



Cumulative weekly and monthly rainfall prediction with Growth models

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Abstract: Uneven distribution and erratic nature of rainfall creates problem in taking quantitative crop production due to limited availability of soil moisture. The knowledge of cumulative rainfall can be of great help in deciding crop varieties suiting to prevalent climatic conditions. With salient information about cumulative rainfall at a place, various agricultural operations can be done at appropriate time. In this study, modelling of cumulative rainfall on weekly and monthly basis for Nainital district of Uttarakhand was done with the help of Gompertz and logistic growth models at various probability levels and it was found that cumulative rainfall on weekly and monthly basis can be predicted fairly accurately with these growth models.

Keywords: Modelling, cumulative rainfall, Gompertz, logistic, growth models.

I. Introduction

Rainfall is the primary source of water for agricultural production. Dependency on rainfall for future crop production has become a major constraint for sustainable food production especially in developing countries. The information about distribution and amount of rainfall with respect to time plays a very important role in deciding timing for various agricultural operations and crop planning. For precise utilization of rainfall, its close estimation is essential and its impact on crop production at a place can be linked to its total seasonal amount or its intra-seasonal distribution. In the present study, Gompertz and logistic growth models were used to predict cumulative rainfall for Nainital district of Uttarakhand, India on weekly and monthly basis at 1, 10, 20, 25, 30, 40, 50, 60 and 70 percent probability level of occurrence.

II. Materials and Methods

Daily rainfall values recorded for a period of 21 years (1992-2012) for Nainital district of Uttarakhand (India) was used in this study. The cumulative rainfall for all 52 standard meteorological weeks (SMWs) and 12 months for the period under investigation were analyzed to get expected rainfall at different probability levels by using widely accepted Weibull's equation as [1] observed that this equation fits closely to linear relationship between rainfall and probability position. The behaviour of a curve relating cumulative values with time may also be called as "Growth curve" and can be easily expressed with the help of Gompertz and logistic growth models described as:

(a) Gompertz growth model: The values of cumulative rainfall (Y_c , mm) on weekly and monthly basis at different percent probability levels by Gompertz growth model can be predicted by using mathematical equation, $Y_c = K \cdot A^{B^x}$ where K , A & B are model constants and " x " is SMW number (1-52). In this model, calculated values of cumulative rainfall for each probability level were chronologically divided into three almost equal segments and subtotal of logarithms of individual observations in each segment was designated as S_1 , S_2 and S_3 . The difference between two successive subtotals i.e. $S_2 - S_1$ and $S_3 - S_2$ were represented as D_1 and D_2 , whereas, number of observations in each segment was denoted by N .

The fitting of Gompertz growth curve is to the logarithms of observed data and may be accomplished in a manner exactly paralleling the fit of modified exponential [2]. The values of model constants (K , A & B) were obtained by mathematical relationship given by [3] as:

$$\begin{aligned} K &= \text{antilog} [(1/N) \cdot (S_1 - (D_1 / (C^N - 1)))] \\ A &= \text{antilog} [(D_1(C-1)) / (C^N - 1)^2]; \text{ and} \\ B &= (D_2 / D_1)^{(1/N)}. \end{aligned}$$

Noting whether D_2 is greater than or less than D_1 , it may ascertain possibility of upper asymptote for an increasing series. If first difference (D_1) exceeds the second difference (D_2), C^N (and, therefore, C) is greater

than one, and there is no upper asymptote for the increasing series, whereas, with D_1 less than D_2 , value of C will be less than one.

(b) Logistic growth model: By this model, values of cumulative weekly or monthly rainfall (Y_c , mm) can be predicted by using mathematical relation, $Y_c = K/(1 + e^{-(A + BX)})$ where X is time variable and K , A & B are the model constants. To fit data in this growth model, “method of selected points”, as suggested by [2] was used by choosing three points equidistant from each other as X_0 (near the beginning), X_1 (in middle), and X_2 (near the end) in cumulative rainfall data series with corresponding values of Y as Y_0 , Y_1 and Y_2 with origin on x-axis at week or month designated as X_0 . The time variable X and plotting of origin may be defined by $X = x - o$ where “ x ” is week or month number and “ o ” is number of week or month where X_0 is taken. The values of model constants were obtained by using following mathematical relationships:

$$K = [2 * Y_0 * Y_1 * Y_2 - Y_1^2 (Y_0 + Y_2)] / [Y_0 * Y_2 - Y_1^2];$$

$$A = \log [(K - Y_0) / Y_0]; \text{ and}$$

$$B = (1/N) * \log [Y_0 (K - Y_1) / Y_1 (K - Y_0)].$$

where N is number of weeks or months from X_0 to X_1 or from X_1 to X_2 .

III. Results and Discussion

The equations for predicting cumulative weekly and monthly rainfall at various probability level of occurrence with the help of growth models were developed by calculating model constants (Table 1). With the help of these developed equations, values of cumulative rainfall on weekly and monthly basis were calculated for both growth models and their accuracy was evaluated by computing coefficient of determination (R^2) between observed and predicted values at various percent probability levels (Table 1) from which it is clear that higher values of R^2 were obtained with both Gompertz and logistic growth models.

IV. Conclusions

From the foregoing, it can be concluded that both Gompertz and logistic growth models can be utilized for predicting cumulative rainfall fairly accurately on weekly and monthly basis at Nainital district of Uttarakhand (India).

References

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Table 1: Equations developed for predicting cumulative rainfall with growth models

With Gompertz growth model		With logistic growth model	
<u>For cumulative weekly rainfall</u>			
Developed equation	R^2	Developed equation	R^2
$Y_{cw1} = 7762.077 * 0.00392^{X-1} * 0.91796^X$	0.980	$Y_{cw1} = 24067.969 / [1 + e^{-(1.47946 - 0.03935(X-13))}]$	0.963
$Y_{cw10} = 6950.575 * 0.00397^{X-10} * 0.91898^X$	0.931	$Y_{cw10} = 21714.138 / [1 + e^{-(1.48860 - 0.03939(X-13))}]$	0.963
$Y_{cw20} = 6046.195 * 0.00402^{X-20} * 0.92029^X$	0.935	$Y_{cw20} = 19108.722 / [1 + e^{-(1.50195 - 0.03946(X-13))}]$	0.962
$Y_{cw25} = 5592.238 * 0.00404^{X-25} * 0.92103^X$	0.937	$Y_{cw25} = 17811.697 / [1 + e^{-(1.51042 - 0.03950(X-13))}]$	0.961
$Y_{cw30} = 5137.135 * 0.00406^{X-30} * 0.92184^X$	0.939	$Y_{cw30} = 16520.013 / [1 + e^{-(1.52057 - 0.03955(X-13))}]$	0.960
$Y_{cw40} = 4223.170 * 0.00408^{X-40} * 0.92378^X$	0.945	$Y_{cw40} = 13960.851 / [1 + e^{-(1.54834 - 0.03970(X-13))}]$	0.958
$Y_{cw50} = 3303.818 * 0.00404^{X-50} * 0.92633^X$	0.951	$Y_{cw50} = 11461.271 / [1 + e^{-(1.59415 - 0.03997(X-13))}]$	0.954
$Y_{cw60} = 2380.153 * 0.00383^{X-60} * 0.93008^X$	0.958	$Y_{cw60} = 9107.952 / [1 + e^{-(1.68332 - 0.04059(X-13))}]$	0.946
$Y_{cw70} = 1468.964 * 0.00292^{X-70} * 0.93706^X$	0.956	$Y_{cw70} = 7130.994 / [1 + e^{-(1.91617 - 0.04288(X-13))}]$	0.918
<u>For cumulative monthly rainfall</u>			
Developed equation	R^2	Developed equation	R^2
$Y_{mw1} = 9896.328 * 0.01007^{X-1} * 0.83209^X$	0.952	$Y_{mw1} = 4533.086 / [1 + e^{-(0.85921 - 0.34370(X-4))}]$	0.960
$Y_{mw10} = 9102.295 * 0.00964^{X-10} * 0.83233^X$	0.950	$Y_{mw10} = 4121.112 / [1 + e^{-(0.86960 - 0.34805(X-4))}]$	0.959
$Y_{mw20} = 8223.024 * 0.00905^{X-20} * 0.83264^X$	0.947	$Y_{mw20} = 3663.549 / [1 + e^{-(0.88421 - 0.35412(X-4))}]$	0.958
$Y_{mw25} = 7784.948 * 0.00872^{X-25} * 0.83282^X$	0.946	$Y_{mw25} = 3434.861 / [1 + e^{-(0.89318 - 0.35781(X-4))}]$	0.958
$Y_{mw30} = 7348.203 * 0.00834^{X-30} * 0.83301^X$	0.944	$Y_{mw30} = 3206.250 / [1 + e^{-(0.90362 - 0.36207(X-4))}]$	0.957
$Y_{mw40} = 6479.962 * 0.00743^{X-40} * 0.83341^X$	0.939	$Y_{mw40} = 2749.312 / [1 + e^{-(0.93064 - 0.37298(X-4))}]$	0.955
$Y_{mw50} = 5621.668 * 0.00625^{X-50} * 0.83378^X$	0.932	$Y_{mw50} = 2292.895 / [1 + e^{-(0.97092 - 0.38889(X-4))}]$	0.952
$Y_{mw60} = 4777.171 * 0.00467^{X-60} * 0.83378^X$	0.921	$Y_{mw60} = 1837.258 / [1 + e^{-(1.03772 - 0.41446(X-4))}]$	0.948
$Y_{mw70} = 3926.999 * 0.00256^{X-70} * 0.83124^X$	0.901	$Y_{mw70} = 1382.859 / [1 + e^{-(1.17247 - 0.46336(X-4))}]$	0.943

where Y_{cwi} and Y_{mwi} are values of cumulative weekly and monthly rainfall at $i\%$ probability level respectively.