



Exploring Complex Networks

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Abstract: *There exists a wide variety of complex networks, ranging from biology to sociology. Various real world networks can be depicted as complex networks. These networks have different structures and communication patterns. In this paper we have discussed their structures and corresponding examples that demonstrates the behavior of the networks. The structure is helpful to recognize the communication in a network. Earlier, there were small networks with few vertices and edges. But nowadays large complex networks have come into existence. Many researchers are trying to unfold the characteristics which will help to understand the complex networks in a better way. In this paper we have discussed various properties and their effects on different networks. These properties define the non-random nature of complex networks. They have significant impact on the network's structure. We have tried to describe complex network structures from various researchers' point of view.*

Keywords: complex networks, graphs, complex network types, network properties.

I. Introduction

A network consists of a set of items, called vertices or nodes (represents individuals) and connections between them called edges. Systems that take the form of network are known as graphs. Examples of such kind of graphs are Internet, the World Wide Web, social networks of acquaintance or other connections between individuals, organizational networks and networks of business relations between companies, neural networks, metabolic networks, food webs, distribution networks such as blood vessels, postal delivery routes, networks of citations between papers, and many others [1].

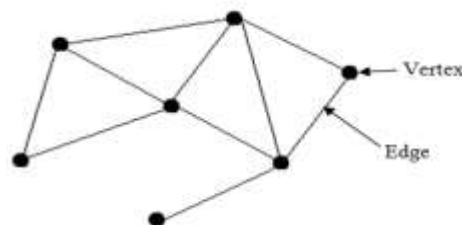


Fig. 1 Small network consisting a few vertices and edges

Recently a new movement have witnessed in network research, with the center of attention drifting away from the study of single small graphs to large-scale graphs having millions or billions of vertices and edges. This new approach has motivated the accessibility of computers and communication networks. With this approach users can congregate and examine data on a scale extremely bigger than previously possible [4]. Earlier studies used to review networks of approximately tens or in extreme cases hundreds of vertices as represented in fig. 1, it is not unusual nowadays to review networks with millions or even billions of vertices as depicted in fig. 2 described by M.E.J Newman.

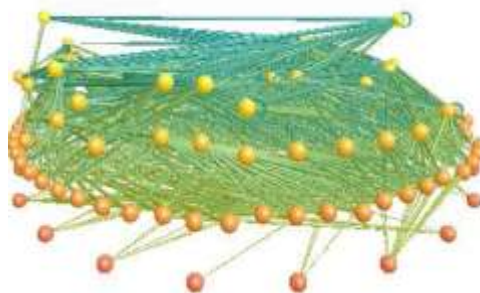


Fig. 2 Complex network consisting of millions of vertices and edges by M.E.J Newman

In social sciences, networks have also been studied extensively. Network studies distinctively in sociology engross the circulation of questionnaires; respondents are asked to specify their communications with others. The responses can be used to restructure a network in which vertices or nodes represent individuals and edges represent the interactions between them. Social network studies represent matter of centrality (which individuals are best connected to others or have most influence) and connectivity (whether and how individuals are connected to one another through the network) [8].

II. The Beginning of Complex Network Studies

The very first review was given by Leonhard Euler in 1736, and the paper was known as Seven Bridges of Konigsberg. Euler's mathematical description of vertices and edges was the groundwork of graph theory. It was a branch of mathematics that studies the characteristics of pair wise relations in a network structure.

Jacob Moreno (1930), a psychologist in the Gestalt tradition, developed the socio-gram and presented it in April 1933. Moreno in 1953 claimed that “*before the advent of sociometry no one knew what the interpersonal structure of a group 'precisely' looked like*”. The sociogram was a depiction of the social configuration of a cluster of elementary school students. The sociogram has found many applications and has grown into the field of social network analysis [2].

Paul Erdos and Alfred Renyi's eight famous papers on random graphs enhanced probabilistic theory in network science. The idea of a Meta-network with the PCANS Model was introduced in 1998 by David Krackhardt and Kathleen Carley. The result of which concluded that there are three types of domains responsible for the structure of the organizations and they are: Individuals, Tasks and Resources. Their work introduced the thought that networks occur across multiple domains and they are interrelated, which resulted to another field of network science called dynamic network analysis. Different network topologies have been described mathematically by various recent network science efforts. Albert-Laszlo Barabasi and Reka Albert developed the loosely defined scale free network topology that consists of hub vertices with many connections. For example, the Internet appears to maintain this characteristic [8].

III. Types of Networks

Complex networks are of many kinds. They can be represented as large graphs that appear in various contexts. For instance in computer science Internet maps, web graphs or data exchanges can be cited. Apart from this, many examples among social, biological or linguistic networks can also be cited like co-authoring networks, protein interactions, or co-occurrence graphs [6]. Different networks are described below:

A. Information Networks

Information networks are also known as “Knowledge Networks”. A very appropriate example of this kind of network is a network of citations between academic papers. These citations form a network in which the vertices are articles and a directed edge from article A to article B indicates that A cites B. The structure of the citation in network reflects the structure of the information stored at its vertices, hence the term “information network”. Citation networks as described in fig. 3 are acyclic because papers can only cite other papers that have already been written, not those that have yet to be written. The very first effort on citation patterns was conducted in the 1960s as huge citation databases became available through the work of Eugene Garfield. Using enhanced resources available in citation databases, further studies of citation networks have been performed [1].

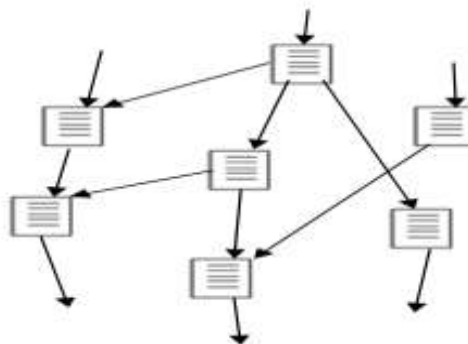


Fig. 3 Citation Network

B. Social Networks

When a set of persons or different groups of persons communicates or interacts with each other, a social network is formed. The communication can be between friends, between business persons, and intermarriages between families. The social networks have the extensive history of the substantial quantitative study of real-world networks. Traditional social networks often have problems of inaccuracy, subjectivity and small sample size. Because of these problems many researchers have turned to various other methods for probing social

networks. Collaboration networks are a source of abundant and moderately reliable data in which participants collaborate in groups of one kind or another, and the links between individuals are established by similar group membership [5]. An example of social network is a network of company directors, in which only those directors are linked that, belong to the same company is depicted in fig. 4 described by Bonanno.

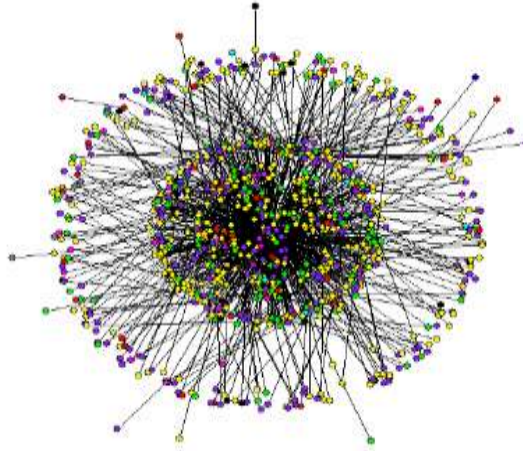


Fig. 4 Network of Company Directors by Bonanno

C. Technological Networks

Man-made networks that are intended for distribution of some commodity or resource, such as electricity or information come under the category of technological networks. Some examples of this kind of network are the network of airline routes, and networks of roads and railways. The telephone network and delivery networks such as those used by the post-office or parcel delivery companies also come under this category of network [6].

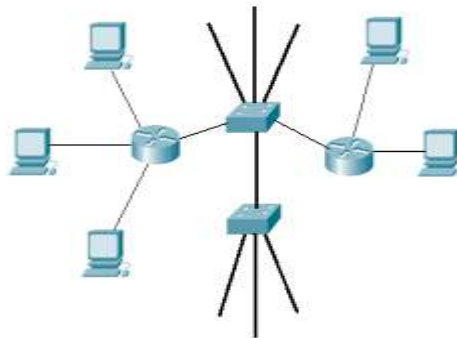


Fig. 5 Internet

One more very good example of technological network is the Internet, i.e. the network of physical connections between computers shown in fig 5. Because the configuration of the Internet is constantly varying thus it is usually examined at a coarse- grained level. Also known as the level of “routers”, i.e. special-purpose computers used to direct the movement of data.

D. Biological Networks

Various biological systems can be suitably represented as networks. An excellent example of a biological network is the network of metabolic pathways.

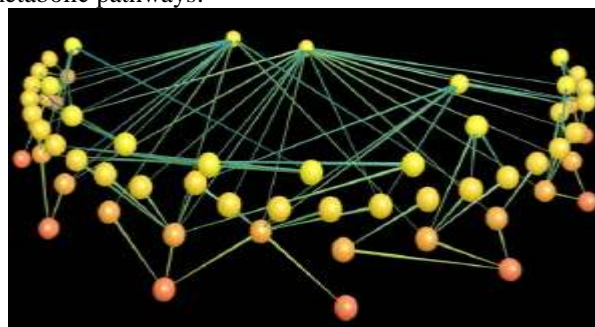


Fig. 6 Food Web by R.J. Williams N.D. Martinez

It is an illustration of metabolic substrates and products with directed edges. Statistical properties of metabolic networks have been studied by Jeong, Fell and Wagner, and Stelling. Food Web (fig. 6) is another much studied example of a biological network in which the vertices represent species in an ecosystem and a directed edge from species A to species B indicates that A preys on B. Statistical properties of the topologies of food webs have been studied by Montoya, Camacho and Dunne [2].

IV. Properties of Complex Networks

Many researchers in the field of complex networks are fascinated by the non-random nature of complex network. In real world, networks are non-random. There are mechanisms for network formation and certain rules to study the structure of the complex network. Different properties of complex networks are discussed in this section that highlights similar features in different networks.

A. The Small-World Effect

The idea of small world explains the fact that even though networks have large size, there is a comparatively shorter path between the two nodes in the network. The number of edges between the two nodes represents the distance between the two nodes. So, the shortest distance s between the vertices in a graph having v vertices can be explained as below:

$$s = \frac{1}{\frac{1}{2}v(v+1)} \sum_{i \geq j} d_{ij}$$

Here d_{ij} is the shortest distance from vertex i to j . Stanley Milgram introduced the concept of “six degrees of separation” in 1967. The concept concluded that among large numbers of pairs of people in the United States, there exists a path of connections having distance of value six [2].

B. Scale-free Networks

Scale-free networks are different from small world networks and random graphs because of different modeling approach. The idea used in small world networks and the random graphs was to create a graph having precise topological features; however the scale-free networks work on the network dynamics. Scale free networks have power-law degree distributions. In these networks, only the degree is scale-free. Some examples of the networks are Price’s network, citation networks, World Wide Web, Internet, metabolic networks and many more. A scale-free network having degree k with exponent r is a sparse as well as random graph and follows below equation.

$$\lim_{k \rightarrow \infty} \frac{\log p_k}{\log(1/k)} = r$$

C. Transitivity or Clustering

The property of transitivity or clustering refers to connection of a single node to maximum possible number of nodes. For instance, if a vertex A is connected to vertex B and vertex B further connected to vertex C, then there exists a possibility of vertex A connected to vertex C. This intrinsic tendency to cluster is measured by the clustering coefficient (Watts and Strogatz, 1998). It is popularly known as “fraction of transitive triples” (Wassermann and Faust, 1994) in social networks [9]. A network having clustering coefficient T is the average of whole network, it can be calculated as:

$$T = \frac{1}{n} \sum_i T_i$$

D. Degree Distributions

In terms of degree distribution, real-world networks are generally different from the random graphs. Degree distributions can be categorized as homogeneous and heterogeneous. In homogeneous degree distribution all the values of degrees are very near to the average value and in heterogeneous degree distribution, degree values are of variable magnitudes. The variation in degrees of vertices is characterized by a distribution function $P(k)$, which gives the probability that a randomly selected node has exactly k edges [9]. It can be calculated as:

$$P_K = \sum_{k'=k}^{\infty} p_{k'}$$

E. Network Resilience

Network resilience is a property of complex networks to discard the vertices that have greater attention in the network. Different networks have different degree of resilience. There exist several methods to remove the vertices from network. One of the methods is to delete the vertices at random. Another method is to remove the vertices having highest degree [3]. The effect of vertex deletion on a network has been studied by Albert. He studied the two well known networks, and they are Internet and World Wide Web [7] and the results are discussed in table 1.

Method	Network	Result
Random vertex removal	Internet	Distance remains unaffected
	World Wide Web	Distance remains unaffected
Highest degree vertex removal	Internet	Distance raises very abruptly
	World Wide Web	Distance raises very abruptly

Table 1. Effect of vertex deletion on networks

F. Community Structure

Community structure is defined as the group of vertices having high density of central vertices and low density of outer vertices. Outer vertices connect different groups together. For ease in evaluation the vertices are divided into groups; a very good example is a network of friends. Colors can be added to the nodes for better visualization of the graph and to differentiate the groups in a graph. Similar communities may exist in different networks. For example, citation networks have similar community as in World Wide Web. Cluster analysis, also known as hierarchical clustering, can be used to extract community structure from a network.

V. Conclusion

A network with millions or billions of vertices and edges is known as a complex network. The emergence of complex networks from earlier simple networks has led to the ease in study of real world networks. Various scientists have shared their work on complex networks; it provides help to understand the network. The structure of a network is of great importance which is discussed in this paper. The structure aids to recognize the communication between the vertices. Various kinds of networks are discussed in this paper and they are social network, information network, technological network, biological network. All these networks differ in their structure and communication flow. Every network is examined along with suitable examples. The real world examples define the type of network in a best suited way. Apart from this, different properties of complex networks are discussed. These properties define the non-random nature of complex networks. The properties also highlight the same features in different networks. Some of the properties that are argued in this review are small world effect, transitivity or clustering, degree distribution and network resilience.

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