American International Journal of Research in Science, Technology, Engineering & Mathematics

Available online at http://www.iasir.net

ISSN (Print): 2328-3491, ISSN (Online): 2328-3580, ISSN (CD-ROM): 2328-3629

AIJRSTEM is a refereed, indexed, peer-reviewed, multidisciplinary and open access journal published by International Association of Scientific Innovation and Research (IASIR), USA
(An Association Unifying the Sciences, Engineering, and Applied Research)

A STUDY ON TWO PARALLEL QUEUES WITH COMMON EXIST SERVICE WITHIN RETRIALS

M.Reni Sagayaraj[1], D.Moganraj[2]
[1]Sacred Heart College, Tirupattur, Vellore (Dist), Tamil Nadu. India
[2]Arunai Engineering College, Tiruvannamalai, Tamil Nadu. India

***Abstract**: Consider a system of two parallel queues where each customer must leave after service through a common gate G. In this paper discussed the simplifying assumption that the queues for station I and station II are inexhaustible. After completion of the service (II) exist the common gate G, the gate will be occupied the customer those who are complete the II (I) the completion of the customer go to orbit. The service will be noncustomer the retrial customer go to the existence gate. So that there is always one customer from each queue either in service, waiting for exist, or in the exist process. Since an I customer (II customer) will always enter as soon as the previous I (customer II) has left gate G.*

***Keywords**: Queueing model, retrials queue, inexhaustible, common gate*

I. Introduction

Several researchers have studied the Problem of Parallel Queues with customers jockeying. F.Haight (1958) [1] originally introduced the problem on two queues in parallel. Konigsberg (1996) [2] studied a queuing model with two servers where each server has its own queue; an incoming customer joins the shortest queue. if the queues are of unequal length otherwise joins one specified queues exceeds 1, the last customer in the longer queue jockeys (switch over) to the shorter queue. Zhao and Grassmann (1990)[3] also studied the shortest queue model with n servers ($n \geq 2$) with jockeying and obtained explicit solutions of the equilibrium probabilities, the expected numbers of customers and the expected waiting time of a customer. Recently, Wang et al (1995) [4] analyzed a modification of the two server shortest queue model with jockeying where one server is primary and the other is secondary in the sense that the primary server is always available for service. R.Reynald susainathan (2014) [5] analyzing the steady state behaviour of a two server parallel queue with customer s jockeying to the shortest queue and also to the primary queue when the secondary service leaves the service at the epoch at which the system level becomes a threshold level N.In this paper we extended the result of R.Reynald susainathan (2014) after finishing his service like primary or secondary how to leave the total system in the common gate.

II. Model Descriptions

The system considered two identical queues that we denote by Q_1 and Q_2 .Each queue has infinite storage capacity and the same service rate μ.Arrivals to the customer are Poisson with rate λ.A system of two parallel queues where each customer must leave after service through a common gate G, before the occupied of the gate customer go to orbit and SEEK COMMAND which allows to be initiated can be issued only if they are in gate service is free and the orbit customer leave the system. Assume that service times at the two stations I and II are independent and identically distributed with density function f(w) on $[0,\infty)$ and exist service takes a fixed length of time $\gamma > 0$ .suppose that a I-customer may be served at station I only if the pervious I –Customer has completely exist service.

III. Retrail policy

We assume that the access from the orbit to the service facility follows the exponential distribution with rate $n\sigma$ which may depend on the current number n ($n \geq 0$) the number of customers in the orbit .That is the probability of the repeated attempt during the interval $(t, t+\Delta t)$ given that there are n customers in the orbit at time t is $n\sigma \Delta t$.It is classical retrial rate policy .the input flow of primary calls interval between repetition and service time in phases are mutually independent.

The waiting time of W_I of a I customer arriving at the queue is the time taken for station I to become ready to accept him, and the total service time is the sum of $S_I + W_G + \upsilon$ where S_I is the service time at station I and W_G is the time spent waiting for exist service .Even the arrivals are Poisson ,the times W_I and W_G are not mutually independent ,because the interference between the two queues at G force both W_I and W_G to depend on the progress of the II queue. However in assuming the performance of the system .If appears at least for small υ to be suitable approximation to regard the set up as consisting of two parallel queues having independent Poisson arrivals and having independent total service times with a suitable distribution. Thus the determination of a distribution for S_I+W_G is of interest in itself.

At the same time of entry of a I-Customer to station-I the distribution of S_I+W_G namely his total service time less υ depends both on location of the II customer currently being served(if any) and also on the number of II customer waiting in the queue. However the dependence on the size of the II queue will be negligible if there is sufficiently many II customers waiting that the possibility of this queue vanishing during the time S_I+W_G is unlikely .If this dependence can in fact be neglected the distribution of S_I+W_G will dependent primarily on the location of the current II Customer ,and if we average the conditional distribution of S_I+W_G given this location, using the steady-state distribution for the location we shall have obtained a suitable distribution for S_I+W_G to be used in approximating the whole system.

Thus in the main part of this paper we shall make the simplifying assumption that the queues for station I and II we inexhaustible, so then there is always are customer from each queue either in service waiting for exist or in their exist process. Since a I customer [II customer] will always enter as soon as the previous I-Customer [II customer] has left gate G.

IV. The Embedded Markov Process

To begin with, let us consider the possible condition which may obtain as a

I customer enter I service. One possibility is that the II customer currently in the system has been waiting for exist service at the time of departure of the previous I customer. Thus as the current I customer enters, the II customer is being exist service at the gate G. Let us call this state of the system G_I.The other possibility is that the current customer is in II service, and has been for some length of time

$S\geq \upsilon$ .Let we call this state of the system S_I.

Analogously, we may define states G_{II} and S_{II} which the system may occupy at the epochs of entry of II customers into the system. Now we define

X_n=State of the system at the epoch of entry of the n[th] customer after observation begins.

It is clear that $(X_n,n\geq1)$ can be modeled as a Markov process on the state space $£=[\upsilon_I,\infty)U[\upsilon_{II},\infty)U\{G_I\}U\{G_{II}\}$

Where $£=[\upsilon_I,\infty)=\{S_I;\ S\geq \upsilon v\}$ and $[\upsilon_{II},\infty)$ are uncorrelated copies of the real interval $[\upsilon,\infty)$ and $\{G_I\},\{G_{II}\}$ are isolated points .If $P[X,A)$ denotes the probability that $X_{n+1}\in A$ given $X_n=x$,where $x\in£$ and $A\in B(£)$ the σ-algebra of Borel subsets of £ ,then defining

$$£(w)=\int_{\omega}^{\infty} f(r)\,dr$$

$$P(G_I,\{G_{II}\})=P(G_{II},\{G_I\})=1-£(\upsilon)$$

$$P(G_I,\{\upsilon_{II}\})=P(G_{II},\{\upsilon_I\})=£(\upsilon)$$

$$P(S_I,\{G_I\})=P(S_{II},\{G_{II}\})=\int_{v=0}^{\infty} f(v)\,\frac{£(S+v)-£(S+v+\upsilon)}{£(S)}\,dv$$

and

$$P(S_I,\{G_{II}\})=P(S_I,\{G_I\})=\int_{v=0}^{\infty} f(s+v)\,\frac{£(v)-£(v+\upsilon)}{£(S)}\,dv$$

If $£(S)>0$.More over for each x in $£=[\upsilon_I,\infty)U[\upsilon_{II},\infty)$the measure $P(x,A)$ restricted to $[\upsilon_I,\infty)$ or $[\upsilon_{II},\infty)$ has a density $P(x,y)$ with respect to Lebesque measure ,and this is given by

$$P(S_I,v_I)=P(S_{II},v_{II})$$

$$= f(v-S-\upsilon)\,\frac{£(\upsilon)}{£(S)}.$$

(OR)

$$P(S_I,v_I)=P(S_{II},v_{II})$$

$$= f(v+S-\upsilon)\,\frac{£(\upsilon)}{£(S)}.\ \text{for } v\geq\upsilon \qquad =0,\ v<\upsilon$$

Again provided that $£(s)>0$(Note that for negative arguments the service time density is o)

Now the Doeblin condition for the existence of an invariants probability measure for the process $(X_n,n\geq1)$ will hold if it can be shown that

$P(S_I,\{G_I\})+P(S_{II},\{G_{II}\})$ is bounded as S various ,that is the following suffices condition D

$$\int_{v=0}^{\infty} f(v)\,\frac{£(S+v)-£(S+v+\upsilon)}{£(S)}\,dv+\int_{v=0}^{\infty} f(s+v)\,\frac{£(v)-£(v+\upsilon)}{£(S)}\,dv>\epsilon$$

For some $\epsilon > 0$ and all S. Since condition D is easily seen to be satisfies for the particular cases of f treated in this chapter, let us assume hence forward that it is satisfied in the general formulation. For $(X_n, n \geq 1)$ there is only one ergodic set.ie, only one set S such that

$$P(x,S)=1 \text{ if } x \in S$$
$$=0 \text{ if } x \notin S.$$

And this ergodic set is £.Also if £(υ)>0 as we shall also assume that there are no cyclically moving sets. Ie, we cannot find the disjoint $A_1, A_2, A_3, \ldots\ldots\ldots A_r$, r>2 such that

$$P(x,A_j)=1 \text{ if } x \in A_{j-1}, j=1,2,3\ldots\ldots r$$
$$=1, \text{if } j=1 \text{ and } X \in A_r$$

If then follows that there is a unique probability measure π on(£,B(£)) such that

$$\Pi(A)=\int \pi(dx)P(x,A) \text{ and that for this measure } \pi,$$
$$\lim_{m\to\infty} \|P_m(x,.)-\pi(.)\| = 0$$

Where $P_m(x,A)=\Pr\{X_{m+1} \in /X_1=x\}$ and $\|.\|$ denotes total variation of the set function. Thus the invariant measure π may be interpreted as a steady state distribution of X_{II}

V. The Associated semi-Markov processes:

Define the processes (X(t);n≥1) in a real time taking values in £ as follows .Let ι_n be the time of the n^{th} transition epoch and let

$$X(t)=X(\iota_n)=X_n, \text{if } \iota_n \leq t < \iota_{n+1}$$
$$X(t)=X(0) \text{ if } 0 \leq t < \iota_1$$

that is X(t) is the stat obtained at the most recent transition epoch prior to time t. The processes (X(t);n≥1) may evidently be regarded as a semi-markov processes on (£,B(£)) the theory of semi –Markov process as a general state space is discussed by Cilnar.

Now let $T_n+\upsilon$ be the time after ι_n of first passage of the $(X(t); t \in R^+)$ process into $[\upsilon,\infty)\cup\{G_I\}$ .If ι_n Corresponds to the entry of a I customer T_n may be regarde as the time to exist service of that customer or S_I+W_G in the notation of the introduction.

$$\text{Let } G_S(\omega)=P(T_n \leq \omega/x(\iota_n)=S_{II})$$
$$F_S(\omega)=P(T_n \leq \omega/x(\iota_n)=S_I)$$
$$H(\omega)=P(T_n \leq \omega/x(\iota_n)=G_I) \quad (1)$$

Then the quantity of interest namely the transition of the steady state distribution of S_I+W_G ,will be

$$£^-_+(\zeta)=\int_0^\infty e^{i\xi\omega} d\Im(\omega)$$

Where

$$\Im(\omega)=2\int_{[\upsilon_I,\infty)} F_{S_I}(\omega)d\pi(S_I)+2\pi(\{G_I\})H(\omega) \quad (2)$$

And π denotes the steady state distribution of X_{II}. Since $(X(t); t \in R^+)$ is a semi Markov process on (£,B(£)),the following equations for first =-passage –time probabilities are satisfied

$$P(T_n \leq \omega / X(\tau_n)=x) = \int_{[\upsilon_{II},\infty)\cup\{G_{II}\}}^{\infty} P(x,dy)\int_{u=\phi}^{\omega} d\phi_{xy}(u)\, p(T_{n+1} \leq \omega-u / X(\tau_{n+1}=y)$$
$$+\int_{[\upsilon_{II},\infty)\cup\{G_{II}\}} P(x,dy)\phi_{xy}(\omega-\gamma) \quad (3)$$

Where $\phi_{xy}(u)$ is the conditional distribution of $\tau_{n+1}-\tau_n$ given that

$$x(\tau_n)=x, x(\tau_{n+1}=y),$$

Clearly from this equation

$$H(\omega)=I(\omega-\gamma)(1-\Im(\gamma))+\Im(\gamma)G_\gamma(\omega-\gamma).$$

Where $I(x) = 0, x < 0$
$\quad 1, x \geq 0$

It is also easily seen under condition C to be described below, that G_I and F_S have densities $g_S(\omega)$ and $F_S(\omega)$ which are bounded uniformly in S for ω in finite intervals. Then $\Im(\omega)$ for $\omega \neq \gamma$ will also have a density, given by

$$2 \int_{[\upsilon_I,\infty)}^{\infty} F_{S_I}(\omega) d\pi(S_I) + 2\pi(\{G_I\}) g_\gamma(\omega - \gamma) \quad (4)$$

Moreover, the densities

$g_S(\gamma)$ and $f_S(\gamma)$ will satisfies a density version of (3) which will yield.

$$\Im(S) g_S(\omega) = \Im(\omega) f(S+\omega) + f(\omega-\gamma)[\Im(S+\omega-\gamma) - \Im(S+\omega)]$$
$$+ \int_{\gamma=0}^{\omega-\gamma} f(r) \Im(r+S+\gamma) g_{r+S+\gamma}(\omega - S - \gamma) dr \quad (5)$$

And

$$\Im(S) g_S(\omega) = \Im(S+\omega) f(\omega) + f(\omega+S-\gamma)[\Im(\omega-\gamma) - \Im(\omega)] I(\omega-\gamma)$$
$$+ \int_{\gamma=0}^{\omega-\gamma} f(r+S) \Im(r+\gamma) g_{r+\gamma}(\omega - S - \gamma) dr \quad (6)$$

Thus

$$\Re(S,\omega) = \Im(S) g_S(\omega)$$
$$and$$
$$\wp(s,\omega) = \Im(S) f_S(\ \omega).$$

We have

$$\Im(S,\omega) = \wp(S+\omega) + \int_{\gamma=0}^{\omega-\gamma} f(r) \Im(r+S+\gamma, \omega - r - \gamma) dr$$

$$\mathrm{M}(S,\omega) = \mathrm{M}(S+\omega) + \int_{\gamma=0}^{\omega-r} f(r+S) \Im(r+\gamma, \omega - r - \gamma) dr \quad (8)$$

Where

$$\Re(S,\omega) = \Im(S+\omega) f(\omega) + f\left(\omega + S - \gamma\right)[\Im(\omega-\gamma) - \Im(\omega)] I(\omega-\gamma) \quad (9)$$

Now suppose that on $(\gamma_I, \infty) = [\gamma_I, \infty) - \{\gamma_I\}$ the measure $\pi(A)$ has a density $D(S_I - \gamma_I)\Im(S_I)$
Where D is a positive real –valued function on (0, ∞) from symmetry it follows that
π also has a density $D(S_{II} - \gamma_{II})\Im(S_{II})$. with respect to Lebesgue measure of (γ_{II}, ∞).

VI. Conclusion

In this paper analyzed the simplifying assumption that the queue for station I and station II are inexhaustible. After completion of the service I (II) exist the common gate G, the gate will be occupied the customer those who are complete the II (I) the completion of the customer go to orbit. The service gate will be no customer the retrial customer go to the existence gate.

VII. References

[1]. Frank A.Haight two queues in parallel Biometrika ,Vol 45 ,No 3/4 (Dec 1958).PP401-410.
[2]. Konigsberg , On Jockeying in queues,Management Sci.,12(1966) 412-436.
[3]. Zhao and Grassmann,W.K.:The shortest queue Model with jockeying Naval Res.Logistics.,37(1990),773-787.
[4]. Wang,P.P., Gray,W.J.and Scott,M:A two-server parallel queuing model in which one server availability depends on deman j.App.Stat.,Sci.,3(1995),75-91.
[5]. R.Reynald Susainathan "A Stochastic model of interconnected two queus with a balancing policy,Mathematica Aeterna Vol.4,2014,no.4,345-349.
[6]. Abate J. and W.White (1887) :Transient behavior of the M/M/1 queue:Starting at the origin ,Queueing system,Vol.2,41-65.
[7]. Abate J. and W.White (1887) :Transient behavior of the M/M/1 queue via Laplace Transform Adv.appl.Probability,Vol.20,145-178.

[8]. Artalejo. J.R, A queueing system with returning customers and waiting line. Operations Research Letters, 1995, 17, 191-199.
[9]. Artalejo. J.R, A classi_ed bibliography of research on retrial queues: Progress in 1990-1999. Top,1995b, 7, 187-211.
[10]. B.T. Doshi, Queueing systems with vacations{a survey, Queueing Systems 1 (1986) 29-66.
[11]. B.T. Doshi, Conditional and unconditional distributions for M/G/1 type queues with server vacation, Queueing Systems7(1990) 229-252.
[12]. Falin. G.I, A survey of retrial queues. Queueing Systems, 1990, 7,127-167.
[13]. Falin. G.I, J.G.C. Templeton, Retrial Queues. Chapman and Hall, London, 1997
[14]. Keilson. J, L.D. Servi, Oscillating random walk models for GI/G/1 vacation systems with Bernoulli schedules, Journal of Applied Probability 23 (1986),pp 790-802
[15]. Marcel F. Neuts, Matrix Geometric Solutions in Stochastic Models an algorithmic approach.

VIII. Acknowledgments

The authors are very grateful to the referees for their valuable suggestions to improve this paper in the present form.