Optimization of sound transmission loss and prediction of insertion loss of single chamber perforated plug muffler with straight duct

Shantanu V. Kanade¹, A. P Bhattu²
¹M.Tech (Design) Student, Mechanical Engineering Dept., College of Engineering, Pune, India.
²Associate Professor, Mechanical Engineering Dept., College of Engineering, Pune, India.

Abstract: With the advancement of technology it has become important to develop mufflers which satisfy space and noise constraints. Therefore, the focus of this paper is not only to analyze the sound transmission loss (STL) and insertion loss of a one-chamber perforated plug muffler but also to optimize the best design shape within a limited space. A numerical scheme for analyzing concentric perforated tube plug muffler has been developed. Coupled differential equations describing one-dimensional acoustic wave propagation in the perforated pipes and cavities of straight-through silencer elements are used from earlier study [5] and then decoupled numerically. In addition, the acoustical performance of mufflers with perforations is found to be superior to the traditional mufflers. In this paper, optimization of perforated lengths, thickness, porosities of both expansion and contraction chamber of single chamber concentric perforated plug muffler with straight duct is carried out using Genetic Algorithm in order to achieve high transmission loss over a wide range of frequency. FEM analysis is also carried to out to validate the results. Insertion loss for the same muffler is calculated using mathematical modeling. Effect of source impedance on Insertion Loss [IL] is also observed [13].

Keywords: transmission loss, insertion loss, plane wave, four-pole matrices, FEM, internal perforated plug tube, genetic algorithm

I. Introduction

Although active noise control techniques are developing fast, the reactive muffler is still the main component in the exhaust silencer system of modern vehicles. In order to attenuate the engine exhaust noise, a few muffler elements with various geometrical configurations have been developed. The silencer system for a road vehicle has to maintain sufficient acoustic performance. Much work has been done to analyze the performance of mufflers consisting of area discontinuity or extended tube under the assumption of plane wave propagation with or without mean flow. A common feature of the exhaust silencers of road vehicles is the use of perforated pipes. Frequently, they are used to contain the mean flow, thus reducing the back-pressure and flow-generated noise of the silencer, while allowing for acoustic coupling to an outer cavity through the perforations.

In 1978, Sullivan and Crocker [1] presented the first mathematical model for perforated element mufflers to analyze the tube resonator by coupling the wave propagation in the center tube and outer cavity. Sullivan [2] then developed a segmentation analysis procedure for modelling all types of perforated element mufflers. However, numerical instability occurs when modelling muffler elements with high porosity. Jayaraman and Yam [3] presented a decoupling approach for the perforated tube muffler components to obtain a closed form solution. The major drawback of this method is that it is based on an unreasonable assumption that the mean flow Mach numbers in the ducts must be equal. Rao and Munjal [4] have sought to overcome this problem with a generalized decoupling analysis which does allow for different flow Mach numbers in the inner pipe and outer casing. The decoupling methods mentioned above, however, are all based on plane wave acoustic theory and are suitable for mufflers with geometrical configurations, such as the plug muffler, perforated reverse flow muffler and concentric-tube resonator. It is assumed that the flow transfer through the perforated portion is uniformly distributed over the length, and therefore the perforate impedance is constant along the length. Sullivan and Crocker's [1] one-dimensional equations have been adopted.

In the present paper concentric perforated plug tube with end inlet/outlet is considered. Mathematical modelling done by Munjal [4, 6] is taken for formulation of problem. Optimization of length of perforated plug tube, thickness, porosities of both expansion and contraction chamber is done using Genetic Algorithm in order to achieve maximum Transmission Loss [11]. FEM analysis is carried out using software package COMSOL for validation of results.

In order to calculate Insertion Loss (IL) source impedance for exhaust tube is taken as \( Z_0 \cdot (0.7 - 0.7j) \) where \( Z_0 \) is characteristic impedance of tube [13].
II. Mathematical model

In this paper straight tube muffler with perforated tube was adopted. As shown in Fig.1 single expansion chamber with perforated plug tube consist of four acoustical elements straight inlet duct, expansion perforated duct, contraction perforated duct & straight outlet duct.

Here \((P_1, u_1) & (P_6, u_6)\) represent pressure and velocity at point 1 & 6 respectively. \((P_2, u_2) & (P_{3a}, u_{3a})\) gives pressure & velocity at the boundary of expansion perforated tube. \((P_{3a}, u_{3a}) & (P_5, u_5)\) gives pressure & velocity at the boundary of expansion perforated tube. \((P_5, u_5) & (P_6, u_6)\) represent pressure & velocity inside perforated tube at point 5 & 6.

Fig. 1. Dimensions and acoustical mechanism of perforated plug muffler with straight end tube.

Fig. 2. Acoustic elements of perforated plug muffler with straight end tube.

Individual transfer matrixes with respect to each case of inlet straight ducts (I), expansion perforated tube (II), contraction perforated tube (III) and outlet straight duct (IV) are described as follows

A. Transfer Matrix for section I [6, 14]:

Equation for pressure & sound particle velocity are as follows.

\[
p(x,t) = e^{i\omega t} \left( A_1 e^{-ikx} + B_1 e^{ikx} \right)
\]

\[
u(x,t) = \left( \frac{A_1}{\rho c_0} e^{-ikx} + \frac{B_1}{\rho c_0} e^{ikx} \right) e^{i\omega t}
\]

Substituting boundary conditions as \(x=0 \) & \(x=L\), using Equation (1) & (2) we get,

\[
\begin{bmatrix}
p_1 \\
\rho_0 c_0 u_1
\end{bmatrix}
= 
\begin{bmatrix}
\cos(kL_1) & i\sin(kL_1) \\
i\sin(kL_1) & \cos(kL_1)
\end{bmatrix}
\begin{bmatrix}
p_2 \\
\rho_0 c_0 u_2
\end{bmatrix}
\]

(3)

B. Transfer Matrix for section IV [6]:

Similar to system matrix in section I we can relate node 5 & node 6 by using following matrix.

\[
\begin{bmatrix}
p_5 \\
\rho_0 c_0 u_5
\end{bmatrix}
= 
\begin{bmatrix}
\cos(kL_2) & i\sin(kL_2) \\
i\sin(kL_2) & \cos(kL_2)
\end{bmatrix}
\begin{bmatrix}
p_6 \\
\rho_0 c_0 u_6
\end{bmatrix}
\]

(4)

C. Transfer Matrix for section II [5,6 & 9]:

Transfer matrix for expansion perforated tube can be derived as mentioned below.

Inner tube:
Continuity equation

\[
V \frac{\partial p_2}{\partial x} + \rho_0 \frac{\partial u_2}{\partial x} + 4 \rho_0 \frac{\partial u_2}{di} u + \frac{\partial p_2}{\partial t} = 0
\]

Momentum equation

\[
\rho_0 \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) u_2 + \frac{\partial p_2}{\partial x} = 0
\]

Outer tube:
Continuity equation

\[ \rho_0 \frac{\partial u_{2a}}{\partial x} - \frac{4d_0 \rho_0}{d_m - d_i^2} u + \frac{\partial p_{2a}}{\partial t} = 0 \]  \hspace{1cm} (7)

Momentum equation

\[ \rho_0 \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) u_{2a} + \frac{\partial p_{2a}}{\partial x} = 0 \]  \hspace{1cm} (8)

Assuming that the acoustic wave is a harmonic motion

\[ p(x,t) = P(x) \cdot e^{i \omega t} \]  \hspace{1cm} (9a)

Under the isentropic processes in ducts, it has

\[ p(x,t) = P(x) \cdot c_0^2 \]  \hspace{1cm} (9b)

Assuming that the perforation along the inner tube is uniform (i.e. \( d_\varsigma / dx = 0 \)). The acoustic impedance of the perforation \((\rho_0 c_0 \varsigma)\) is

\[ \rho_0 \varsigma = \frac{p_2(x) - p_{2a}(x)}{u(x)} \]  \hspace{1cm} (10)

where \( \varsigma \) is the specific acoustical impedance of the perforated tube.

Empirical relations have been developed from experience. According to the experience, formula of \( \varsigma \) [8, 9 & 10] is given by,

\[ \varsigma = \frac{7.337 \times 10^{-3} (1 + 72.23M) + j 2.2245 \times 10^{-5} (1 + 51t)}{1 + 204dh} f \]  \hspace{1cm} (11)

where \( t \) is the thickness of the muffler; \( dh \) is the diameter of perforated holes for section II; \( f \) is the Frequency; \( \eta \) is the porosity of perforated tube for section II. Partic

icle velocity is comparatively much smaller than sound velocity so in further development of equations Mach number is taken as zero. Selected parameters:

\( t = 0.0015; \; dh = 0.003; \; \eta = 0.15; \; M = 0 \)

By substituting Equations (9-10) into (5-8)

\[ \left[ \frac{d^2}{dx^2} + k_a^2 \right] p_2 = -\left( k_a^2 - k_b^2 \right) p_{2a} \]  \hspace{1cm} (12)

\[ \left[ \frac{d^2}{dx^2} + k_a^2 \right] p_{2a} = -\left( k_b^2 - k^2 \right) p_2 \]  \hspace{1cm} (13)

\( k = \frac{a_0}{c}; \; k_a^2 = k^2 - i \frac{4k}{\varsigma d_i}; \; k_b^2 = k^2 - i \frac{4kd_a}{\varsigma (d_m^2 - d_i^2)} \)

Eliminating \( u_2 \) & \( u_{2a} \) by differentiation & substitution of Equation (12) & (13) we have:

\[ \begin{bmatrix} D^2 + \alpha_2 D + \alpha_2 & \alpha_3 D + \alpha_4 & \alpha_5 D + \alpha_6 \\ \alpha_5 D + \alpha_6 & D^2 + \alpha_7 D + \alpha_8 \end{bmatrix} \begin{bmatrix} p_2 \\ p_{2a} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]  \hspace{1cm} (14)

Where,

\( \alpha_2 = k_a^2; \; \alpha_3 = k^2 - k_a^2; \; \alpha_4 = k^2 - k_b^2; \; \alpha_5 = \alpha_3 = \alpha_5 = \alpha_7 = 0; \; \alpha_8 = k_b^2; \)

Developing Equation (14) yield:

\[ p_2 + \alpha_2 p_2 + \alpha_3 p_2 + \alpha_5 p_{2a} + \alpha_4 p_{2a} = 0 \]  \hspace{1cm} (15a)

\[ p_{2a} + \alpha_6 p_2 + \alpha_5 p_{2a} + \alpha_7 p_{2a} + \alpha_8 p_{2a} = 0 \]  \hspace{1cm} (15b)

Let \( \frac{\partial p_2}{\partial x} = y_1 \); \( \frac{\partial p_{2a}}{\partial x} = y_2 \); \( p_2 = y_3 \); \( p_{2a} = y_4 \)

According to (15a) to (15c), the new matrix between \( \{y^\prime\} \) and \( \{y\} \) is

\[ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} c \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \]  \hspace{1cm} (16b)

\[ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \]  \hspace{1cm} (16a)
Let \( \{ y \} = [S][\Gamma] \) \hspace{1cm} (17a)

which is

\[
\begin{bmatrix}
\frac{dp_2}{dx} \\
\frac{dp_{2a}}{dx} \\
p_2 \\
p_{2a}
\end{bmatrix} =
\begin{bmatrix}
S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\
S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\
S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\
S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4}
\end{bmatrix}
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\Gamma_3 \\
\Gamma_4
\end{bmatrix}
\hspace{1cm} (17b)
\]

\([S]_{4x4}\) is the model matrix formed by four sets of Eigen vectors \([S]_{4x4}\) of \([C]_{4x4}\). Substituting Equation (17) into (16) and then multiplying \([S]^{-1}\) on both sides,

\[
[S]^{-1}[S]^{-1}[\Gamma] = [S]^{-1}[C][S][\Gamma]
\]

\[
[S]^{-1}[\Gamma] = [S]^{-1}[C][S][\Gamma]
\]

Set \( \{\chi\} = [S]^{-1}[C][S] =
\[
\begin{bmatrix}
\varepsilon_1 \\
0 \\
0 \\
0
\end{bmatrix}
\hspace{1cm} (19)
\]

where \( \varepsilon_i \) is the Eigen value of \([C]\). We can write Equation (17) as:

\[
\{\Gamma^*\} = [\chi][\Gamma]
\]

This Equation(19) is a decoupled equation. The related solution can then be obtained as:

\[
\Gamma_j = k_j e^{\varepsilon_j x}
\]

Using these equations relation in acoustic pressure and particle velocity can be obtained by:

\[
\begin{bmatrix}
p_2(x) \\
p_{2a}(x) \\
\rho_0\phi_0u_2(x) \\
\rho_0\phi_0u_{2a}(x)
\end{bmatrix} =
\begin{bmatrix}
H_{1,1} & H_{1,2} & H_{1,3} & H_{1,4} \\
H_{2,1} & H_{2,2} & H_{2,3} & H_{2,4} \\
H_{3,1} & H_{3,2} & H_{3,3} & H_{3,4} \\
H_{4,1} & H_{4,2} & H_{4,3} & H_{4,4}
\end{bmatrix}
\begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4
\end{bmatrix}
\hspace{1cm} (22)
\]

where \( H_{1,j} = S_{1,j}e^{\beta_j x}; H_{2,j} = S_{2,j}e^{\beta_j x}; H_{3,j} = \frac{iS_{3,j}e^{\beta_j x}}{k}; H_{4,j} = \frac{iS_{4,j}e^{\beta_j x}}{k} \)

Substituting \( x = 0 \) and \( x = L_c \) into Equation (22)

\[
\begin{bmatrix}
p_2(0) \\
p_{2a}(0) \\
\rho_0\phi_0u_2(0) \\
\rho_0\phi_0u_{2a}(0)
\end{bmatrix} =
\begin{bmatrix}
p_2(L_c) \\
p_{2a}(L_c) \\
\rho_0\phi_0u_2(L_c) \\
\rho_0\phi_0u_{2a}(L_c)
\end{bmatrix}
\hspace{1cm} (23a)
\]

where \( [T] = [H(0)] [H(L_c)]^{-1} \)

(23b)

Boundary Condition:

\[
\frac{p_{2a}(0)}{-u_{2a}(0)} = -i\rho_0\phi_0 \cot(l_a^* k)
\]

(24a)

\[
\frac{p_2(L_c)}{-u_2(L_c)} = -i\rho_0\phi_0 \cot(0^* k)
\]

(24b)

Substituting these boundary conditions (21a) & (24b)

\[
\begin{bmatrix}
p_2 \\
\rho_0\phi_0u_2 \\
\rho_0\phi_0u_{2a}
\end{bmatrix} =
\begin{bmatrix}
T_a & T_b \\
T_c & T_d
\end{bmatrix}
\begin{bmatrix}
p_{3a}
\end{bmatrix}
\hspace{1cm} (25)
\]

**D. Transfer Matrix for section III [5,6 & 9]:**

Transfer matrix for contraction perforated tube can be derived on the similar lines as mentioned below.

Inner tube:

Continuity equation

\[
V \frac{\partial \rho_4}{\partial x} + \rho_0 \frac{\partial u_4}{\partial x} + 4\rho_0 \frac{\partial h}{\partial t} + \frac{\partial p_{3a}}{\partial t} = 0
\]

Momentum equation
\[ \rho_0 \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) u_x + \frac{\partial p}{\partial x} = 0 \]  
(27)

**Outer tube:**  
Continuity equation
\[ \rho_0 \frac{\partial u_3}{\partial x} = \frac{4d_n \rho_0}{d_m^2 - d_i^2} u_x + \frac{\partial p_3}{\partial t} = 0 \]  
(28)

Momentum equation
\[ \rho_0 \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right) u_3 + \frac{\partial p_3}{\partial x} = 0 \]  
(29)

For perforates with grazing flow, we have
\[ \varsigma = [7.337 \times 10^{-3} (1 + 72.23 M) + j2.2245 \times 10^{-5} (1 + 51 t_1) (1 + 204 dh_1)] / \eta_1 \]  
(30)

where ‘t’ is the thickness of the muffler; ‘dh_1’ is the diameter of perforated holes for section III; ‘f’ is the Frequency; ‘\eta_1’ is the porosity of perforated tube for section III.

Likewise, as derived in Eqs. (8-22) adopting the similar process as in expansion perforated tube below set of equations can be obtained

\[
\begin{bmatrix}
    p_4(0) \\
    p_{3a}(0) \\
    \rho_0 \rho_0 c_0 u_3(0) \\
    \rho_0 \rho_0 c_0 u_{3a}(0)
\end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix}
    p_4(L_c) \\
    p_{3a}(L_c) \\
    \rho_0 \rho_0 c_0 u_3(L_c) \\
    \rho_0 \rho_0 c_0 u_{3a}(L_c)
\end{bmatrix}
\]  
(31)

where \[ \begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} H(0) \end{bmatrix} \begin{bmatrix} H(L_c) \end{bmatrix}^{-1} \]
(32)

**Boundary Condition:**
\[ \frac{p_{4a}(0)}{-u_{4a}(0)} = -i \rho_0 c_0 \cot(0^\circ k) \]  
(33)

\[ \frac{p_{3a}(L_{c1})}{-u_{3a}(L_{c1})} = -i \rho_0 c_0 \cot(L b^\circ k) \]  
(34)

Substituting these boundary conditions (33) & (34)
\[ \begin{bmatrix}
    p_{3a} \\
    \rho_0 \rho_0 c_0 u_{3a}
\end{bmatrix} = \begin{bmatrix}
    T_{a1} & T_{b1} \\
    T_{c1} & T_{d1}
\end{bmatrix} \begin{bmatrix}
    \rho_0 \rho_0 c_0 u_4 \\
    \rho_0 \rho_0 c_0 u_{5}
\end{bmatrix} \]  
(35)

**E. Assembly of the Matrices:**

Using Equations (3), (4) & (35) we get,
\[ \begin{bmatrix}
    p_1 \\
    \rho_0 \rho_0 c_0 u_1
\end{bmatrix} = \begin{bmatrix}
    \cos(k L_c) & i \sin(k L_c) \\
    i \sin(k L_c) & \cos(k L_c)
\end{bmatrix} \begin{bmatrix}
    T_a & T_b \\
    T_c & T_d
\end{bmatrix} \begin{bmatrix}
    \rho_0 \rho_0 c_0 u_{6}
\end{bmatrix} \]  
(36)

Simplified Matrix is given by:
\[ \begin{bmatrix}
    p_1 \\
    \rho_0 \rho_0 c_0 u_1
\end{bmatrix} = \begin{bmatrix}
    PTM_a & PTM_b \\
    PTM_c & PTM_d
\end{bmatrix} \begin{bmatrix}
    \rho_0 \rho_0 c_0 u_6
\end{bmatrix} \]  
(37)

**F. Calculation of Transmission Loss:**
\[ STL = 10 \cdot \log_{10}(PTM_a + PTM_b + PTM_c + PTM_d) \]  
(38)

In order to get results by mathematical modelling entire model is built in MATLAB. It is used for TL prediction and optimization using Genetic Algorithm.

**G. Validation of Results:**
The above mathematical formulation is compared with result obtained by K. S. Peat [7] by using dimensions \( L_c = 0.3 \) m; \( L_a = 0 \) m; \( L_b = 0 \) m; \( L_c1 = 0.3 \) m; \( L_a1 = 0 \) m; \( L_b1 = 0 \) m; \( d_1 = d_0 = 0.075 \) m; \( d_m = 0.25 \) m; \( t = 1.5 \) mm; \( \eta = 0.15 \); \( \eta_1 = 0.15 \); \( dh = 3 \) mm. It’s observed that results above obtained for STL are precisely comparable with those presented by K. S. Peat [7].

FEM model is also and analysed in COMSOL for prediction of transmission loss. Fig. 4 shows the comparison of mathematical and FEM results.
Fig. 4. Single chamber perforated plug muffler comparison of STL of mathematical modelling and FEM [7] (Lc=0.300 m; La=0; Lb=0; Lc1 =0.3 m; La1 =0 m; Lb1 =0 m; di=do= 0.075m; dm= 0.25m)

H. Calculation of Insertion Loss

Insertion loss can be calculated with the below mentioned mathematical formulation. Here Z_S is source impedance [13]; Z_T is radiation impedance [6, 12]; A, B, C, D are four poles of acoustic elements; A_0, B_0, C_0, D_0 are four poles of straight pipe.

\[ IL = 20 \log_{10} \left( \frac{A/Z_S + B/Z_T + C/Z_T + D/Z_T}{A_0/Z_S + B_0/Z_T + C_0/Z_T + D_0/Z_T} \right) \]

III. Optimization of transmission loss using genetic algorithm

Model shown in Fig. 1 is used for optimization. Here La, Lb1, thickness of pipe (t) & porosities of both pipes (η, η1) are varied in bounded region. For optimization, optimization toolbox in MATLAB is used. Optimization is carried out by using Genetic Algorithm for frequency range of 1-2000 Hz.

The objective function in maximizing the STL at the puretone (f) is given as follows

OBJ = STL (f, La, Lb1, t, η, η1)

Objective function is defined in MATLAB in order to optimize Transmission Loss along with boundary condition of maximum STL in range of (700-750 Hz). Diameter of holes, diameter of the inlet duct, outlet duct & perforated duct are kept constant from manufacturing and spacing constraints. Below table gives the variable bound for the variables used in optimization and their optimized values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Optimized value</th>
</tr>
</thead>
<tbody>
<tr>
<td>La (m)</td>
<td>0</td>
<td>0.29</td>
<td>0.093</td>
</tr>
<tr>
<td>Lb1 (m)</td>
<td>0</td>
<td>0.29</td>
<td>0.094</td>
</tr>
<tr>
<td>t (m)</td>
<td>0.001</td>
<td>0.005</td>
<td>0.004</td>
</tr>
<tr>
<td>η</td>
<td>0.1</td>
<td>0.3</td>
<td>0.245</td>
</tr>
<tr>
<td>η1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

IV. Results & Conclusion

Optimized model using parameters mentioned in Table No. 1: is built up in COMSOL and results are compared with mathematical modelling results. It can be seen that in Fig. 5 both results are matching well.

Fig. 5. Comparison of plot of STL (dB) using mathematical modelling and FEM for above optimized values
In FEM end correction effect is considered and also singularity effect in MATLAB due to inverse of matrices is not there. So FEM results can be considered as more realistic.

**Table 2: Maximum STL & corresponding frequencies**

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Transmission Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>65.12519</td>
</tr>
<tr>
<td>725</td>
<td>84.99796</td>
</tr>
<tr>
<td>750</td>
<td>61.36597</td>
</tr>
<tr>
<td>728</td>
<td>86.03456</td>
</tr>
<tr>
<td>700-750</td>
<td>75.6629 (Average)</td>
</tr>
</tbody>
</table>

It has been observed that global maxima exist at frequency 728 Hz (COMSOL). It can be seen that maximum transmission loss is achieved over entire range from 700 Hz to 750 Hz. The average Transmission Loss over entire range of 700 Hz to 750 Hz is 75.66 dB. So this wide range makes it suitable for both four stroke four cylinders and six cylinders generator setups which operate at 1500 rpm constant speed.

Insertion Loss results are obtained by using mathematical modeling given by M. L. Munjal for muffler mentioned in Fig.1 [7]. IL is also calculated for optimized muffler & also effect of source impedance is observed by changing its value.

**Fig. 6. Comparison of plot of IL (dB) using mathematical modelling for different source impedances for optimized model and base model**

This study demonstrates a quick and economical approach to optimize the design for a single-chamber perforated plug muffler with straight inlet/outlet under space constraints without redundant testing.

It can be seen that STL is above 60dB for entire range of 700 – 750 Hz & hence suitable for different applications. Insertion loss is also calculated using mathematical modeling & it can be seen that IL is weak function of source impedance.

**References**


