN}.\,T_B\,(b-s)]).\,ds + \int T_B(b-s)\,(1 - \exp[-\sigma_{incl}^{NN}.\,T_A\,(b)]).\,ds \quad (11)$$

Hence the number of participants and the number of binary collisions decrease with increasing impact parameter.

From geometric consideration it is estimated as –

$$N_{coll}\ \alpha\ N_{part}^{x} \quad (12)$$

Here

N_{Coll}: Number of binary collisions.

N_{Part}: Number of participating nucleons.

When the value of x is calculated theoretically and experimentally the values are matched well.

To calculate the integration required for the calculation of N_{coll} and N_{part} *in* Glauber Model, trapezoidal integration and Monte Carlo integration [10] methods are used. These quantities can be measured directly by experiments, so this method helps to connect the theoretical calculations and experimentally measured observables. But it is failed to locate the nucleons at specific coordinates.

ii. Monte Carlo based Model

In this model individual nucleons are considered to be randomly distributed event by event and collision quantities are calculated by averaging over multiple events.(Event means collision of two nuclei.)

The following steps are used to relate the number of participating nucleons and the number of binary collisions with the impact parameter (b),of the nucleon-nucleon collision.

1. The impact parameter is randomly selected from the distribution, when N is the number of events and (b) is impact parameter-

$$\frac{dN}{db}\ \alpha\ b \quad (13)$$

2. The nucleons are distributed in accordance with the given nuclear density distribution in nucleus, for a given impact parameter. The radial part is $r^2\rho(r)$, where r is the radial distance of nucleon from the centre of nucleus. The polar part is weighted bysin θ, where θ is the polar angle of nucleon have range (0,π), the azimuthal part is from (0, 2π). So the elementary volume in spherical polar coordinates system is given as 4 $\pi r^2 dr \sin\theta\, d\theta$ d$\emptyset$.

3. The centres of two nuclei are shifted to (-b/2, 0, 0) and (b/2, 0, 0) respectively.

4. Two nucleons from different nuclei collide if transverse distance (d) between them is

$$d \leq \sqrt{\sigma_{inel}^{NN}/\pi} \quad (14)$$

5. For each event the total number of binary collisions N_{coll} is calculated by the sum of individual number of collisions, and the total number of participating nucleons N_{part} is the number of nucleons that interact only once.

Here the calculations are done in two steps; first the position of nucleons are determined stochastically, and the nucleons are assumed to be moving in straight line along the beam axis for collisions(such that nucleons are participating and spectators.)In quantum mechanical picture the position of each nucleon is determined according to a probability density function, and for that a minimum inter-nucleon separation is required between the centers of nucleons.

The nuclear charge density is usually estimated by a Fermi distribution function with parameters like nucleon density, the nuclear radius and the skin depth. The inelastic nucleon-nucleon cross-section which is a function of collision energy is extracted from p+p collision.

The nucleon from two nuclei is assumed to collide if relative transverse distance is less than ball-diameter (D).

And D is expressed as D = $\sqrt{\frac{\sigma_{NN}}{\pi}}$

Where σ_{NN} is total inelastic nucleon-nucleon cross-section.

II. Summary & Conclusions

This model is very much helpful in understanding the geometrical configuration of nuclei quantitatively, when they collide at high energy considering the collision as a sequence of nucleon-nucleon collision. For a given impact parameter and a given value of centre of mass energy, the number of participating nucleons and the number of binary collisions can be estimated theoretically by this model.

The nuclear density profile and inelastic nucleon-nucleon cross-section are given as an input to the model for comparing it to the experimental data. It is assumed that the static cross-section is same as that for p+p collision and does not depend on nuclear environment. In Optical Model the number of participating nucleons and binary collisions are analytically derived but in Monte Carlo Model it is counted.
For the application, this model is mapped to the number of charged particles produced by defining centrality classes, and it is seen that for Au+Au collision at RHIC For $|n| \leq 0.5$ and energy range 7 GeV to 200 GeV, the charged particle multiplicity is obtained as explained by this simple geometrical model. In non-central collision the reaction volume is elliptic in shape just after the collision, and the pressure gradient decreases from the centre to the ends.
The initial spatial anisotropy is characterized by eccentricity and translates to the momentum anisotropy of produced particles. Experimentally the momentum anisotropy i.e. V_2 is proportional to eccentricity. So with the help of this model by calculating eccentricity the anisotropy in momentum space can be established theoretically.
Glauber model suggests that these system carry very large angular momentum, and from the conservation of momentum this must be transferred the initial angular momentum of Quark-Gluon Plasma. This uncompensated angular momentum may affect the initial longitudinal flow velocity. But still a sound theoretical analysis which includes the effect of large initial angular momentum is required.

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