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BOUNDARY LAYER FLOW AND HEAT TRANSFER OF FLUID-PARTICLE SUSPENSION OVER A VERTICAL STRETCHING SHEET WITH THERMAL CONDUCTIVITY AND RADIATION

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***Abstract**: A boundary layer analysis is presented for the effect of viscous dissipation, thermal conductivity and thermal radiation of a convective flow of a Newtonian dusty fluid past a vertical stretching sheet. The governing partial differential equations are reduced to non linear ordinary differential equations by using similarity transformation. The resultant equations are then solved numerically using Runge-Kutta fourth order method along with shooting technique. The non-dimensional boundary layer profiles as well as the shear stress and rate of wall heat transfer have been investigated with the effect of different physical parameters like Grashof number, Prandtl number, Eckret number and the diffusion parameter. The physics of the problem is well explored for the embedded material parameters through graphs and tables.*

***Keywords**: Boundary layer flow, Two-phase flow, Stretching sheet, Heat transfer, Thermal radiation.*

I. Introduction

Momentum and heat transfer in a boundary layer over a linear stretching /shrinking sheet has considerable interest in the recent and past years because of its over increasing industrial applications such as extrusion of plastic sheets, wire drawing, power and cooling industry for drying chemical industry hot rolling, glass fiber production and important bearings on several technological processes. In particular, in the extrusion of a polymer in a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a thin sheet, and then solidified through quenching or gradual cooling by direct contact with water or coolant liquid. Viscous dissipation changes the temperature distribution by playing a role like an energy source, which leads to affect the rate of heat transfer. The merit of viscous dissipation depends on whether the sheet is being cooled or heated. Such processes occur when the effect of buoyancy forces in free convection becomes significant. If the temperature of the surrounding fluid becomes high, then the thermal radiation effect play a vital role in the case of space technology. So the study of two-dimensional boundary layer viscous flow and heat transfer over a stretching surface with affect of buoyancy force and thermal radiation is very important as it finds a large scale of practical applications in different areas.
The boundary layer flow on a continuously solid stretching surface with various aspects was first investigated by Sakiadis(1961). He considered the boundary layer flow over a flat surface moving with a constant velocity and formulated a boundary layer equation for two dimensional, axisymmetric flows. Due to entertainment of ambient fluid, this phenomenon represents a different type of boundary layer problem having solution substantially different from that of boundary layer flow over semi-infinite flat plate. Crane(1970) extended the work of Sakiadis by considering a moving strip, the velocity of which is proportional to the distance from the slit and obtained closed form exponential solution. Subsequently, many investigators taking the advantage of simplicity of geometry and its exact solution attempted the problem with variety of assumptions. Gupta and Gupta(1977), Carragher and Crane(1982), Dutta et al.(1985) studied the heat transfer in the flow over a stretching surface with different aspects taking into account. Pal and Mondal (2014) analyzed the effects of temperature dependent viscosity and variable thermal conductivity on mixed convection problem by considering the wall heating conditions namely prescribed surface temperature and prescribed wall heat flux over a stretching sheet. They have solved numerically by using the fifth-order Runge-Kutta Fehlberg method with shooting technique.
In the context of space technology and in processes involving high temperature the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers, power plants for interplanetary flight and gas cooled nuclear reactors, have focused attention on thermal

radiation as a mode of energy transfer, and emphasize the need for improved understanding of radiative transfer in these processes. The interaction of radiation with laminar free convection heat transfer from a vertical plate was investigated by Cess (1966)for an absorbing, emitting fluid in the optically thick region, using the singular perturbation technique. Arpaci (1968) considered a similar problem in both the optically thin and optically thick regions and used the approximate integral technique and first order profiles to solve the energy equation. Cheng and Ozisik (1972)considered a related problem for an absorbing, emitting and isotropically scattering fluid and treated the radiation part of the problem exactly with the normal mode expansion technique. Raptis (1998) has analyzed the thermal radiation and free convection flow through a porous medium by using perturbation technique. Hossain and Takhar(1996) studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using Keller Box finite difference method. Raptis and and Perdikis (1999) studied the effects of thermal radiation past a moving vertical plate. Das et al. (1996) have analyzed the radiation effects on the flow past a impulsively started infinite isothermal vertical plate and the governing equations are solved by Laplace transform technique. The natural convection flow with radiation effects past a semi infinite plate was studied by Chamkha et al(2001).

The presence of dust particles in the flow of a viscous fluid has significant effect. The dust particles tend to retard the flow and to decrease the fluid temperature. Such flows are uncounted in a wide variety of engineering problems such as nuclear reactor cooling, rain erosion, paint spraying, transport, waste water treatment and combustion. Saffman (1962) initiated the study of dusty fluids and discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Dutta and Mishra (1980, 1982)have investigated the boundary layer flow of a dusty fluid over a semi infinite flat plate and an oscillating plate. Vajravelu et al (1992)have studied hydro magnetic flow of a dusty fluid over a stretching sheet including the effects of suction. Nandkeolyar and Sibanda(2013)investigated the two-dimensional boundary layer flow of a viscous, incompressible and electrically conducting dusty fluid past a vertical permeable stretching sheet under the influence of transverse magnetic field with the viscous and joule dissipation. Gireesha et. al.(2013) have studied the two-dimensional unsteady mixed convective flow of a dusty fluid over a stretching sheet with thermal radiation and space dependent internal heat generation /absorption. They used the well known RKF45 method to solve the governing equations of both fluid and dust phases. MHD flow and heat transfer of an incompressible dusty fluid over a stretching sheet was investigated by Gireesha et al(2012). The similarity transformations are used to reduce the governing equations and are solved numerically by using Runge – Kutta – Fehlberg fourth – fifth order method(RKF45 Method). Heat transfer effects on dusty gas flow past a semi infinite inclined plate was studied by Palani et al.(2007). Recently Mishra and Tripathy (2011, 2013) have studied the boundary layer flow and heat transfer of two phase flow over a flat plate and wedge.

Motivated by all these investigations, the present study explores the effects of the thermal radiation, thermal buoyancy, thermal conductivity, heat transfer due to fluid-particle interaction, heat added to the system to slip-energy flux in the energy equation of particle phase, heat due to conduction and viscous dissipation in the energy equation, effective volumetric force, fluid -particle interaction, particle –particle interaction, the momentum equation for particulate phase in normal direction have been considered in both the phases for better understanding of the boundary layer characteristics, which was not investigated by previous investigators. The effects of thermal conductivity and thermal radiation are included here; as this is true in some polymer solutions: thermal radiation plays a significant role in controlling the heat transfer in the polymer processing industries. The quality of the final product depends to a great extent on the heat controlling factors, and the knowledge of radiative heat transfer in the system can perhaps lead to a desired product with sought qualities. The governing coupled, non-linear partial differential equations of the flow and heat transfer problem are transferred into non-linear coupled ordinary differential equations by using a similarity transformation. These coupled non-linear ordinary differential equations with variable coefficients subject to the appropriate boundary conditions are solved numerically by using Runge – Kutte fourth order scheme with shooting technique for several sets of values of the physical parameters like finite volume fraction(φ), Grashoff number(Gr), Prandtl number(Pr), Eckret number (Ec), radiation parameter (Ra), diffusion parameter (ϵ) on shear stresses (c_f), wall heat transfer(Nu) and other boundary later characterstics.

II. Mathematical Modeling and solution of the problem

Consider a steady, two- dimensional laminar boundary layer of an incompressible viscous two-phase flow over a vertical linearly stretching sheet in the presence of radiation field. The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow. The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperatureT_∞. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow. The radiative heat flux in the energy equation of both the phases is approximated by Rosseland approximation. Under these above assumptions, the governing equations of the flow and energy fields are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$
$$\frac{\partial}{\partial x}\left(\rho_p u_p\right)+\frac{\partial}{\partial y}\left(\rho_p v_p\right)=0 \quad (2)$$
$$(1-\varphi)\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=(1-\varphi)\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)-(1-\varphi)\rho g\beta^*(T-T_\infty) \quad (3)$$
$$\varphi\rho_s\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+\varphi(\rho_s-\rho)g \quad (4)$$
$$\varphi\rho_s\left(u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(v-v_p\right) \quad (5)$$
$$(1-\varphi)\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=(1-\varphi)\,k\frac{\partial^2 T}{\partial y^2}+\frac{1}{\tau_T}\varphi\rho_s c_s\left(T_p-T\right)$$
$$\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+(1-\varphi)\mu\left(\frac{\partial u}{\partial y}\right)^2-(1-\varphi)\frac{\partial q_{rf}}{\partial y} \quad (6)$$
$$\varphi\,\rho_s c_s\left(u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_T}\varphi\rho_s c_s\left(T_p-T\right)$$
$$-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+\varphi\mu_s\left[u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right]-\varphi\frac{\partial q_{rp}}{\partial y} \quad (7)$$

Where (u,v) and (u_p,v_p) are the velocity components of the fluid and particle phases along the x and y directions respectively and (T,T_p) are the temperature of fluid and particle phase respectively. (ρ,ρ_p) (μ,μ_s) and (κ,κ_s) are the density, coefficient of viscosity and thermal conductivity of the fluid and particle phase respectively. (τ_p,τ_T) are the velocity and thermal equilibrium time of the particle cloud i.e. the time required by the particle cloud to adjust its velocity and temperature relative to the fluid respectively. (c_p,c_s) are the specific heat of fluid and suspended particulate matter(SPM) respectively. $(q_{rf,}\ q_{rp})$ are the radiative heat flux of the fluid and particle phase respectively. φ is the finite volume fraction, ρ_s is the material density of the particle and β^* is the co-efficient of thermal expansion.

Using Rosseland approximation, the radiation heat flux for the fluid phase q_{rf} (Brewster(1972)) is given by

$$q_{rf}=-\frac{4\sigma^*}{3\,\kappa^*}\frac{\partial T^4}{\partial y} \quad (8)$$

Where σ^* and κ^* are Stephan Boltzman constant and mean absorption coefficient respectively.

Here the temperature difference within the flow is assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature T_∞, using a truncated taylor series about the free stream temperature T_∞ to yield

$$T^4=4T_\infty^3\,T-3T_\infty^4 \quad (9)$$

Substituting equation (9) in equation (8), we obtain

$$q_{rf}=-\frac{16T_\infty^3\ \sigma^*}{3\,\kappa^*}\frac{\partial^2 T}{\partial y^2} \quad (10)$$

Similarly, the radiation heat flux for the particle phase (q_{rp}) is given by

$$q_{rp}==-\frac{16T_\infty^3\ \sigma^*}{3\,\kappa^*}\frac{\partial^2 T_p}{\partial y^2} \quad (11)$$

The boundary conditions for the flow problem are given by

$$u=U_w(x),\ v=0,\quad T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2;\quad \text{as } y\to 0 \quad (12a)$$
$$\rho_p=\omega\rho,\ u=0,u_p=0,v_p=v,\ T=T_p=T_\infty;\quad \text{as } y\to\infty \quad (12b)$$

Where $U_w(x)=cx$ is a stretching sheet velocity, c is the initial stretching rate being a positive constant and ω is the density ratio in the main stream. T_w is the wall temperature and A is a positive constant. A is a positive constant, $l=\sqrt{\nu/c}$ is a characteristic length. For most of the gases $\tau_p\approx\tau_T$ if $\frac{c_s}{c_p}=\frac{2}{3Pr}$ and $k_s=k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$.

The equation (1) is identically satisfied through introducing the stream function $\psi(x,y)=\sqrt{c\nu}\,xf(\eta)$, such that $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. We further introduce the following transformations in the equations (2) to (7), to convert the governing equations into a set of similarity equations,

$$u=cxf'(\eta),\ v=-\sqrt{c\nu}f(\eta),\ \eta=\sqrt{c/\nu}\ y,$$
$$u_p=cxF(\eta),\ v_p=\sqrt{c\nu}\,G(\eta),\ \rho_r=\rho_p/\rho=H(\eta)$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}, \quad (13)$$

Where $T-T_\infty=A\left(\frac{x}{l}\right)^2\theta(\eta),\ T_p-T_\infty=A\left(\frac{x}{l}\right)^2\ \theta_p(\eta)$

Substituting the above non-dimensional transformations (13) in (2) to (7), we obtain the following non-linear ordinary differential equations.

$$HF+HG'+GH'=0 \quad (14)$$
$$f'''+ff''-f'^2+\frac{1}{1-\varphi}\beta H(F-f')+Gr\,\theta=0 \quad (15)$$

$F^2 + GF' - \epsilon F'' - \beta(f' - F) + \frac{1}{Fr}\left(1 - \frac{1}{\gamma}\right) = 0$ (16)

$GG' - \epsilon G'' + \beta(f + G) = 0$ (17)

$(1 + Ra)\theta'' - Pr(2f'\theta - f\theta') + \frac{2}{3}\frac{1}{1-\varphi}\beta H(\theta_p - \theta) - \frac{1}{1-\varphi}\beta\, Pr\, Ec\, H(F - f')^2 - PrEcf''^2 = 0$ (18)

$\left(\frac{\epsilon}{Pr} + \frac{3}{2}\frac{Ra}{\gamma}\right)\theta_p'' - 2F\theta_p + G\theta_p' - \beta(\theta_p - \theta) - \frac{3}{2}\beta\, Pr\, Ec\,(f' - F)^2 + \frac{3}{2}\epsilon\, Pr\, Ec\left(FF'' + F'^2\right) = 0$ (19)

with boundary conditions

$G'(\eta) = 0, f(\eta) = 0, f'(\eta) = 1, F'(\eta) = 0, \theta(\eta) = 1, \theta_p' = 0;\ as\ \eta \to 0$ (20a)

$f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta), H(\eta) = \omega, \theta(\eta) \to 0, \theta_p(\eta) \to 0;\ as\ \eta \to \infty$ (20b)

Where prime denotes differentiation with respect to η, $\gamma = \rho_s/\rho$ is the material density of the particle, $\rho_r = \rho_p/\rho$ is the relative density and

$\beta = \frac{1}{c\tau_p}$, is the fluid-particle interaction parameter

$\epsilon = \frac{v_s}{v}$, is the diffusion parameter

$Gr = \frac{g\beta^*(T - T_\infty)}{c^2 x}$, is the local Grassoff number

$Fr = \frac{c^2 x}{g}$, is the local Froude number

$Pr = \frac{\mu c_p}{k}$, is the Prandtl number

$Ec = \frac{c^2 l^2}{A c_p}$, is the Eckret number

$Ra = \frac{16\,\sigma^*\, T_\infty^3}{3\,\kappa^*\,\kappa}$, is the radiation parameter.

The important physical parameter of the present investigation and the boundary layer flow is the local skin friction coefficient c_f is defined as,

$c_f = \frac{\tau_w}{\rho U_w^2}$, (21)

where the skin friction τ_w is given by

$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ (22)

Using the non-dimensional variables,

$c_f\sqrt{Re_x} = f''(0)$ (23)

And the wall heat transfer rate i.e. the local Nusselt number Nu_x is defined as

$Nu_x = \frac{x q_w}{\kappa(T_w - T_\infty)}$ (24)

Where the heat transfer from the sheet q_w is given by

$q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}$ (25)

And using the non-dimensional variables, one obtain

$\frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0)$ (26)

Where $Re_x = \frac{UL}{v}$ is the local Reynolds number.

III. Numerical Simulation

In order to integrate (14) to (19) as an initial value problem, one require the values of $f''(0)$, $F(0)$, $G(0)$, $H(0)$, $\theta'(0)$, $\theta_p(0)$. But no such values are given at the boundary. The most important factor of shooting method is to choose the appropriate finite values for η_∞. In order to determine η_∞, we start with some initial gauss value for some particular set of physical parameters and obtain values for unknown boundary conditions differ only by a specified significant digit. The last value of η_∞ is finally chosen to be the appropriate value of η_∞ for that particular set of parameters. The value of η_∞ may different for another set of physical parameters. Once the finite value of η_∞ is determined then the integration is carried out. We compare the calculated values for unknown boundary values at $\eta = 10$. (say) with the given boundary conditions

$$f'(\infty) = 0,\ F(\infty) = 0,\ G(\infty) = -f(\infty),\ H(\infty) = \omega,\ \theta(\infty) = 0,\ \theta_p(\infty) = 0$$

and adjust the estimate values for unknown values of boundary conditions, to give a better approximation for the solution. i.e. We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula described in equations (A). Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ and $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to determine the correct values of $F(0), G(0), H(0), \theta'(0), \theta_p(0)$. We take series values for unknown boundary conditions, and apply fourth order Runge-Kutta method with step size $h = 0.01$. the above procedure is repeated until we get the results up to the desired accuracy, with an error of 10^{-5}.

IV. Results and Discussion

In order to get a physical insight into the problem, a parametric study is conducted to illustrate the effects of different governing physical parameters viz., the finite volume fraction(φ), Grashoff number(Gr), Prandtl number(Pr), Eckret number(Ec), radiation parameter(Ra), diffusion parameter(ϵ) upon the shear stresses (c_f), wall heat transfer(Nu) as well as the nature of flow and transport and the numerical results are depicted through graphs and tables.

The velocity and temperature profiles for both the phases are depicted in Fig.s 1 – 4 for the variation of diffusion parameter(ϵ). It is observed that the carrier fluid velocity increases with the increase of the diffusion parameter , and the skin friction coefficient also increases, which is shown in the Table – 1. But the particle velocity decreases with the increase of ϵ. The carrier fluid temperature decreases with the increase of ϵ and the particle phase temperature increases with increase of the diffusion parameter. Further it can be observed that the rate of wall heat transfer from plate to fluid is more with increase of ϵ, as observed from Table-1.

The variation of non-dimensional velocity and temperature field for both the phases is illustrated for different values of Grashof number (Gr) in Figs. 5 – 8. Here, the positive buoyancy force acts like a favorable pressure gradient and hence accelerates the fluid as well as particle in the boundary layer. This results in higher velocity as the buoyancy parameter increases. From Fig. 7 & 8, it is clear that the thermal boundary layer of both the phases decreases with increase of Gr.

The effect of Eckret number (Ec), which signifies the viscous dissipation of the fluid, on the heat transfer, is exhibited in Fig. 9 & 10. It is observed that an increase in viscous dissipation of the fluid tends to increase in fluid temperature. The reason for this effect is that the viscosity of the fluid takes energy from motion of the fluid and transforms it into the internal energy of the fluid which results in the heating of the fluid temperature is encountered. The thermal boundary layer gets thicker with the increase in the viscous dissipation. Physically it means that the heat energy is stored in the fluid due to the frictional heating. But, due to two-phase interaction the particle temperature decays when the frictional heating is more.

Figs. 11 & 12 represent the thermal boundary layer of both the phases for different values of thermal radiation parameter (Ra). The effect of radiation is also to intensify the heat transfer. Thus the radiation should be minimum in order to facilitate the cooling process. This is agreement with the physical fact that the thermal boundary layer thickness increases with increase of Ra. But the particle phase temperature decrease when radiation is more.

Figs. 13 & 14 depict the non dimensional temperature profiles of both the carrier fluid phase $\theta(\eta)$ and the particle phase $\theta_p(\eta)$ versus η for different values of Prandtl number (Pr). One can infer from these figures that the temperature of fluid and dust particles decreases with the increase in Pr, which implies momentum boundary layer is thicker than the thermal boundary layer. The fluid temperature decays asymptotically and approaches to zero in the free stream region.

Table–1 shows the computations of the skin-friction coefficient (c_f) and Nusselt number(Nu) for various physical parameters in terms of $f''(0)$ and $-\theta'(0)$ respectively. The magnitude of shear stress increases with increase of finite volume fraction (φ), diffusion parameter(ϵ), Grashof number(Gr), Eckret number $(Ec)$and radiation parameter(Ra) and decrease with the increase of the Prandtl number(Pr). The rate of wall heat transfer significantly increases with the increase of diffusion parameter(ϵ), Grashof number(Gr), Prandtl number (Pr) , and decreases with increase of Eckret number (Ec) and radiation parameter (Ra) .

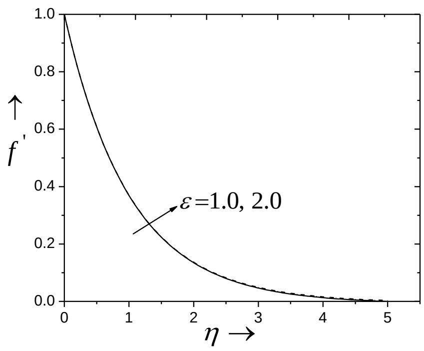

Fig. 1: Effect of diffusion parameter on velocity profiles of carrier fluid phase.

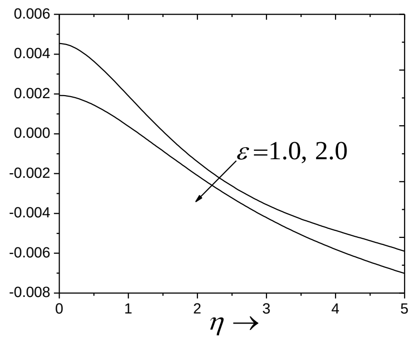

Fig. 2: Effect of diffusion parameter on velocity profiles of particle phase.

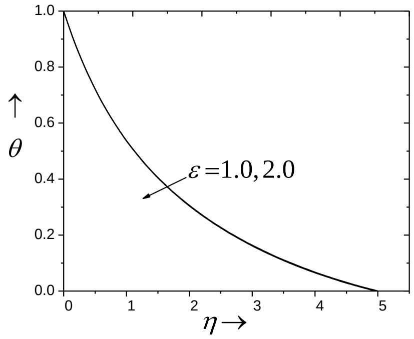

Fig. 3: Effect of diffusion parameter on temperature profiles of carrier fluid phase

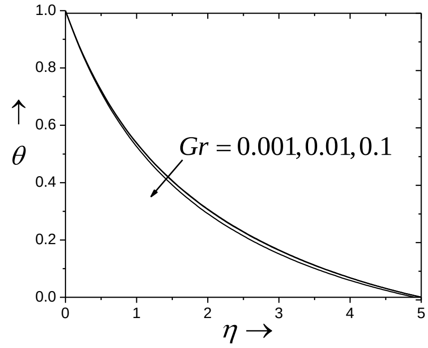

Fig. 7: Effect of Grashof number on temperature profiles of carrier fluid phase.

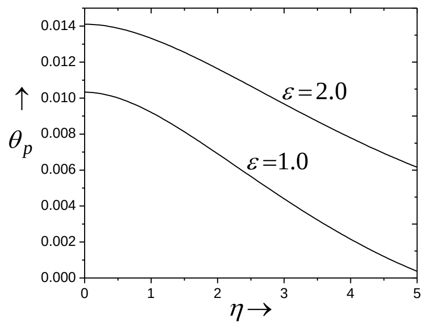

Fig. 4: Effect of diffusion parameter on temperature profiles of particle phase.

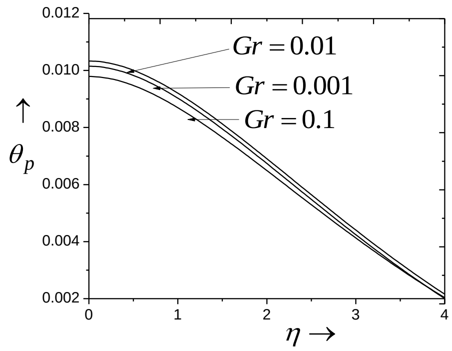

Fig. 8: Effect of Grashof number on temperature profiles of particle fluid phase.

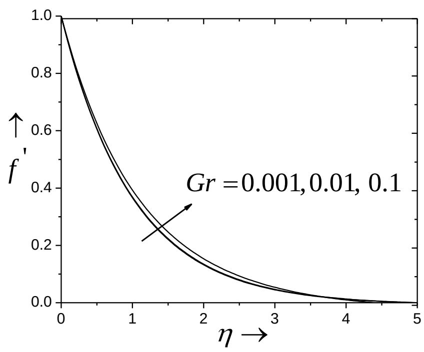

Fig. 5: Effect of Grashof number on velocity profiles of carrier fluid phase.

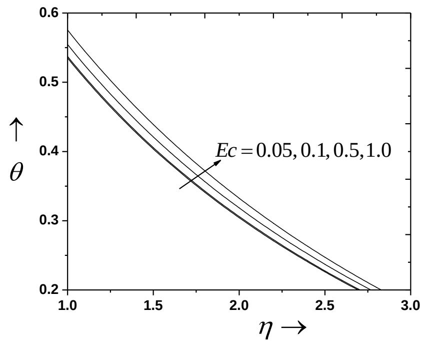

Fig. 9: Effect of Eckret number on temperature profiles of carrier fluid phase.

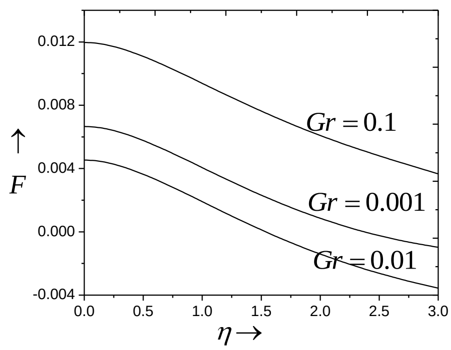

Fig. 6: Effect of Grashof number on velocity profiles of particle phase.

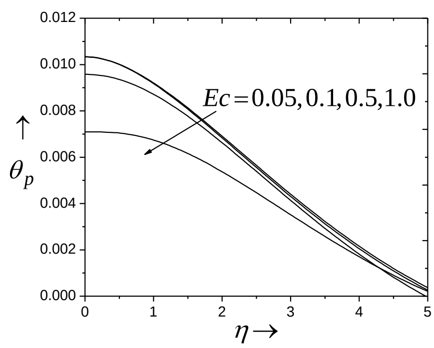

Fig. 10: Effect of Eckret number on temperature profiles of particle phase.

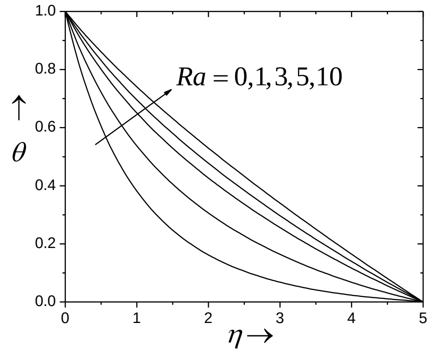

Fig. 11: Effect of Radiation parameter on temperature profiles of carrier fluid phase

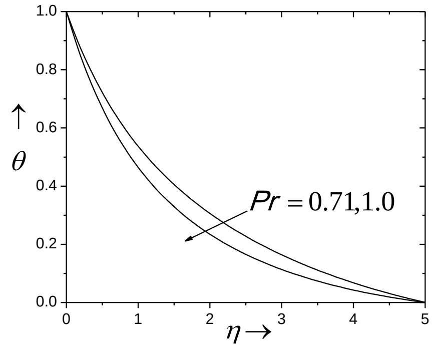

Fig. 13: Effect of Prandtl number on temperature profiles of carrier fluid phase.

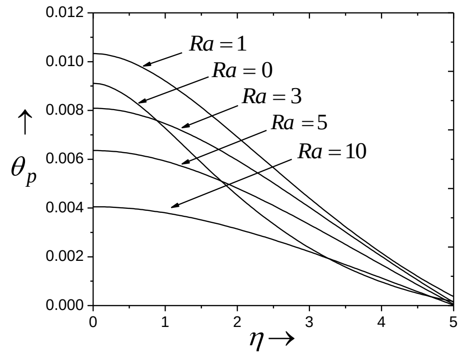

Fig. 12: Effect of Radiation parameter on temperature profiles of particle phase.

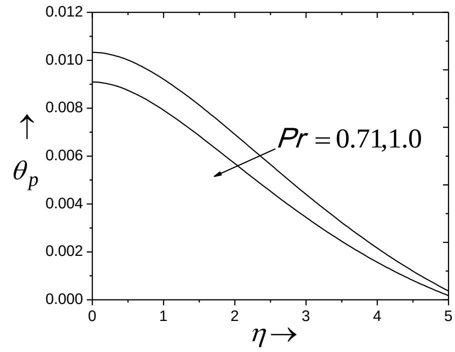

Fig. 14: Effect of Prandtl number on temperature profiles of particle phase.

Table 1: Effect of finite volume fraction, diffusion parameter, Grashof number,Prandtl number, Eckret number, Radiation parameter on shin friction and Nusselt number.

φ	ϵ	Gr	Pr	Ec	Ra	$f''(0)$	$-\theta'(0)$
0.0	1.0	0.01	0.71	0.1	1.0	-0.99591	0.69439
0.001						-0.99596	0.69397
0.01						-0.99625	0.69537
0.1						-0.99607	0.69388
0.001	1.0	0.01	0.71	0.1	1.0	-0.99596	0.69397
	2.0					-0.99528	0.69516
0.001	1.0	0.001	0.71	0.1	1.0	-1.0017	0.69223
		0.01				-0.99625	0.69537
		0.1				-0.94147	0.70933
0.001	1.0	0.01	0.71	0.1	1.0	-0.99596	0.69397
			1.0			-0.99664	0.85434
0.001	1.0	0.01	0.71	0.05	1.0	-0.99597	0.70105
				0.1		-0.99596	0.69397
				0.5		-0.99597	0.63826
				1.0		-0.99562	0.56948
0.001	1.0	0.01	0.71	0.1	0.0	-0.99735	1.05861
					1.0	-0.99596	0.69397
					3.0	-0.99489	0.46863
					5.0	-0.99442	0.38464
					10.0	-0.99395	0.30367

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