CONSUMER SURPLUS AND PRICES IN PERFECT COMPETITION AND MONOPOLY

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Abstract: It is well known that existence of monopoly is accompanied by economic inefficiency in the form of dead weight loss and reduced consumer surplus. The consumer surplus that exists in case of perfect competition gets reduced in case of monopoly; as a part of it goes to the monopolist in the form of monopoly profit, a part of it is lost in the form of deadweight loss while the rest remains as consumer surplus in monopoly. This paper shows that under specific conditions there is a definite relationship (in case of monopoly) between monopoly profit, dead weight loss and consumer surplus and between prices in perfect competition and monopoly.

JEL Codes: D41, D42

Key Words: Perfect Competition; Monopoly; Consumer Surplus; Dead Weight Loss; Monopoly Profit; Price.

I. Introduction:

Economic literature is replete with papers that engage in welfare analysis through measuring economic surplus. Welfare is non-decreasing in economic surplus. Magnitude as well as the change in the economic surplus is indicative of welfare. For instance, consumer welfare is measured by the magnitude of consumer surplus. Consumer welfare too is non-decreasing in consumer surplus. Perfect competition is the market form in which consumer surplus is the greatest in magnitude, thus most favorable to the consumers, as it leads to the highest level of consumer welfare.

Monopoly is characterized by economic inefficiency, which is in the form of reduced consumer surplus and deadweight loss. The exception to the above said inefficiency is the case of monopoly (usually natural monopoly) where marginal cost pricing is practiced. As, in case of marginal cost pricing, the resulting consumer surplus is the same as that of perfect competition.

If we consider perfect competition and the case monopoly (simple monopoly pricing) under linear demand and constant marginal and average cost conditions, then we find that the consumer surplus that existed in case of perfect competition (henceforth CS\text{PC}) gets divided into consumer surplus in monopoly (henceforth CS\text{M}), monopoly profit (henceforth \(\pi\text{M}\)), and deadweight loss (henceforth DWL\text{M}). The same can be put forth numerically as follows:

\[ \text{CS}\text{PC} = \text{CS}\text{M} + \pi\text{M} + \text{DWL}\text{M} \]

The above stated division of CS\text{PC} is shown by the following diagrams:

Figure 1. (a) Perfect Competition
Figure 1. (b) Monopoly

The above diagram in panel (a) depicts perfect competition, where the demand curve is characterized by the property of linearity and marginal cost by constancy. The diagram shows equilibrium price in perfect competition (henceforth \( P^{PC} \)) and equilibrium quantity in perfect competition (henceforth \( Q^{PC} \)). Here, \( CS^{PC} \) is the area of the triangle \( P^{PC}DC \).

The diagram in panel (b), in addition, shows equilibrium price in monopoly (henceforth \( P^M \)) and equilibrium quantity in monopoly (henceforth \( Q^M \)). As can be seen, \( CS^{PC} \), the area of the triangle \( P^{PC}DC \), gets divided into three main parts, i.e., triangle \( P^MDB \) (consumer surplus, i.e., \( CS^M \)), rectangle \( P^{PC}P^MBA \) (monopoly profit, i.e., \( \pi^M \)), and triangle \( ABC \) (deadweight loss, i.e., \( DWL^M \)).

\[
CS^{PC} = CS^M + \pi^M + DWL^M
\]

**Claim - 1:**
In case of linear, downward sloping demand curve and constant (and equal) marginal and average costs, the following relationship would be true:

\[
CS^M = DWL^M = \frac{1}{2} \pi^M = \frac{1}{4} CS^{PC}
\]

**Claim - 2:**
In case of linear, downward sloping demand curve and constant marginal cost, the following relationship would be true:

\[
P^{PC} = \{(2)*(P^M) - a\}; \text{ (where ‘}a\text{’ is the y-intercept of the demand curve)}
\]

The above results can be shown (and generalized) with the help of an illustration. Let us consider a linear demand curve, of the form, \( Q = \frac{a-P}{b} \) (i.e., \( P = a - bQ \)), (where ‘\(a\)’ and ‘\(b\)’ are the intercept and slope parameters respectively) and the total cost function to be given by, \( TC = CQ \). Marginal cost in this case would be \( C \) (which is a constant). With the help of the above information, cases of both, perfect competition as well as monopoly can be analyzed. Diagrammatic illustration of both the market forms will depict the stated quantities in the claim. Following diagrams depict perfect competition and monopoly.

Figure 2. (a) Perfect Competition
The diagram in panel (a) shows perfect competition. The diagram shows $P_{PC} = C$ and $Q_{PC} = \frac{(a - C)}{b}$. Here, $CS_{PC}$ is the area of the triangle $P_{PC}aD$.

Numerically $Q_{PC}$, $P_{PC}$ and $CS_{PC}$ would be as follows:

$Q_{PC} = \frac{(a - C)}{b}$;

$P_{PC} = a - b \left( \frac{(a - C)}{b} \right) = C$; and

$CS_{PC} = \frac{(a - C)^2}{2b}$.

The diagram in panel (b) shows monopoly. As can be seen, $CS_{PC}$ is divided into three parts, namely, consumer surplus ($CS_{M}$), the area of the triangle $P_{MA}E$; monopoly profit ($\pi_{M}$), the area of the rectangle $P_{PC}P_{M}EF$; and dead weight loss ($DWL_{M}$), the area of the triangle $FED$. The diagram also shows $P_{M} = \frac{(a - C)}{2}$ and $Q_{M} = \frac{(a - C)}{2b}$.

Numerical magnitudes of $Q_{M}$, $P_{M}$, $CS_{M}$, $\pi_{M}$ and $DWL_{M}$ would be as follows:

$Q_{M} = \frac{(a - C)}{2b}$;

$P_{M} = \frac{(a - C)}{2}$;

$\pi_{M} = \frac{(a - C)^2}{4b}$;

$CS_{M} = \frac{(a - C)^2}{4b}$; and

$DWL_{M} = \frac{(a - C)^2}{4b}$.

The above quantities are coherent with the claims, that,

(1). $CS_{M} = DWL_{M} = \frac{1}{2} \pi_{M} = \frac{1}{4} CS_{PC}$ and

(2). $P_{PC} = \{(2)(P_{M}) - a\}$; (where ‘a’ is the y-intercept of the demand curve)

Explanations of the Claims:

Claim - 1:
In case of linear, downward sloping demand curve and constant (and equal) marginal and average costs, the following relationship would be true:

$$CS_{M} = DWL_{M} = \frac{1}{2} \pi_{M} = \frac{1}{4} CS_{PC}$$

Explanation:
Consider a demand curve, of the form, $Q = \frac{(a - P)}{b}$ (i.e., $P = a - bQ$) and total cost, of the form, $TC = CQ$. The notations are defined as follows: ‘a’ and ‘b’ are the intercept and slope parameters of the demand curve; ‘P’ and
‘Q’ are the price and quantity respectively. ‘C’ in the total cost is a constant. Following diagrams illustrate the cases of perfect competition (panel (a)) and monopoly (panel (b)).

**Figure 3. (a) Perfect Competition**

Quantities in perfect competition would be as follows:

**Equilibrium output (Q^PC):**

\[ P = MC \text{ (the profit maximizing condition)} \Rightarrow P = a - bQ = C \Rightarrow Q = \left(\frac{a-C}{b}\right) \Rightarrow Q^{PC} = \left(\frac{a-C}{b}\right) \]

Substituting the value of the \(Q^{PC}\) in the demand function would give the value of **equilibrium price (P^{PC}):**

\[ P = a - bQ \Rightarrow P = a - bQ^{PC} \Rightarrow P = a - \left(\frac{a-C}{b}\right) \Rightarrow P = C \Rightarrow P^{PC} = C \]

**Consumer Surplus:**

Consumer surplus is the area of the triangle CaD

\[ CS^{PC} = \frac{1}{2} \left( a - C \left( \frac{a-C}{b} \right) \right) = \left(\frac{(a-C)^2}{2b}\right) \Rightarrow CS^{PC} = \left(\frac{(a-C)^2}{2b}\right) \]

**Figure 3. (b) Monopoly**

Quantities in monopoly would be as follows:

**Equilibrium output (Q^M):**

\[ TR = P*Q \Rightarrow (a - bQ)*Q \Rightarrow TR = aQ - bQ^2 \]
MR = a – 2bQ
MR = MC (the profit maximizing condition)
⇒ a – 2bQ = C ⇒ Q = \left(\frac{a - C}{2b}\right)
⇒ Q^M = \left(\frac{a - C}{2b}\right)

Equilibrium price (P^M):
Substituting the value of the Q^M in the demand function would give the value of equilibrium price (P^M):
P = a – bQ ⇒ P = a – bQ^M ⇒ P = a – \left(2b\left(\frac{a - C}{2b}\right)\right)
⇒ P = C ⇒ P^M = \left(\frac{2a + C}{2}\right)

Monopoly profit (π^M):
Profit = TR – TC = \{(P^M*Q^M) – (C*Q^M)\} = area of the rectangle P^MPC^M
⇒ \left(\frac{(a + C)}{2}\right) \left(\frac{a - C}{2b}\right) – \left(\frac{(a - C)}{2b}\right) \left(\frac{(a + C)}{2}\right) - C
⇒ \frac{1}{2} \left[\left(\frac{(a + C)}{2}\right) \left(\frac{a - C}{2b}\right) - C\right]
⇒ \frac{1}{2} \left[\left(\frac{(a + C)}{2}\right) \left(\frac{a - C}{2b}\right)\right] – \left(\frac{(a - C)}{2b}\right) \left(\frac{(a + C)}{2}\right) - C
⇒ \pi^M = \left(\frac{(a - C)^2}{4b}\right)

Consumer surplus (CS^M):
Consumer surplus is the area of the triangle P^MCE,
⇒ \frac{1}{2} \left(\frac{(a - C)^2}{2b}\right) ⇒ CS^M = \left(\frac{(a - C)^2}{8b}\right)

Dead Weight Loss (DWL^M):
Dead Weight Loss is the area of the triangle FED,
⇒ \frac{1}{2} \left[\left(\frac{(a + C)^2}{2}\right) - C\right] ⇒ DWL^M = \left(\frac{(a - C)^2}{8b}\right)

It can be seen that,
(a). CS^PC = CS^M + π^M + DWL^M
(b). CS^M = DWL^M = \frac{1}{2} \pi^M = \frac{1}{4} CS^PC

Claim - 2:
In case of linear, downward sloping demand curve and constant marginal cost, the following relationship would be true:
P^PC = \left(\frac{(2a)(P^M) - a}\right); \text{ (where ‘a’ is the y-intercept of the demand curve)}

Proof:
P = a – bQ; TC = CQ
TR = aQ – bQ^2 ⇒ MR = a – 2bQ
MR = MC (the profit maximizing condition)
⇒ a – 2bQ = C ⇒ Q = \left(\frac{a - C}{2b}\right)
⇒ Q^M = \left(\frac{a - C}{2b}\right)
Substituting the value of the Q^M in the demand function would give the value of equilibrium price (P^M):
P^M = a – bQ ⇒ P = a – b\left(\frac{a - C}{2b}\right)
⇒ P = C ⇒ P^M = \left(\frac{2a + C}{2}\right)
Since C = MC = P^PC, substituting P^PC for C and solving further yields the following result:
P^PC = \left(\frac{(2a)(P^M) - a}\right)

Hence, the claim is proved.

References: